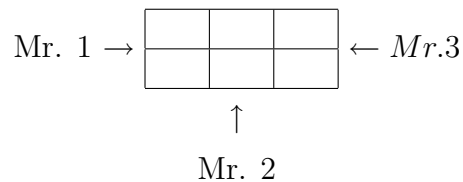


**Caput Logic Language and Information – First semester;
2004/2005
Homework set #9 – DUE TUESDAY NOV 30**

1. Modify the magic box scenario (with full view) by adding a third observer, Mr.3, who can see the box from the back. That is, when we see the box from above, then Mr. 1, Mr. 2 and Mr.3 are located thus:



Just as in class, we take it that L_1 and L_2 are the languages used by Mr.1 and Mr.2, where L_1 is the propositional language with propositional letters in $\{l, r\}$ and L_2 is the propositional language with propositional letters in $\{l, c, r\}$. L_3 , the language used by Mr.3, is also the propositional language with propositional letters in $\{l, r\}$.

- (a) **(.6 pts)** Give two examples of compatibility sequences for this scenario.
- (b) **(.7 pts)** Definition 1 in slide 14 says what it means to be a model for the magic box scenario with full view, when the observers are Mr.1 and Mr.2 only.

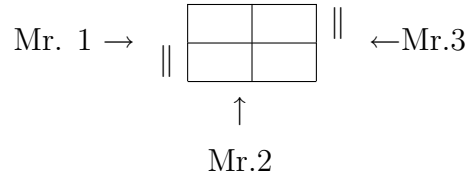
Formulate a new definition saying what it means to be a model for the magic box scenario with full view, when the observers are Mr.1, Mr.2 and Mr.3 as depicted above. Call the class of all these models V^+ .

- (c) **(.7 pts)** Show that $3 : \neg(l \wedge r)$ is a logical consequence of $1 : \neg(l \wedge r)$ in the class V^+ . That is, prove that

$$1 : \neg(l \wedge r) \models_{V^+} 3 : \neg(l \wedge r)$$

Continue in next page...

2. Consider the magic box scenario (with partial view) which looks as follows from above:



The parallel bars outside the box indicate which parts of the views are blocked. Thus, Mr.1 and Mr. 3 can only see whether there is a ball at the left (from their point of view) and they do not know whether there is a ball at the right. Instead, *Mr.2* can form a complete view. The languages, L_1 , L_2 and L_3 in this case are all equal to the propositional language with propositional letters in $\{l, r\}$.

- (a) **(.6 pts)** Give two examples of compatibility sequences for this scenario.
- (b) **(.7 pts)** Formulate a new definition saying what it means to be a model for the magic box scenario with partial full view with observers M.1, Mr.2 and Mr.3 as depicted above. Call the class of all these models PV .
- (c) **(.7 pts)** Prove that

$$2 : l \vee r, 1 : \neg l \models_{PV} 3 : l$$

That is, for any model in PV , if Mr.2 can see at least one ball but *Mr.1* cannot see any ball, then *Mr.3* can see at least one ball.