Mathematical Framework of Quantum Field Theory

September 3, 2009

The goal of this course is to give a general mathematical outline of the framework and structures which appear in quantum field theory. More specifically, the course will start with an outline of classical mechanics and classical field theory. Both Lagrangian and Hamiltonian approaches will be discussed. Then the general structure of local quantum field theory will be explained. We will see the similarities of these structures with statistical mechanics. In this setting the Feynman path integral will appear as a natural proposal for constructing quantum field theories. We will discuss several reasons why it does not work literally in many interesting cases. At this point we will have a short discussion of the program of constructive field theory.

Then we will focus on the ways how make the Feynman integral work in the semiclassical setting. In this part of the course main heroes will be Feynman diagrams. At the end of the course we will learn how to quantize gauge theories in a semiclassical setting. If time permits I will discuss some remarkable cases when a quantum field theory can be constructed non-perturbatively.

There is a number of textbooks where various aspects of the course are represented. There are also lecture notes. The bulk of the course will follow the lectures notes which you can find on the web-site of the course.