TOPOLOGICAL QUANTUM FIELD THEORY AND RELATED ALGEBRAIC STRUCTURES

The course will be focused on combinatorial approach to topological quantum field theory and on the algebraical aspects of conformal field theory.

Anyone who attended the course "Quantum groups and knot theory" taught by Arjeh Cohen, Jasper Stokman, and Eric Opdam last semester should have all prerequisites. Some basic knowledge of Lie algebra and differential geometry is advisable in the second part. I will try to assume minimum of specialized knowledge from the audience.

Here is the outline of the course. Last parts are tentative. As the semester will progress, it will be clear how much of the outline will be covered.

1. Combinatorial topological quantum field theory

1.1. **Modular categories.** The course will start with an overview of ribbon categories. Then modular categories will be introduced. It will be shown that truncated category of modules over quantized universal enveloping algebras at roots of unity is a modular category.

1.2. **Topological quantum field theories.** Here we will outline basic ideas of a topological quantum field theory is the spirit of Atiyah and Segal.

1.3. **Combinatorial 3D-TQFTs from a modular category.** Here we will start with a short overview of the description of 3-manifolds via a surgery on framed links. Then invariants of 3-manifolds will be constructed as invariants of framed links which are constant on links describing 3-manifolds from the same topological class.

1.4. **Combinatorial 3D-TQFTs for cell complexes.** A topological field theory can also be constructed as a state sum on a cell decomposition of a 3-manifold. If the weights in such sum satisfy certain relations, the sum depends only on a topological class of the manifold, and not a particular cell decomposition. We will see that 6j-symbols (associativity constraints in a monoidal category with couple of extra properties) provide such weights. Truncated category of modules over quantum universal enveloping algebras at roots of unity give examples of such weights.

2. Algebraical structures in conformal quantum field theories

2.1. **Quantum field theory and statistical mechanics.** Here we will return to the discussion of concepts of a local quantum field theory in a more general setting, when the theory does not have to be topological. Two examples will be discussed: a finite dimensional example of a quantum field theory on cell complexes, and the Gaussin quantum field theory on Riemannian manifolds. We will focus on two dimensional quantum field theories on a planar region, on a cylinder, and on a torus and will see that quantum local fields can be regarded as operator-valued distributions.

2.2. **Two-dimensional conformally invariant quantum field theories.** Here we will discuss global and local conformal invariance, conformal invariance of a quantum field theory. We will focus on a two-dimensional case, where the Lie algebra of local conformal transformations is infinite-dimensional.

2.3. **Virasoro algebra and affine Kac-Moody algebras.** Here we will recall some basic facts about the Virasoro algebra and affine Kac-Moody algebras. We will discuss the representation theory and the intertwining operators between highest weight modules and evaluation modules. We will also see that matrix elements of compositions of such intertwiners satisfy a system of linear differential equations, known as the Knizhnik-Zamolodchikov (KZ) system.

2.4. **Examples of conformal field theories on a plane.** Here we will use the representation theory of affine Kac-Moody algebra and the KZ-equations to construct conformal blocks and will use conformal blocks to correlation functions of a conformally invariant quantum field theory on a plane.

2.5. **Braiding, fusion, and modularity in conformal field theory.** Here we will see that the combinatorial data behind a conformal field theory can be described by modular categories. We will also discuss conformally invariant theories on a torus and the modular invariance.