Content and context in reasoning

How to decide the truth of a statement

β holds whenever Γ holds

Examples.

- A nurse or a doctor will give a talk tomorrow. It will not be a nurse. Therefore, it will be a doctor.
- If $A \vee B$ and not A, then it follows that B.
- If Peter is a bachelor, then he is not married.
- Whenever it is sunny out there, it is also warm.

Plan

Part I – Relevant disciplines.

Part II – Content and context based inference: the S^3 engine.

Part III – Heterogenous inference formalized.

Studies of inference in cognitive psychology

- *Formal Rules* – Rips, Braine, O'Brien.
- *Mental Models* – Johnson-Laird, Byrne.
- *Domain sensitive rules and schemas* – Cheng, Holyoak.
- *Heuristics and biases* – Pollard, Evans.

The study of conditionals

Use structures of the form $\langle Worlds, (R_\phi)_{\phi \in \mathfrak{L}} \rangle$

Idealized instructions for accepting/rejecting $\Gamma \vdash \beta$
in the occasion s:

1. Set $S(\Gamma)$:= all occasions in which all of the sentences in Γ hold.
2. Set $R(\Gamma) := S(\Gamma)$ minus all occasions that do not satisfy some minimal reasonableness criteria.
3. Set $C(\Gamma, B) := R(\Gamma)$ minus all occasions where B holds true. The conditional is accepted if and only if $C(\Gamma, B)$ is empty.

Usual conditions on the reasonableness function

These properties are often taken to hold true of the reasonableness assignment $(A, s) \rightarrowtail R(A, s)$, for all sentences A, B, and situation s :

(R1) If it is impossible for sentence B to be true, then $R(A, s)$ does not contain any situation in which B is true, i.e., $R(A, s) \cap S(B) = \emptyset$.

(R2) $R(A, s)$ is empty if and only if A is impossible, that is, if and only if $S(A) = \emptyset$.

(R3) $R(A, s)$ is empty if and only if $R(\neg\neg A, s)$ is empty.

(R4) If $R(A, s) \subseteq S(B)$, then $R(A \wedge B, s) \subseteq S(B)$.

(R5) $R(A \vee B, s) \subseteq R(A, s) \cup R(B, s)$.

Our framework complies with R1, R3, and R4 only.

Conceptual Spaces (Gardenfors 2000)

Three layered view of cognition:

- Subsymbolic level: neural networks.
- Conceptual level: manipulation of regions in a product space. Geometrical approach.
- Symbolic level: the usual real of logical systems.

Informationalism (Barwise 1997)

In this pragmatic, information-based account of inference:

- The focus is the insider's perspective.
- The formalization is based on the Information Flow Theory (IF) proposed by Barwise and Seligman (1997). Basic concepts are:
  - Classifications $\mathbf{A} = \langle \Omega_{\mathbf{A}}, \Sigma_{\mathbf{A}}, \models_{\mathbf{A}} \rangle$, and
  - Local logics $\mathfrak{L} = \langle cla(\mathfrak{L}), th(\mathfrak{L}), \Phi_{\mathfrak{L}} \rangle$.

The heating system example

- Pre-linguistic level: the spaces Ω and Ω_0.

Observable	**Name**	**Domain**
Thermostat setting	thermo	$[10, 30]$
Room temperature (in Celcius)	rtemp	$[-30, 45]$
Power to house (0=off, 1= on)	pow	$\{0, 1\}$
Exhaust vents (0= blocked, 1= clear)	vent	$\{0, 1\}$
Cool/Off/Heat (-1= cooling, 0 = off, 1 = heating)	mode	$\{-1, 0, 1\}$
Running? (0=no, 1= yes)	run	$\{0, 1\}$
Output air temperature (in Celcius)	otemp	$[-30, 45]$

with
$$\begin{aligned} run &= f_{run}(thermo, rtemp, pow, rtemp, mode) \\ otemp &= f_{otemp}(thermo, rtemp, pow, rtemp, mode) \end{aligned}$$

- Linguistic level: the propositional language $\mathfrak{L}$.

Atomic types: roomTemp[x], thermoBetween[x,y], hotAir
Running, blocked/clear.

Grounding the language in Ω

For each atomic (or simple) type α we have:

- Its characteristic function $\alpha : \Omega \to \{0, 1\}$. Also,

$$\mathit{LivesOn}_\alpha = \textit{set of dimensions needed in the function } \alpha$$

- An outer meaning-approximation function Nec_α.

$$[\![\alpha]\!] \cap \Phi \subseteq \mathit{Nec}_\alpha(\Phi) \subseteq \Phi$$

- An inner meaning-approximation function Suf_α.

$$\mathit{Suf}_\alpha(\Phi) \subseteq [\![\alpha]\!] \cap \Phi$$

The conditions Φ are simple, easy to construct sets.

An example: the type Running

Type: Running

Characteristic function $\mathit{Running}(\sigma) = \begin{cases} 1 & \text{if } \mathit{run} = 1 \\ 0 & \text{otherwise} \end{cases}$

Necessary conditions

$\mathit{Nec}_{\mathit{Running}}(\Phi) = \{\sigma \in \Phi \mid \mathit{mode} \in \{-1, 1\} \wedge \mathit{pow} = 1 \wedge \mathit{vent} = 1\}$

Sufficient conditions

$\mathit{Suf}_{\mathit{Running}}(\Phi) = \{\sigma \in \Phi \mid \mathit{run} = 1\}$

This type draws attention to dimensions $\mathit{LivesOn}_{\mathit{Running}} = \{\mathit{run}\}$.

Backgrounds

$$\begin{pmatrix} thermo & [10,30] \\ rtemp & [10,45] \\ pow & \{1\} \\ vent & \{1\} \\ mode & \{-1\} \\ run & \{0,1\} \\ otemp & [-30,45] \end{pmatrix} \qquad \begin{pmatrix} thermo & [10,30] \\ rtemp & [-30,30] \\ pow & \{1\} \\ vent & \{1\} \\ mode & \{1\} \\ run & \{0,1\} \\ otemp & [-30,45] \end{pmatrix}$$

DefaultSummerConditions *DefaultWinterConditions*

Queries

$\ll \Gamma, \beta \mid \Phi \gg$: *Does* β *hold whenever* Γ *holds?*

S^3 not-so-idealized instructions for accepting/rejecting the query:

1. **Interpret** the query as the question of whether the *sequent* $\Gamma \vdash \beta$ has counterexamples in a condition set $\Phi \subseteq \Psi$.
2. **Construct and approximation** of the set of possible counterexamples for $\Gamma \vdash \beta$ that exist in Ψ.

Interpreting the query $\ll \Gamma, \beta \mid \Phi \gg$

Does $\Gamma \vdash \beta$ have counterexamples in Ψ?

$$\Phi = \begin{pmatrix} dim_1 & I_1 \\ \vdots & \vdots \\ dim_N & I_N \end{pmatrix} \mapsto \begin{pmatrix} dim_1 & L_1 \\ \vdots & \vdots \\ dim_N & L_N \end{pmatrix} = \Psi$$

where

$$L_i = \begin{cases} Dom_i & \textit{if } i \leq k \textit{ and } dim_i \in \bigcup_{\alpha \in \Gamma} LivesOn_\alpha \\ I_i & \textit{otherwise} \end{cases} .$$

Non-monotonicity

`Current background:`

The default winter conditions.

- `Question 1:` $roomTemp[10],\ thermoBetween[15, 17] \vdash hotAir?$

  S^3`says:`**Yes.**

- `Question 2:`

  $roomTemp[10],\ thermoBetween[15, 17], blocked \vdash hotAir?$

  S^3`says:`**No.**

Context dependence

`The question:`

$$roomTemp[10],\ thermoBetween[15,17] \vdash hotAir?$$

- `Background 1:` The default winter conditions.

  S^3`says:` **Yes.**

- `Background 2:` The default summer conditions.

  S^3`says:` **No.**

The Frame problem

`Current background:` The default winter conditions.

- $roomTemp[10],\ thermoBetween[15,17] \vdash hotAir?$

  S^3`says:` **Yes.**

- $roomTemp[10],\ thermoBetween[15,17] \vdash Running?$

  S^3`says:` **Yes.**

What if the thermostat is lowered to be between 5 and 7 degrees?

Search guided by logical form

We work with extended sequents

$$\Gamma \vdash \Delta \ (on\ \Phi)$$

We fix a set of *Givens*, and present two formalisms:

1. A standard deductive system SD^*.
2. A proof-search *reduction* system RED based on the relation defined next.

The reduction relation =>

Let $Seq, Seq_1, Seq_2, \ldots, Seq_n$ be extended sequents. Then

$$Seq => \{Seq_1, Seq_2, \ldots, Seq_n\}$$

if and only if the set of counterexamples to Seq is the union of the sets of counterexamples to $Seq_1, Seq_2, \ldots, Seq_n$.

I'll not explain the reduction system RED because of time constraints. I'll just say that it is at least as powerful as CPL (described next) and better fitting with the S^3 style of search for counterexamples.

The deductive system SD^*

(Identity Axioms) $\Gamma, \alpha \vdash \Delta, \alpha \text{ (on } \Phi)$

(Null Set Axioms) $\Gamma \vdash \Delta \text{ (on } \emptyset)$

(Standard Cut) $$\frac{\Gamma, \alpha \vdash \Delta \text{ (on } \Phi) \quad \text{and} \quad \Gamma \vdash \Delta, \alpha \text{ (on } \Phi)}{\Gamma \vdash \Delta \text{ (on } \Phi)}.$$

(Weakening) $$\frac{\Gamma \vdash \Delta \text{ (}on\text{ } \Phi)}{\Gamma, \Gamma' \vdash \Delta, \Delta' \text{ (on } \Phi)}$$

(Restriction) $$\frac{\Gamma \vdash \Delta \text{ (on } \Phi)}{\Gamma \vdash \Delta \text{ (on } \Phi')} \quad \textit{provided that } \Phi' \subseteq \Phi$$

(Divide and Conquer – DC) $$\frac{\Gamma \vdash \Delta \text{ (on } \Phi_1) \qquad \Gamma \vdash \Delta \text{ (on } \Phi_2)}{\Gamma \vdash \Delta \text{ (on } \Phi_1 \cup \Phi_2)}$$

Plus some meaning approximation rules!

(**L-Shrink**) $$\frac{\Gamma, \alpha \vdash \Delta \text{ (on } Nec_\alpha(\Phi))}{\Gamma, \alpha \vdash \Delta \text{ (on } \Phi)}$$

(**R-Shrink**) $$\frac{\Gamma \vdash \Delta, \beta \text{ (on } \Phi - Suf_\beta(\Phi))}{\Gamma \vdash \Delta, \beta \text{ (on } \Phi)}$$

(**L-Absorb**) $$\frac{\Gamma, \alpha \vdash \Delta \text{ (on } \Phi)}{\Gamma \vdash \Delta \text{ (}on\text{ } \Phi)} \quad \text{(if } Suf_\alpha(\Phi) = \Phi)$$

(**R-Absorb**) $$\frac{\Gamma \vdash \Delta, \beta \text{ (on } \Phi)}{\Gamma \vdash \Delta \text{ (}on\text{ } \Phi)} \quad \text{(if } Nec_\beta(\Phi) = \emptyset)$$