Homework 4.

Note Title 2/27/2008

① Let $f(z)$ be a function $S^1 = \{|z|=1, z\in\mathbb{C}\}$ and $f \in L_2(S^1)$ then the mappings

$$\{f \in L_2(S^1)\} \rightleftarrows \{\hat{f} \in L_2(\mathbb{Z})\}$$

$$f(z) = \sum_{n\in\mathbb{Z}} \hat{f}(n) z^n$$

$$\hat{f}(n) = \frac{1}{2\pi i} \int_{|z|=1} f(z) z^{-n-1} dz$$

are inverse to each other.

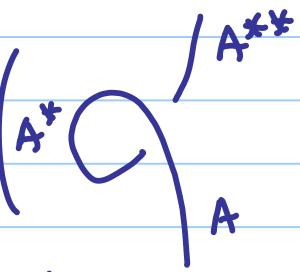
② Prove that $S(\tau) = \tau$ for $U_h sl_2$

Here $\tau = u\, e^{-\frac{hH}{2}}$, where $u = \sum_i S(\beta^i)\alpha_i$

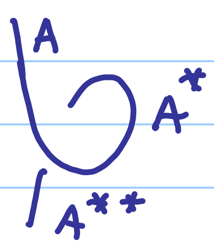
and $R = \sum_i \alpha_i \otimes \beta^i$ is the universal

R-matrix for $U_h sl_2$.

Hint: Use two elements

$\tilde{F}\Big($ 

$\Big) = u_A$

$\tilde{F}\Big($ 

$\Big) = \tilde{u}_A$

Prove that

$\tilde{u}_A u_A = \pi_A(\tau^2)$

$u_A = \pi_A(u)$

③ Prove that the evaluation functor $\tilde{F} : \tilde{\mathcal{D}}(\mathcal{C}) \longrightarrow \mathcal{C}$ for diagrams decorated by $\mathcal{C}$ satisfies the cabling identity:

$$\widetilde{F}\left(D_{A\otimes B} \right) = \widetilde{F}\left(D^{(2)}_{A,B} \right)$$

Here $D_{A\otimes B}$ is a diagram where one of the components is decorated by $A\otimes B$ (and appropriate multiple duals), $D^{(2)}_{A,B}$ is the diagram where this component is replaced by 2 parallel components colored by A and B respectively

$$A\otimes B \Big| \rightsquigarrow A\Big| \quad B\Big|$$

4 Let $V(a,b)$ be a representation of $U_q(sl_2)$ with the basis $\{v_j\}_{j\in\mathbb{Z}}$ with

$$K v_j = a q^j v_j, \quad E v_j = v_{j+1}, \quad F v_j = b_j v_{j-1}$$

with the Casimir element

$$C = EF + \frac{1}{(q-q^{-1})^2}\left(q^{-1}K + q\,K^{-1}\right)$$

acting on $V(a,b)$ as

$$c\,V(a,b) = b\,V(a,b)$$

Here $a \in \mathbb{C}^\vee = \mathbb{C}\setminus\{0\}$, $b \in \mathbb{C}$.

Find $\{b_j\}_{j\in\mathbb{Z}}$ in terms of b,

compare with the formula in terms of b_0.