The Four Principles of Information Flow

First principle: Information flow results from regularities in a distributed system.

Second principle: Information flow crucially involves both types and its particulars.

Third principle: It is by virtue of regularities among connections that information about some components of a distributed system carries information about other components.

Fourth principle: The regularities of a given distributed system are relative to its analysis in terms of information channels.

Classifications

A *classification* $\mathbf{A} = \langle \Omega_{\mathbf{A}}, \Sigma_{\mathbf{A}}, \models_{\mathbf{A}} \rangle$ consists of

$$\begin{array}{c} \Sigma_A \\ \Big| \models_{\mathbf{A}} \\ \Omega_A \end{array}$$

1. A set of tokens to be classified, called $\Omega_{\mathbf{A}}$.

2. A set $\Sigma_{\mathbf{A}}$ of *types* used to classify tokens.

3. A binary categorization relation $\models_A$, between $\Omega_{\mathbf{A}}$ and $\Sigma_{\mathbf{A}}$.

Infomorphisms

Definition 1 *If* $\mathbf{A} = \langle \Omega_{\mathbf{A}}, \Sigma_{\mathbf{A}}, \models_{\mathbf{A}} \rangle$ *and* $\mathbf{B} = \langle \Omega_{\mathbf{B}}, \Sigma_{\mathbf{B}}, \models_{\mathbf{B}} \rangle$ *are classifications, an infomorphism* $f : \mathbf{A} \rightleftarrows \mathbf{B}$ *is a pair* $\langle f^{\wedge}, f^{\vee} \rangle$ *of functions*

$$\begin{array}{ccc} \Sigma_A & \xrightarrow{f^{\wedge}} & \Sigma_B \\ \Big| \models_A & & \Big| \models_B \\ \Omega_A & \xleftarrow[f^{\vee}]{} & \Omega_B \end{array}$$

such that for all tokens $b \in \mathbf{B}$ *and all types* $\alpha \in \Sigma_{\mathbf{A}}$,

$$f^{\vee}(c) \models_{\mathbf{A}} \alpha \text{ iff } c \models_{\mathbf{B}} f^{\wedge}(\alpha)$$

Information Channels

An *information channel* $\mathcal{C}$ is an indexed family $\mathcal{C} = \{f_i : \mathbf{A}_i \rightleftarrows \mathbf{C}\}$ of infomorphisms whose common codomain $\mathbf{C}$ is called the *core* of $\mathcal{C}$. Consider a channel $\mathcal{C}$ of the form

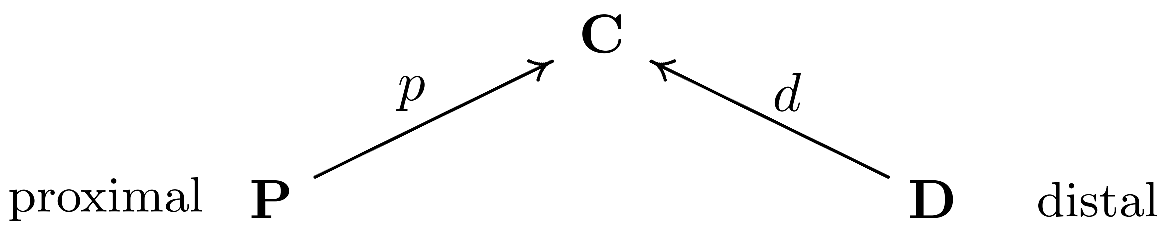
$$\mathbf{A} \xrightarrow{\ f\ } \mathbf{C} \xleftarrow{\ g\ } \mathbf{B}$$

We say that tokens $a \in \Omega_A$ and $b \in \Omega_B$ are *connected* in $\mathcal{C}$ if there is a token $c \in \mathbf{C}$ such that $f^{\vee}(c) = a$ and $g^{\vee}(c) = b$.

INFORMATION FLOW (FIRST ATTEMPT)

Token a being of type α in $\mathbf{A}$ carries the information that token b is of type β in $\mathbf{B}$, relative to channel $\mathcal{C}$, if a and b are connected in $\mathbf{C}$ and $\langle f^{\wedge}(\alpha), g^{\wedge}(\beta)\rangle$ is a constraint supported by $\mathbf{C}$.

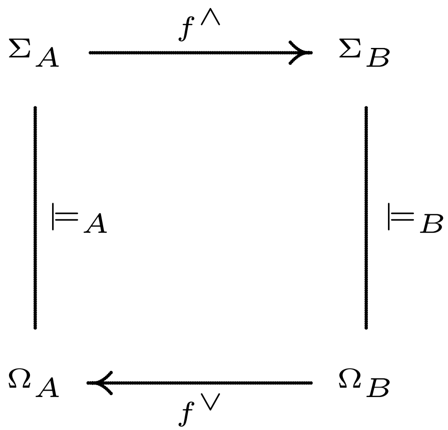
Reasoning at a distance



What kind of (perhaps unsound/incomplete) theory of the distal classification is available to someone with complete knowledge of the proximal one? Suppose $f : \mathbf{A} \rightleftarrows \mathbf{B}$ is an infomorphism.

- If Γ is a set of types of $\mathbf{A}$, we denote by Γ^f the set of translations of types in Γ, that is, $\Gamma^f = \{f^\wedge(\alpha) \mid \alpha \in \Gamma\}$.
- If Γ is a set of types of $\mathbf{B}$, we denote by Γ^{-f} the set of types in $\mathbf{A}$ whose translations are in Γ, that is, $\Gamma^{-f} = \{\alpha \in \Sigma_A \mid f^\wedge(\alpha) \in \Gamma\}$.

Rules for reasoning at a distance



The following "rules of inference" give natural ways of "moving constraints" along the infomorphisms in a channel:

$$f - \textbf{Intro} : \frac{\Gamma^{-f} \vdash_A \Delta^{-f}}{\Gamma \vdash_B \Delta} \qquad f - \textbf{Elim} : \frac{\Gamma^{f} \vdash_B \Delta^{f}}{\Gamma \vdash_A \Delta}$$

- f-**Intro** preserves validity, but does not preserve non-validity.
- f-**Elim** does not preserve validity, but does preserve non-validity.

Information Flow –other proposals (I)

Dretske's Information Content: To a person with prior knowledge k, r being F carries the information that s is G if and only if $Pr(s \text{ is } G \mid r \text{ is } F)$ is 1 (and less than 1 given k alone).

Possible worlds Information Content: To a person with prior knowledge k, r being F carries the information that s is G if in all possible worlds compatible with k and in which r is F, s is G (and there is at least one possible world compatible with k where r is F).

Information Flow –other proposals (II)

Possible worlds Information Content: To a person with prior knowledge k, r being F carries the information that s is G if in every state compatible with k and in which r is F, s is G (and there is at least one state compatible with k where r is F).

Inferential Information Content: To a person with prior knowledge k, r being F carries the information that s is G if the person could legitimately infer that s is G from r is F together with k (but could not from k alone).

Local logics

Intuitively, local logics allows us to track what f-**Intro** and f-**Elim** do.

Definition 2 *A* local logic

$$\mathfrak{L} = \langle cla(\mathfrak{L}), th(\mathfrak{L}), N_{\mathcal{L}} \rangle$$

consists of

1. *a classification* $cla(\mathfrak{L}) = \langle tok(\mathfrak{L}), typ(\mathfrak{L}), \models_{\mathfrak{L}} \rangle$*,*
2. *a theory* $th(\mathfrak{L}) = \langle typ(\mathfrak{L}), \vdash_{\mathfrak{L}} \rangle$ *closed under the usual inference rules: Identity, Weakening and Global Cut (see next slide).*
3. *a set* $N_{\mathcal{L}} \subseteq tok(\mathfrak{L})$*, called* normal tokens *of* $\mathfrak{L}$*, satisfying all the constraints of* $th(\mathfrak{L})$*.*

Regular theories

A regular theory $\mathcal{T}$ over a set of types **typ** is a collection of constraints such that $f$or all $\alpha \in$ **typ**, and $\Gamma, \Delta, \Gamma', \Delta', \Sigma \subseteq$ **typ**,

- Identity: $\alpha \vdash \alpha$ is in $\mathcal{T}$.
- Weakening: If $\Gamma \vdash \Delta$ is in $\mathcal{T}$, then $\Gamma \cup \Gamma' \vdash \Delta \cup \Delta'$ is in $\mathcal{T}$.
- Global Cut: If $\Gamma \cup \Sigma_1 \vdash \Delta \cup \Sigma_2$ is in $\mathcal{T}$ for every pair of disjoint sets Σ_0, Σ_1 such that $\Sigma_1 \cup \Sigma_2 = \Sigma$, then $\Gamma \vdash \Delta$ is also in $\mathcal{T}$.

Thus, item (2) in the previous slide says that the theory of a local logic is regular.