**Recursion Theory**
**First semester; 2004/2005**
**Practice set #3**

For all of the following problems you can use the (valid) fact that every TM-computable function can be computed by a Turing machine with only one halting state (that is only one state with no arrows going out of it). Remember also that in the definition of Turing machine (program) we included a clause saying that there is a special state designated as the starting state.

1. **Problem 1:** It is a fact that TM-computable functions are closed under composition. Prove the following particular case: if $f(n), g(n), h(m, n)$ are TM-computable, then $h(f(n), g(n))$ is also TM-computable.

2. **Problem 2:** It is a fact that TM-computable functions are closed under primitive recursion. Prove the following easier result: if $g$ and $h$ are TM-computable, then $f(n)$ defined by $f(0) = g(0)$ and $f(n+1) = h(f(n))$ is also TM-computable.

3. **Problem 3:** Let $f : \mathtt{N} \to \mathtt{N}$ be a bijective TM-computable function. Show that $f^{-1}$ (the inverse of $f$) is also TM-computable. If you need, you can use the previous exercises, of course, and also problem 2 from the current homework (TM-computable functions are closed under the $\mu$ operator).

4. **Problem 4 (Exercise 2.5.3):** From now on, we will say that a set $S \subseteq \mathtt{N}^k$ is *computable* if and only if its characteristic function $\chi_S$ is TM-computable (from what we have proven up to now, TM-computability is the more general notion of computability we have so far). Prove that $S$ is computable if and only if its complement is computable.