PLAYING SAVITCH

Peter van Emde Boas

ILLC-FNWI-Univ. of Amsterdam

Bronstee.com Software and Services

SOFSEM 2000 - MILOVY

© Games Workshop

© Games Workshop

References and slides available at: http://turing.wins.uva.nl/~peter/teaching/thmod00.html

Walter Savitch

© Peter van Emde Boas

© Peter van Emde Boas

© Peter van Emde Boas

ICSOR; CWI, Aug 1976

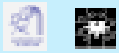
San Diego, Oct 1983

THE STATE OF THE ART

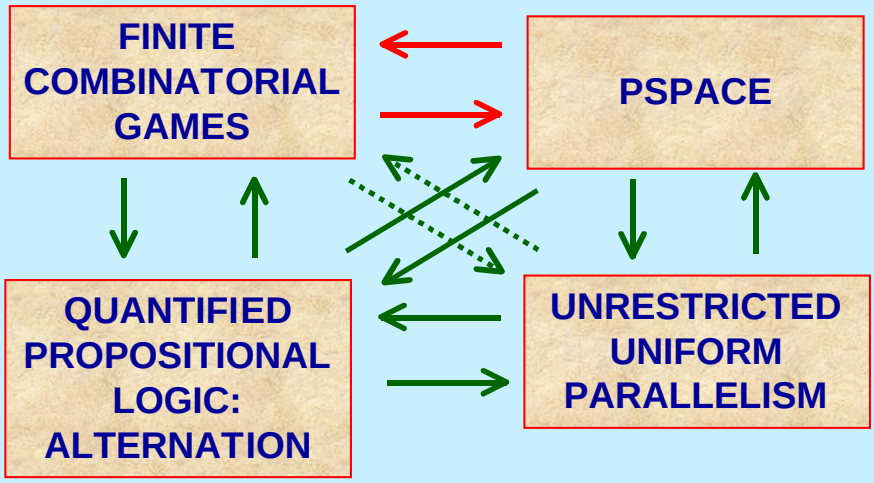
So far the connection between PSPACE and Games is established indirectly, using either Alternation or QBF (or both) as intermediate.

In the Literature occasionally the same holds for the reverse connection: (this game can be analysed in PSPACE because it can be analyzed in Polynomial Time on a ATM ....)

A direct approach is possible !

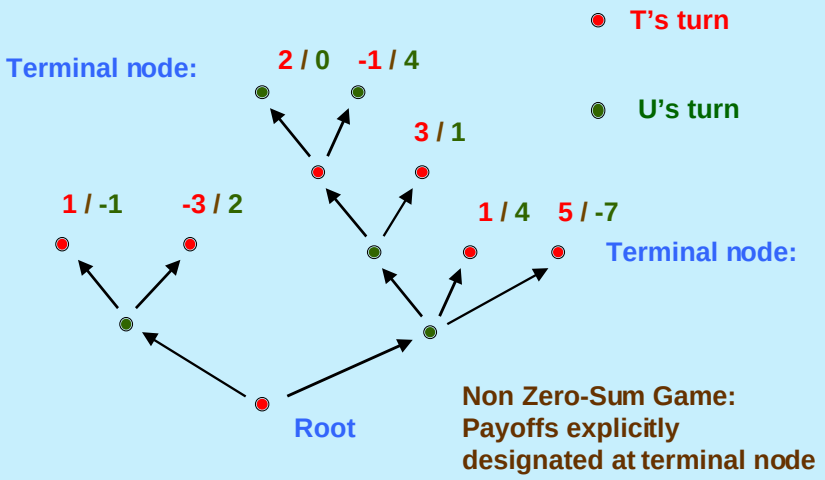


A DIRECT CONNECTION



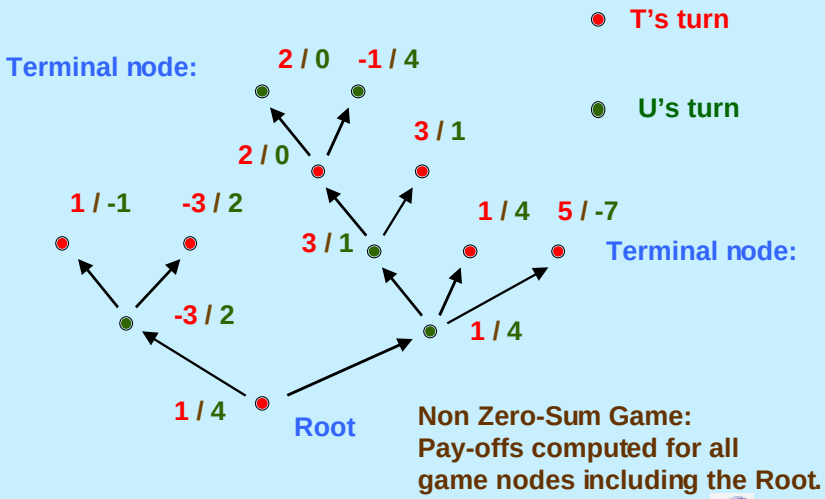
Game Trees

(*Extensive Form - close to Computation*)



Non Zero-Sum Game:
Payoffs explicitly
designated at terminal node

Backward Induction



Non Zero-Sum Game:
Pay-offs computed for all
game nodes including the Root.

REASONABLE GAMES

Assumptions for the sequel:

Finite Complete Information Zero Sum Games
Structure: tree given by description,
where deciding properties like
is p a position ?, is p final ? is p starting position ?, who has to move in p ?, and the generation of successors of p are all trivial problems ..... The tree can be generated in time proportional to its size.....
Moreover the duration of a play is polynomial.




Backward Induction in PSPACE?

The Recursive scheme for Backward Induction combines recursion (over move sequence) with iteration (over locally legal moves).

Space Consumption =
O(| Stackframe | . Recursion Depth)

| Stackframe | =
O(| Move sequence | + | Configuration|)

Recursion Depth = | Move sequence | =
O(Duration Game)

Therefore the game duration should be polynomial!




Polynomial Space Configuration Graph

- **Configurations & Transitions:**
  - (finite) State, Focus of Interaction & Memory Contents
  - Transitions are Local (involving State and Memory locations in Focus only; Focus may shift). Only a Finite number of Transitions in a Configuration
  - Input Space doesn´t count for Space Measure




Polynomial Space Configuration Graph

- **Exponential Size Configuration Graph:**
  - input lenght: $|x| = k$; Space bound: $S(k)$
  - Number of States: q (constant)
  - Number of Focus Locations: $k.S(k)^t$ (where t denotes the number of "heads")
  - Number of Memory Contents: $C^{S(k)}$
  - Together: $q.k.S(k)^t.\ C^{S(k)} = 2^{O(S(k))}$ (assuming $S(k) = \Omega(\log(k))$)




The Savitch Game

Given: some input x for a PSPACE acceptor M
(M can be nondeterministic)

To Construct: a 2 person Complete Information reasonable Game $G(M,x)$ such that
x is accepted by M iff the first player has a winning strategy in $G(M,x)$

WLOG: time accepting computation $\leq 2^{(S(|x|))}$




The Savitch Game

Aethis **Thorgrim**

Typical Position:
Configurations C_1 , C_2 and Time Interval $t_1 < t_2$
$|C_1|$, $|C_2| \leq (S(|x|))$, $0 \leq t_1 < t_2 \leq 2^{(S(|x|))}$

ROUND of the Game :
Thorgrim chooses t_3 such that $t_1 < t_3 < t_2$
Aethis chooses C_3 at t_3
Thorgrim decides to continue with either
C_1 , C_3 and $t_1 < t_3$ or C_3 , C_2 and $t_3 < t_2$




The Savitch Game

Initial Position:
C_1 is the starting position and C_2 the (unique) accepting Configuration. $0 = t_1$ and $t_2 = 2^{(S(|x|))}$

Final Position: $t_2 - t_1 = 1$

Aethis wins if $C_1 \dashrightarrow C_2$ is a legal transition; otherwise **Thorgrim** wins the game

Polynomial duration enforced by requiring
$(t_2 - t_1).\varepsilon \leq (t_3 - t_1) \leq (t_2 - t_1).(1- \varepsilon)$ for some fixed ε satisfying $0 < \varepsilon \leq 1/2$




The Savitch Game

Winning Strategies:

If **x** is **accepted** **Aethis** can win the game by being **truthful** (always play the true configuration in some Accepting Computation...)

If **x** is **not accepted** the assertion entailed by the initial position is **false**. Regardless the configuration C_3 chosen by **Aethis** he must make a **false** assertion either on the first or on the second interval (or both). **Thorgim** wins by always attacking the **false** interval....




The Savitch Game

The Punchline:

Endgame Analysis of the Savitch Game is in Deterministic PSPACE, even if the original acceptor was Nondeterministic:

NPSPACE = PSPACE !

an Alternative (direct) proof of the Savitch Theorem....




The Savitch Game

Final remarks:

Aethis can play his winning strategy if he knows the accepting computation.

Thorgrim can play his winning strategy if he can locate errors. Utterly unfeasible....

COMPARE THIS WITH INTERACTIVE PROTOCOLS: PSPACE = IP