A closer look at Critical Points

Slow dynamics, aging and their universal features

Pasquale Calabrese

Instituut voor Theoretische Fysica
Universiteit van Amsterdam

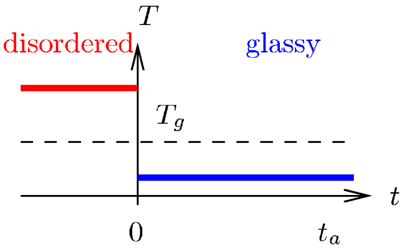
Group meeting ITFA 04/10/2006

mainly in collaboration with A. Gambassi [MPI-Stuttgart]

Based on:

- PC, A. Gambassi, Phys. Rev. E 65, 066120 (2002) [cond-mat/0203096]
- PC, A. Gambassi, Phys. Rev. B 66, 212407 (2002) [cond-mat/0207487]
- PC, A. Gambassi, Phys. Rev. E 66, 066101 (2002) [cond-mat/0207452]
- PC, A. Gambassi, Phys. Rev. E 67, 036111 (2003) [cond-mat/0211062]
- PC, A. Gambassi, JSTAT 0407, P013 (2004) [cond-mat/0406289]
- PC, A. Gambassi, J. Phys. A 38, R133 (2005) [cond-mat/0410357]
- PC, AG, F. Krzakala, JSTAT 0606, P016 (2006) [cond-mat/0604412]
- PC, A. Gambassi, cond-mat/0610XXX

Aging



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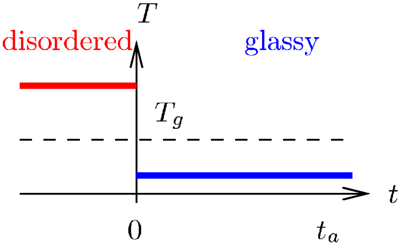

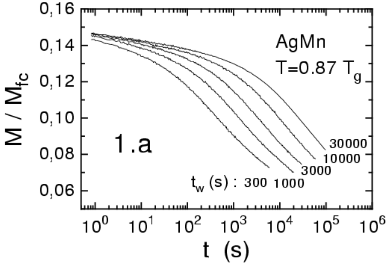

Vincent at al [cond-mat/9607224]

"The older, the slower"

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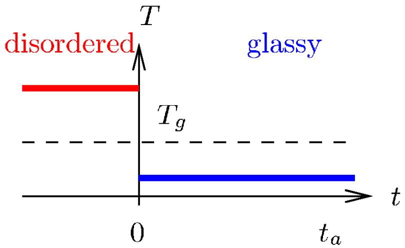

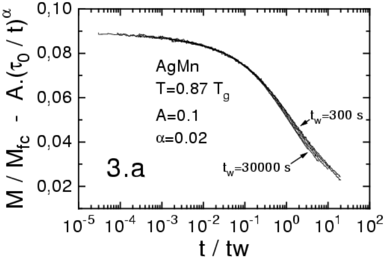

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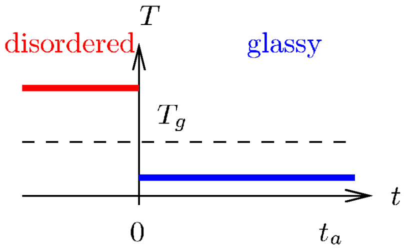

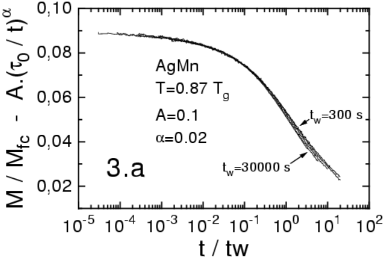

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Is aging a signature of glassiness?

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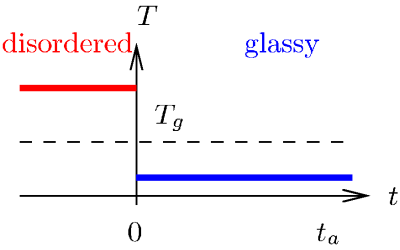

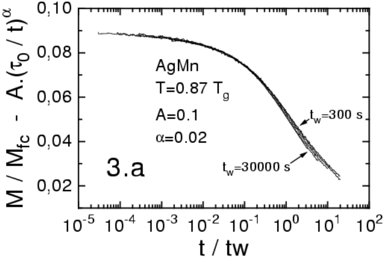

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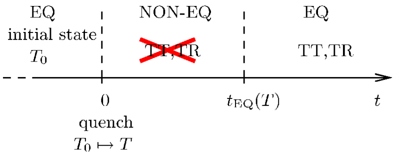
Is aging a signature of glassiness?

No, only of slow dynamics

Example: Ferromagnet relaxing towards equilibrium at T



$T > T_c \qquad t_{\rm EQ} < \infty \implies$ EQ

$T < T_c \qquad t_{\rm EQ} = \infty$

$\implies$ phase ordering dynamics

$T = T_c \qquad t_{\rm EQ} = \infty, \quad \xi(t) \sim t^{1/z}$

$\implies$ Critical Dynamics

Fluctuation-Dissipation theorem/ratio

$$s < t \qquad \begin{cases} C_{x-y}(t,s) \equiv \langle \phi_x(t)\phi_y(s)\rangle_{\rm conn} & \text{Correlation} \\ R_{x-y}(t,s) \equiv \dfrac{\delta}{\delta h_y(s)}\langle \phi_x(t)\rangle\Big|_{\rm h=0} & \text{Response} \end{cases}$$

$$t_{\rm EQ}(T) \ll s < t: \quad C_x(t,s) = C_x^{(eq)}(t-s); \quad R_x(t,s) = R_x^{(eq)}(t-s)$$

$$\boxed{T R_x^{(eq)}(\tau) = -\frac{d C_x^{(eq)}(\tau)}{d\tau}} \qquad \text{FDT}$$

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In general? Fluctuation-Dissipation Ratio **[CK '94]**

$$X(t,s) \equiv \frac{TR(t,s)}{\partial_s C(t,s)}$$

$$X^\infty \equiv \lim_{s\to\infty}\lim_{t\to\infty} X(t,s) \qquad = 1 \text{ if } t_{\rm EQ}(T) < \infty$$

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In general? Fluctuation-Dissipation Ratio **[CK '94]**

$$X(t,s) \equiv \frac{TR(t,s)}{\partial_s C(t,s)} \equiv \frac{T}{T_{\rm eff}}$$

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??? FDT & TD with $T_{\rm eff} \equiv T/X^\infty$ $\overset{YES}{\rightsquigarrow}$ ∞-range glass

Model A

Purely dissipative dynamics of an Ising-like order parameter $\varphi(x,t)$

$$\text{Statics} \Rightarrow \mathcal{H}[\varphi] = \int \mathrm{d}^d x \left[\frac{1}{2}(\partial\varphi)^2 + \frac{1}{2} r\varphi^2 + \frac{u}{4!}\varphi^4\right]$$

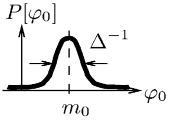
Equilibrium Dynamics:

$$\begin{aligned} \partial_t \varphi(x,t) &= -D\frac{\delta\mathcal{H}[\varphi]}{\delta\varphi(x,t)} + \zeta(x,t) \\ \langle \zeta(x,t)\zeta(x',t')\rangle &= 2D\delta(x-x')\delta(t-t') \end{aligned} \Rightarrow P_{\mathrm{EQ}}[\varphi] \propto \mathrm{e}^{-\mathcal{H}[\varphi]}$$

[MSR] $\Longrightarrow S[\varphi,\tilde\varphi] = \int_0^\infty \mathrm{d}t \int \mathrm{d}^d x \left[\tilde\varphi\partial_t\varphi + D\tilde\varphi\frac{\delta\mathcal{H}}{\delta\varphi} - D\tilde\varphi^2\right]$

$$\langle \mathcal{O}\rangle_\zeta = \int [\mathrm{d}\varphi\mathrm{d}\tilde\varphi]\, \mathcal{O}\, \mathrm{e}^{-S[\varphi,\tilde\varphi]} \qquad \tilde\varphi \leftrightarrow h \Rightarrow R(t,s) = \langle\varphi(t)\tilde\varphi(s)\rangle$$

Out of equilibrium: initial condition at $t=0$



initial cond. $P[\varphi_0] \propto \mathrm{e}^{-H_0[\varphi_0]}$
$H_0[\varphi_0] = \int \mathrm{d}^d x \frac{\Delta}{2}[\varphi_0(x) - m_0]^2$

Δ^{-1} irrelevant

[JSS'88] $\Rightarrow$ FT with action $S[\varphi,\tilde\varphi] + H_0[\varphi_0]$

Example: The Gaussian approximation

$$
\begin{aligned}
R_{\mathbf{q}}(t,s) &= \theta(t-s)e^{-(\mathbf{q}^2+r_0)(t-s)} \\
C_{\mathbf{q}}(t,s) &= \frac{1}{\mathbf{q}^2+r_0}[e^{-(\mathbf{q}^2+r_0)|t-s|} - e^{-(\mathbf{q}^2+r_0)(t+s)}]
\end{aligned}
$$

$$
\mathcal{X}_{\mathbf{q}}(t,s) = \frac{1}{1+e^{-2(\mathbf{q}^2+r_0)s}}
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Conclusions:

- Non critical ($r_0 > 0$) $\Rightarrow X^\infty = 1$, i.e. finite $t_{\rm eq}$
- Critical ($r_0 = 0$), if $\mathbf{q} \neq 0 \Rightarrow X^\infty = 1$
- Zero mode $\mathbf{q} = 0 \Rightarrow X^\infty = 1/2$
- The only aging mode is the zero mode!

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"Theorem": $X^\infty_{\mathbf{x}=0} = \mathcal{X}^\infty_{\mathbf{q}=0} \equiv X^\infty$

$$X^{-1}_{\mathbf{x}=0} = \frac{\int (dq)\partial_s C_{\mathbf{q}}(t,s)}{T\int (dq) R_{\mathbf{q}}(t,s)} = \frac{\int (dq) R_{\mathbf{q}} \frac{\partial_s C_{\mathbf{q}}(t,s)}{T R_{\mathbf{q}}(t,s)}}{\int (dq) R_{\mathbf{q}}(t,s)} = \langle \mathcal{X}_{\mathbf{q}}^{-1} \rangle_{R_{\mathbf{q}}}$$

Scaling forms

$$\mathbf{R_{q=0}(t,s) = A_R\ (t-s)^a \left(\frac{t}{s}\right)^{\theta} \mathcal{F}_R(s/t)}$$

$$\mathbf{C_{q=0}(t,s) = A_C s\,(t-s)^a \left(\frac{t}{s}\right)^{\theta} \mathcal{F}_C(s/t)}$$

At $\mathbf{q} \neq 0$, the new scaling variable is $q^z t$

$a = (2-\eta-z)/z$

θ initial-slip exponent [JSS'88]

A_R, A_C non-univ.

$\mathcal{F}_{R,C}(x)$ universal, provided $\mathcal{F}_{R,C}(0) = 1$

$$\mathcal{X}(t,s) = \frac{R_{\mathbf{q}=0}(t,s)}{\partial_s C_{\mathbf{q}=0}(t,s)}$$ is universal

$$\mathbf{X^{\infty} = \frac{A_R}{A_C(1-\theta)}}$$ is universal too [Godreche & Luck 00]

Model (Glauber dyn)		X^∞
Random Walk	[1]	$1/2$
Spherical	[2]	$1 - 2/d$
1-dim. Ising	[2,3]	$1/2$
2-dim. Ising	[2]	0.26(1)
" "	[4]	0.340(5)
" "	[5]	0.33(2)
" "	[6]	0.33(1)
" "	[7]	0.330(5)
3-dim. Ising	[2]	0.40
3-dim. XY	[8]	0.43(4)

Exact solution, **Monte Carlo simulations**.

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Two-loop $\epsilon = 4-d$ expansion result:

$$\frac{(X^\infty)^{-1}}{2} =$$

$$1+\frac{n+2}{4(n+8)}\epsilon+\frac{n+2}{(n+8)^2}\left[\frac{n+2}{8}+\frac{3(3n+14)}{4(n+8)}-0.041\right]\epsilon^2$$

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- Decreasing function of ϵ
- Decreasing function of n
- For $n=\infty \Longrightarrow$ the spherical model
- At $\epsilon=1$ and $n=1 \Rightarrow X^\infty \sim 0.43$
- At $\epsilon=2$ and $n=1 \Rightarrow X^\infty \sim 0.34$

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First message

Surprisingly accurate

Effective temperature?

If X^{∞} defines an effective temperature

must be observable independent

This has been checked in some simple "glassy" models [ckp97,sf00] and in 1d Ising [mbgs03]

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- At Gaussian level yes!!
- Including fluctuations, not anymore:

$$X_E^\infty = \frac{1}{2}\left(1 - \frac{2}{3}\frac{n+2}{n+8}\epsilon\right)$$

$$X_T^\infty = \frac{1}{2}\left(1 - \frac{1}{12}\frac{3n+16}{n+8}\epsilon\right)$$

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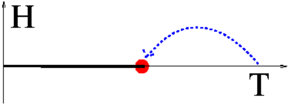
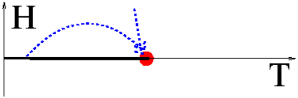
- Again they agree with other approaches

Evolution from an ordered state I: General



$$M(t) = 0$$



$$M(t) \sim t^{-\beta/\nu z}$$

$$R(t,s) = A_R \; (t-s)^a \left(\frac{t}{s}\right)^{\theta} \mathcal{F}_R(s/t)$$

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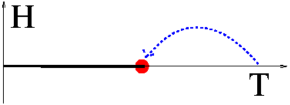

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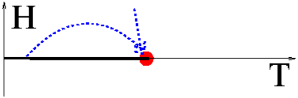

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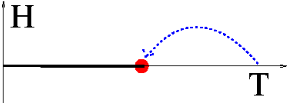
$t_0 \equiv A_m m_0^{-1/\sigma}$ non univ.

$\sigma = \theta + a + \beta/(\nu z)$ univ

If $m_0 = \infty$ ie, $s, t \gg t_0 \Longrightarrow R(t,s) = a_R (t-s)^a \left(\frac{t}{s}\right)^{\bar\theta} f_R(s/t)$

Is $\bar\theta$ a new exponent?

Evolution from an ordered state I: General



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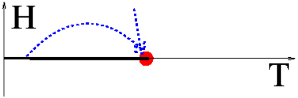

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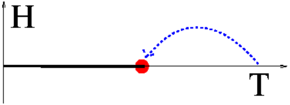
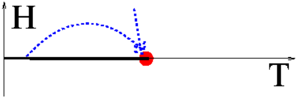


$$M(t) = 0$$



$$M(t) \sim t^{-\beta/\nu z}$$

$$R(t,s) = A_R \;\; (t-s)^a \left(\frac{t}{s}\right)^{\theta} \mathcal{F}_R(s/t)$$

$$R(t,s) = A_R \;\; (t-s)^a \left(\frac{t}{s}\right)^{\theta} \mathcal{F}_R(s/t, t/t_0)$$

$t_0 \equiv A_m m_0^{-1/\sigma}$ non univ.

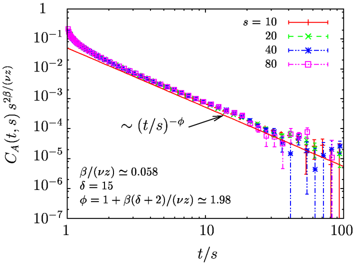
$\sigma = \theta + a + \beta/(\nu z)$ univ

If $m_0 = \infty$ ie, $s, t \gg t_0 \Longrightarrow R(t,s) = a_R (t-s)^a \left(\frac{t}{s}\right)^{\bar\theta} f_R(s/t)$

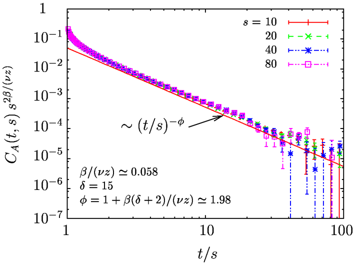
Is $\bar\theta$ a new exponent? No $\bar\theta = -\beta\delta/\nu z$

$\Longrightarrow$ for long times: (a) $m_0 = 0$ (b) $m_0 \neq 0 \rightsquigarrow$ *crossover*

2D Ising


$s = 10$
20
40
80
$\sim (t/s)^{-\phi}$
$\beta/(\nu z) \simeq 0.058$
$\delta = 15$
$\phi = 1 + \beta(\delta+2)/(\nu z) \simeq 1.98$
$C_A(t,s)\, s^{2\beta/(\nu z)}$
t/s

2D Ising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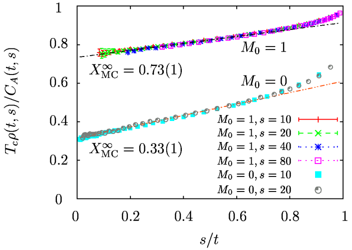
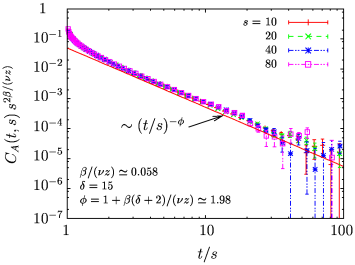
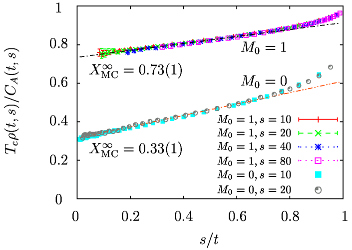

$M_0 = 1$
$X^{\infty}_{\rm MC} = 0.73(1)$
$M_0 = 0$
$X^{\infty}_{\rm MC} = 0.33(1)$
$M_0 = 1, s = 10$
$M_0 = 1, s = 20$
$M_0 = 1, s = 40$
$M_0 = 1, s = 80$
$M_0 = 0, s = 10$
$M_0 = 0, s = 20$
$T_c \rho(t,s)/C_A(t,s)$
s/t

2D Ising





Field Theory

$$X^\infty = \frac{4}{5} - 0.023\epsilon$$

that at $d = 2$ is ~ 0.75 [OK]

The evolution is governed by the competition of two different modes longitudinal ($\| M_0$) and transverses ($\perp M_0$)

The initial distribution $H_0[\varphi_0] = \int \mathrm{d}^d x \frac{\Delta}{2}[\varphi_0(x) - m_0]^2$ does not correspond to a low T state at $h = 0$, but only to a state at any T in a non-zero magnetic field

In the ordered phase the transverse modes are massless (Goldstone theorem)

Longitudinal scaling as for Ising. Transverse scaling for $M_0 \to \infty$:

$$R(t,s) = a_R^\pi (t-s)^a \left(\frac{s}{t}\right)^{\theta\pi} f_R^\pi(s/t)$$

Is θ_π new?

The evolution is governed by the competition of two different modes longitudinal ($\| M_0$) and transverses ($\perp M_0$)

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Gaussian $X_\pi^\infty = 2/3 \neq 4/5$

Evolution from an ordered state: Results II Heisenberg U.C.

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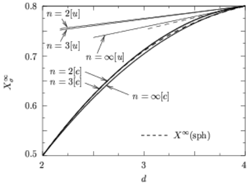
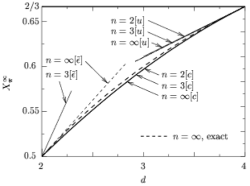
For $n \to \infty$, exactly solvable $X_\pi^\infty = d/(d+2)$





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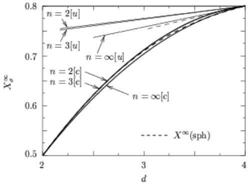

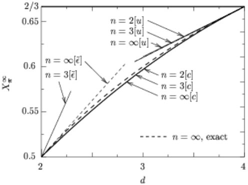

What about low T quench??

- Purely dissipative dyn (Model A), $O(n)$ GLW:
  - $n = 1$ *anisotropic magnets/alloys*
  - $\forall n$ lattice spin models with Glauber dyn
  - $\rightsquigarrow$ $X^\infty(m_0 = 0)$[2L]: OK w. MC Ising 2d & 3d and XY 3d
  - $\rightsquigarrow$ $T_{\rm eff}$???
  - $\rightsquigarrow$ Critical surfaces out-of-equilibrium [only MF]
  - $\rightsquigarrow$ $X^\infty(m_0 \neq 0)$[1L]: OK w. MC Ising 2d
- Conserved dyn (Model B) scalar GLW
  - some *uniaxial ferromagnets*
  - Ising model with Kawasaki dyn
  - $\rightsquigarrow$ [1L] $X^\infty_{\rm Model\ B} > X^\infty_{\rm Model\ A}$: OK with MC Ising 2d **[GKRT'04]**
- Other dyn: Model C [1L], Model A dilute Ising [1L] **[SP'04]**, Model A φ^3

. . . what should be looked at. . .

? More realistic dynamics...

- magnets $\Longrightarrow$ Models J,G (A)
- fluids $\Longrightarrow$ Model H (B)
- helium $\Longrightarrow$ Model F,F'

. . . **waiting for experiments!**

? Purely dynamical critical points (contact processes, direct percolation, Reaction-diffusion etc...)

? Slow dynamics at QCP [slowly]

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...**waiting for experiments!**

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Take-home message:

FT is a viable approach to investigate aging phenomena at critical points!

Ref: P. Calabrese& A. Gambassi, J.Phys.A **38** (2005) R133-R193

And after the "famous last words"...

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At 4 pm at Kriterion for university farewell