

**Caput Logic Language and Information – First semester;
2004/2005**

Homework set #11 – DUE TUESDAY DEC 14

NOTE: This homework set is somewhat shorter than usual. Please do hand in your solutions by Dec 14 at 5:00pm. I will not accept late homework this week, since I need to make sure to be done with the grading on time.

1. **(2pt)** A *theory* \mathcal{T} over a set of types **typ** is simply a set of sequents with types in **typ**. We say that the theory \mathcal{T} is *compact* if whenever $\Gamma \vdash \Delta$ is in \mathcal{T} , there are finite sets of types Γ' and Δ' such that $\Gamma' \subseteq \Gamma$, $\Delta' \subseteq \Delta$, and $\Gamma' \vdash \Delta'$ is also in \mathcal{T} .

Let \mathcal{T} be a compact theory. Show that \mathcal{T} is regular (see slide 10) if and only if it complies with Identity, Weakening, and with

Finite Cut: If Γ, Δ are sets of types, α is a type and the sequents $\Gamma, \alpha \vdash \Delta$ and $\Gamma \vdash \Delta, \alpha$ are in \mathcal{T} , then $\Gamma \vdash \Delta$ is also in \mathcal{T} .

2. **(2pt)** Every classification $\mathbf{A} = \langle \Omega_{\mathbf{A}}, \Sigma_{\mathbf{A}}, \models_{\mathbf{A}} \rangle$ induces a theory $Th(\mathbf{A})$ that contains exactly those sequents $\Gamma \vdash \Delta$ which are satisfied by all the tokens in \mathbf{A} . Show that $Th(\mathbf{A})$ is a regular theory.