Homework 5.

Note Title 3/6/2008

1. Let (A,R) be a finite dimensional quasitriangular Hopf algebra.

Define mappings $R_\pm : A^0 \longrightarrow A$

$$R_+(\ell) = (\ell \otimes id)(R)$$

$$R_-(\ell) = (id \otimes \ell)(R^{-1})$$

(a) <u>Lemma</u>. $R_\pm$ are Hopf algebra homomorphisms.

[Prove it]

Define the mapping $J : A^0 \xrightarrow{\Delta} A^0 \otimes A^0 \xrightarrow{R_+ \otimes R_-} A \otimes A$

Together with the comultiplication

$$\Delta : A \to A \otimes A$$

this defines the mapping of vector spaces

$$I : A \otimes A^0 \longrightarrow A \otimes A$$

$$a \otimes e \longmapsto \sum_{a,e} a^{(1)} \otimes a^{(2)} \otimes e^{(1)} \otimes e^{(2)} \longmapsto \sum_{a,e} a^{(1)} e^{(1)} \otimes a^{(2)} e^{(2)}$$

$$\Delta a = \sum_a a^{(1)} \otimes a^{(2)}, \quad J(e) = \sum_e e^{(1)} \otimes e^{(2)}$$

(b) <u>Proposition</u> $I : \underset{\text{(as a vect. space)}}{D(A) = A \otimes A^{\circ}} \longrightarrow A \otimes A$

is an <u>algebra</u> homomorphism (not a bialgebra homomorphism)

[Prove it]

(c) Assume that the mapping

$$A^{\circ} \longrightarrow A,$$

$$e \mapsto J(e) = \sum_e e^{(1)} \otimes e^{(2)} \longmapsto \sum_e e^{(1)} S(e^{(2)})$$

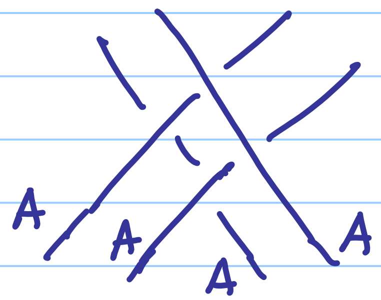
is a linear isomorphism.

Prove:

<u>Proposition</u>. The homomorphism $I : D(A) \to A^{\otimes 2}$ is an isomorphism of algebras

(d) Find the image of $\mathcal{R} \in D(A)^{\otimes 2}$ in $A^{\otimes 2} \otimes A^{\otimes 2}$, i.e. $(I \otimes I)(\mathcal{R})$

<u>Hint</u>: The corresponding braiding is



2. Let $R : V^{\otimes 2} \circlearrowleft$ be a solution to the Yang-Baxter equation:

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{23} \quad (\text{in } V^{\otimes 3})$$

$$R_{12} = R \otimes 1, \quad R_{23} = 1 \otimes R, \text{ etc.}$$

assume $\exists$ $(R^{t_2})^{-1}$, R^{-1}

t_2 = transpos. over the second factor in $V^{\otimes 2}$

Let $\check{R} = P \cdot R$, $\quad P(x \otimes y) = y \otimes x$

Define the braided monoidal category $T(R)$ as:

Objects: $V^{\varepsilon_1} \otimes \cdots \otimes V^{\varepsilon_n}$, $V^+ = V$, $V^- = V^*$

Morphisms: linear maps of vector spaces commuting with commutativity constraints (see below).

Monoidal: $(V^{\varepsilon_1} \otimes \cdots \otimes V^{\varepsilon_n}) \otimes (V^{\sigma_1} \otimes \cdots \otimes V^{\sigma_m}) =$

$$= V^{\varepsilon_1} \otimes \cdots \otimes V^{\varepsilon_n} \otimes V^{\sigma_1} \otimes \cdots \otimes V^{\sigma_m}$$

Rigid: $(V^{\varepsilon_1} \otimes \cdots \otimes V^{\varepsilon_n})^* = V^{\varepsilon_n *} \otimes \cdots \otimes V^{\varepsilon_1 *}$

Braiding:

$c_{V,V} = P \cdot R$, $\quad c_{V,V^*} = P R^{t_2}$

transposed over the second factor in $V \otimes V$

$c_{V^* V} = P R^{t_1}$, $\quad c_{V^* V^*} = P R^{t_1 t_2}$

Extends to $T(R)$ by hexagon axioms

Prove that there exists a ribbon structure on $T(R)$ and $\Rightarrow$ such nondegen.

R defines invariants of framed links.

3. Prove that $\underline{U_q(sl_2)\text{-mod}}^{+}$ generated by irred. f.d. representations with $\varepsilon = +1$ is a braided ribbon category

4. Do the same for $\underline{U_q(sl_2)\text{-mod}}$.