HW 7

Note Title 5/11/2008

① Let $\pi : P \to M$ be principal G-bundle. Prove $\ker(d\pi) \simeq \mathfrak{g}$ where $d\pi : TP \to TM$ and $\mathfrak{g} = \mathrm{Lie}(G)$.

② Prove the theorem:

Connections $\hat{A}$ on P are in bijection with 1-forms $\Omega^1(P, \mathfrak{g})$ which are G-invariant:

$$r_g^*(\hat{A}) = Ad_{g^{-1}}(\hat{A})$$

and $$i_p^*(\hat{A}) = \theta_p$$

(See notes on connections for the definitions)

(3) Consider the category A_ε where ε is ℓ-th primitive root of unity, ℓ-odd.

- A_ε is semisimple with simple objects V_i, $i = 0, \dots, \ell-1$
- $V_i \otimes V_j = V_{i+j \,(\mathrm{mod}\,\ell)}$
- $c_{ij} : V_i \otimes V_j \to V_j \otimes V_i$ acts on $\mathrm{Hom}(V_i \otimes V_j, U)$ and on $\mathrm{Hom}(U, V_i \otimes V_j)$ by multiplication

$$f \mapsto f\, \varepsilon^{ij}$$

(a) Prove that this is a modular category.

(b) Compute the invariant $\tau_{A_\varepsilon}(M)$ in terms of classical invariants $(\pi_1(M), \dots)$.

4 Solve the Ex. from lecture 11.