

Homework 6.

Note Title

4/9/2008

This is the h/w for Lecture 9.

1. Prove that for modular category

$$\sum_j d_j \langle \text{Diagram 1} \rangle = p^+ \langle \text{Diagram 2} \rangle$$

Diagram 1: A circle with a smaller circle inside it. The inner circle is labeled 'G'. The outer circle has an incoming line from the top labeled 'i' and an outgoing line from the bottom labeled 'j'.

Diagram 2: A circle labeled 'G' with an incoming line from the top labeled 'i' and an outgoing line from the bottom.

2. Prove the identity

$$(st)^3 = p^+ s^2 c$$

3. Prove the identity

$$\sum_k d_k \langle \text{Diagram 3} \rangle = \delta_{ij} \frac{p^+ p^-}{d_i} \langle \text{Diagram 4} \rangle$$

Diagram 3: A circle with two vertical lines inside. The left line has an incoming line from the top labeled 'i' and an outgoing line from the bottom labeled 'j'. The right line has an incoming line from the top labeled 'j' and an outgoing line from the bottom labeled 'i'. The circle has an incoming line from the left labeled 'k' and an outgoing line from the right labeled 'k'.

Diagram 4: A circle with an incoming line from the top labeled 'i' and an outgoing line from the bottom labeled 'i'.

4. Prove that

$$N_i N_j = N_j N_i ,$$

i.e. that

$$\sum_b N_{i0}^a N_{jc}^b = \sum_b N_{jb}^a N_{ic}^b$$

5. Prove the theorem about
matrices N_i

6. Prove

Thm. If $M_{ij} \leq 0 \quad i \neq j \quad \sum_{j=1}^n M_{ij} > 0$
then $\det(M) \neq 0$