Chapter 1

Time

In this chapter we discuss some relevant data yielded by psychological investigations of mental time. The reader may wonder what such a chapter is doing in a book ostensibly about linguistics. There is natural tendency to view time as something given in the world, so that the business of language is to come up with a veridical representation of this structure. The related case of space shows that something more may be involved. The difference in acceptability between the sentences

(1) a. The bicycle is next to the garage.
b. *The garage is next to the bicycle.

can best be explained by assuming that space is organized cognitively using landmarks, and that the expression 'x is next to y' is used preferrably with x instantiated by an object whose location is at issue, and y by a landmark. Thus cognitive considerations determine a grammatical construction.

Similarly one may explore the idea that time is not given in perception, but constructed by cognition. This overview is included because one might then expect that the tense and aspect systems of natural language tap into cognitive structures concerned with time and that the relevant part of grammar still bears some traces of its cognitive origin. That is, we hypothesize that tense and aspect do not only relate to the finished product of temporal cognition, but also to some of the cognitive processes involved. We do not wish to suggest that the grammatical manifestations of tense and aspect can be exhaustively explained by reference to cognitive structure, but it is not unreasonable to expect some sort of connection. We will come to the conclusion that tense and especially aspect are related to planning, and one purpose of this chapter is to discuss in what sense time is involved in planning (and conversely!). The next chapters will then develop the connection between language and planning in greater detail.

Again, the tense systems of natural language may at first seem to have something inevitable about them: *of course* tense should be formulated in terms of the 'precedes' relation, because time is in fact a linear order. On such a view, the question is only whether one needs auxiliary points on the time line such as Reichenbach's reference time. Well, not quite. It is of some interest to inquire why, and to what extent, 'precedes' is fundamental, and why time is considered to be a continuous linear order. These questions will occupy us in the second half of the chapter.

Not all of the data summarized below are directly relevant to natural language, especially those data concerned with small timescales. But it is at small timescales that one can see the cognitive processes at work which are involved in the construction of time, and if we are right it is these processes that enter into the semantics of tense and aspect[1].

1.1 Psychology of time

Psychological investigations of time can conveniently be brought under the following headings

1. time as succession
2. time as duration
3. temporal perspective

We will discuss these in turn, gratefully using material summarized in Block's collection [1].

1.1.1 Time as succession

Naively, time is viewed as a continuous linear order. Where does this idea come from? As might be expected, there are limits to the human ability to discriminate order. If there are two visual stimuli with stimulus onset asynchrony (SOA) less than around $44ms$ which are projected on the same retinal area, the stimuli are perceived as simultaneous. With slightly larger SOA's, subjects experience flicker, i.e. they experience successiveness without necessarily being able to encode the order of the events. If in this case

[1] In the remainder of the chapter, passages between '**' indicate elucidations of a technical nature. These can be skipped without loss.

the stimuli are projected on different retinal areas, subjects experience apparent movement of the stimulus. Interestingly, temporal order judgements can be wide off the mark in such a case. The paradigm example is the case where the two asynchronous stimuli have different colors, say red and green: the subject then sees one moving stimulus which changes color from red to green in mid-trajectory! What is paradoxical about this phenomenon is that apparently the subject perceives the color green before the green stimulus has occurred. Only at larger SOA's, in the order of $100ms$, can successiveness proper be perceived, in the sense that the subject can make true judgements about the order of succession.

Data such as briefly described in the previous paragraph are obtained using extremely simple stimuli, such as tones of short duration and color flashes. In actual perception, where many stimuli jostle for attention, the processes leading up to a judgement of successiveness are more complicated, mainly because it then is no longer possible to treat the stimuli as pointlike.

In fact, despite centuries of philosophical discussions on time, starting with Aristotle, in which it was assumed that 'now' or 'the present' should be conceived of as a point, psychological research is pointing in a different direction. Let us define the *psychological present* as the collection of events or items jointly present in awareness. With this definition, it turns out that the (objective) duration of the (subjective) present is about $5s$. The psychological present has been described with an evocative phrase as the 'specious present', an expression that was given currency (although not invented) by William James, who elucidated it as follows:

> [T]he practically cognized present [i.e. the specious present] is no knife-edge, but a saddle-back, with a certain breadth of its own on which we sit perched, and from which we look in two directions of time. The unit of composition of our perception of time is a *duration*, with a bow and a stern, as it were–a rearward- and a forward-looking end. It is only as parts of this *duration-block* that the relation of *succession* of one end to the other is perceived. We do not first feel one end and then feel the other after it, and from the perception of the succession infer an interval of time between, but we seem to feel the interval of time as a whole, with its two ends embedded in it. The experience is from the outset a synthetic datum, not a simple one; and to sensible perception its elements are inseparable, although attention looking back may easily decompose the experience, and distinguish its beginning from its end. [3, p. 574-5]

This picture of the formation of judgements of succession, which has been upheld by modern investigations, clearly emphasizes the constructive character of such judgements. The duration of the specious present is connected to the capacity of working memory, i.e. the number of items that can be in an activated state simultaneously. The transition from present to past then corresponds to a transfer of items from (short term) working memory to (long term) declarative memory. Interestingly, there is no experienced discontinuity between the 'specious present' and the past, apparently because this transfer proceeds smoothly.

It appears from this quotation that the 'precedes' relation is not automatically encoded, but requires conscious attention for explicit encoding. Nevertheless, it also seems to be the case that 'precedes' has a priviledged position in the sense that attention to temporal structure focusses on succession, and not on other temporal relations such as 'begins before', 'ends before' or 'overlap'. This may go some way toward explaining why the latter predicates are not grammaticalized in tense systems. A possible exception is the predicate 'ends before' that may play a role in the perfect. If one compares the two sentences

(2) a. I lost my watch, but I found it again.
    b. I have lost my watch, *but I found it again.

it appears that the contrast between (2-a) and (2-b) is that the latter claims that the state of having a watch has ended before now. In fact, the apparent necessity of an enriched temporal language may account for the widespread opinion that the perfect is not a proper tense.

1.1.2 Time as duration

How do we estimate duration? Since time is not itself a stimulus, measurement of time must be based on experienced and remembered changes; as James put it 'awareness of change is . . . the condition on which our perception of time's flow depends' (James [3, p. 584]). One must distinguish here between duration in passing or experienced duration, and duration in retrospect or remembered duration. What is interesting about the latter is that people appear to measure the duration of an epoch by the ease with which they can retrieve events from that epoch. More events retrieved more easily thus means a longer estimated duration. This is of some relevance to the constructions of time from events sketched below, which give some mathematical substance to this idea. Another possibility, of interest to us because it relates to our technical proposals, is that memory contains a number of scenarios with (realistic) default values of durations of activities. By 'default value' we mean the duration of an activity which proceeds normally, with a minimum number of interruptions. Experiences of time running slow or fast while performing an activity are then due to a comparison with the scenario for that activity. Jones and Boltz [4] demonstrated this by comparing judgements of durations of 'natural' melodies with those of malformed melodies. The latter lead subjects to wrong estimations (either too long or too short) about their durations. It will be seen below that scenarios such as envisaged here, containing temporal information about default values, play a very important role in our semantics for time and aspect.

Apart from this connection, important though it is, temporal duration appears to be less relevant to grammar (as opposed to the lexicon) than time's other two aspects, so we shall say little about it here. But it is plausible to assume that duration plays little role in grammar precisely because judgements of duration are so volatile, and likely to differ wildly between subjects unless supported by some form of clock, represented lexically.

1.1.3 Temporal perspective

The various attitudes that people adopt in relation to past, present and future are what characterize temporal perspective. It includes span of awareness of past, present and future, as well as the relative weight given to these. Temporal perspective is different from the simple 'precedes' ordering of events or instants, because it introduces a vantage point, the deictic *now*. In fact, although it is useful conceptually to separate time as succession and temporal perspective[2], in all probability the encoding of succession makes use of temporal perspective:

> Perhaps in interaction with human cognitive processes, information relating to the ordering of events from earlier to later gives rise to the common idea that the progression of time may be represented as a line or an arrow. The continuously integrated functioning of perceiving, remembering and anticipating processes apparently produces a relatively automatic awareness of the successive ordering of events. This is a fundamental aspect of all temporal experiences beyond those that merely produce an experience of successiveness without the ability to discriminate temporal order. The primary psychological basis for the encoding of order relationships between events relates to the dynamic

[2] These are McTaggart's *B* and *A* series, respectively; see Whitrow [18].

> characteristics of information processing: in the process of encoding an event, a person remembers related events which preceded it, anticipates future events, or both [1].

That is, while encoding an event, one simultaneously recalls related preceding events, and anticipates related future events, where 'precedes' is defined operationally as: 'if I *recall* event e, it must have taken place before now', and similarly for the relation between anticipation and the future. The temporal ordering is thus overlayed with recollections and anticipations, and hence generally with contextual material.

Some residue of this process can still be found in language. With regard to the future, for instance, English uses both the syntactic present tense and the construction with 'will'. The former expresses that an event is certain or at least scheduled

(3) The train leaves at five o'clock tomorrow morning.

the latter that the event is contingent

(4) It will rain/*rains tomorrow[3].

Thus different anticipatory attitudes have been coded into the grammar. We can see from this example that temporal perspective plays a role in language over and above the use of vantage points such as *now* or a past time point in narrative tenses.

Our perspective (remembering the past, anticipating and planning the future) seems so natural that one may well wonder how it could be otherwise. Nevertheless, there exist clinical data which show that temporal perspective can be very different. Melges [8] discusses the case of patients with frontal lobe lesions; the frontal lobes are thought to play an important role in anticipatory processes, hence in the future time perspective. Patients with frontal lobe lesions may become indifferent to the future and a slave to the demand characteristics of the present. Melges cites two examples: one patient who was shown a bed with the sheet turned down, immediately undressed and got into bed[4]; another patient, who was shown a tongue depressor, took it and proceeded to examine the doctor's mouth. What is striking about these

[3] A cynic might well say that in Britain and the Netherlands the actual situation is the reverse.

[4] Just to put things in perspective: this is reminiscent of a famous anecdote about the mathematician David Hilbert, who, when asked by his wife during a party to go upstairs and change his shirt, undressed and went to bed instead of returning to the party.

examples is that, once the motivational or goal structure is lost, patients become dependent on the Gibsonian affordances of their environment, which then act almost like stimulus–response bonds. Affordances (as defined by Gibson) are the functional roles that objects may 'wear on their sleeves': in this sense a door 'affords' to go through it, and a bed with the sheet turned down 'affords' to lie down on it. But healthy humans use an affordance as a possibility, to be used in planning toward a goal, and not as a necessity, i.e. as a condition–action rule.

1.2 Why do we have the experience of time at all?

Summarizing the preceding considerations, the following attempt at a cognitive definition of time emphasizes its constructive nature

> Time is the conscious experiental product of the processes that allow the (human) organism to adaptively organize itself so that its behaviour remains tuned to the sequential (i.e. order) relations in its environment. (Michon [9, p. 40])

We do indeed have a conscious experience of time, and this is of course the *sine qua non* of grammatical representation. But one might ask why this is so: couldn't we have functioned equally well without conscious experience of time? It will be seen that we can learn something about the linguistic representation of time from a consideration of this question.

The above quotation rightly emphasizes that it is fundamental for an animal that its motor action must be 'in tune', 'in sync' with the outside world. In principle there are two roads to achieve this, with and without internal clock. Assume for the moment that activity is governed by a motor-program, stored in procedural memory, which may have a hierarchical structure. The top level of the program is an abstract plan that is implemented in ever greater detail as information seeps down the hierarchy. A striking example of this is provided by handwriting: a person has a characteristic style of handwriting whether done with preferred hand, nonpreferred hand, or even foot. The important point is whether time occurs as a control parameter in the program, as an *internal clock*, to determine the relative timing of the actions to be performed. Sometimes such a parameter is necessary, e.g. in bimanual coordination when tapping nonharmonically related rhythms. But sometimes the time parameter appears otiose, as in handwriting, where apparently only the general order of actions is specified, not their time-course, which depends on interaction with the environment (e.g. type of pen, paper, clothing; constraints of musculature). This interaction is achieved not by synchronizing external and internal clocks, but by manipulating parameters (e.g. by simulation or learning) in which time occurs at most implicitly, such as force or momentum. Here, time itself is not a control parameter. As an example, take the case of a motor skill such as the catching of a ball by a goalkeeper. Here it appears to be unnecessary to compute a trajectory and estimated time of arival at the goal; at least if the ball is shot straight at the keeper, time-to-contact is inversely proportional to dilation of the ball's image on retina, which is directly available. In principle it is possible to act on the basis of that information only. Time is then only a byproduct.

The upshot of these considerations is that in many motor skills there is no explicit time, in the form of a clock, involved which is there to become aware of. More generally, temporal information appears not to be encoded unless explicitly attended to (cf. Michon and Jackson [10]). The experience of time is connected to the reflective mode, and belongs to the declarative domain in high-level cognition. But why do we have this capability, and why is the experience of time necessary or at least useful? Michon [9, p. 42] advances the hypothesis that an impasse in the process of action-tuning may lead us to explicitly lay out the order of actions to be performed, and their duration. Explicit awareness of time is thus useful for planning and problem-solving in cases where no skills have yet been developed; afterwards the plan obtained may then be internalized as a skill. But if that is so, the representation of time in language may still bear some traces of the origin of consious time in planning.

1.3 Events, time, cognition

Above we have repeatedly emphasized the constructive nature of time; as Gibson wrote, 'time is not a stimulus', but it is a product of our interaction with the environment. It then becomes an interesting question whether the process which leads to our notion of time can be modelled formally; and it is here that events make their appearance. There have in fact been several attempts in the literature to derive the structure of time from properties of an event structure, deemed to be somehow more immediate. The motivation for studying this question has often been of a metaphysical nature, aimed at reducing time to something more basic. For instance, one might doubt that 'instants' or time-points are given in experience, or one might think that modelling the time-line by the real numbers introduces unpalatable ontological commitments. In such cases a sensible strategy is to define structures which do not lead to such doubts, and to prove a representation theorem showing that the doubtful structure can be constructed from the simpler structure. We shall review several such constructions, and while we do not share the motivation which originally led to these constructions, we nonetheless believe they shed much light on the notions of time and event, considered from a cognitive point of view. We will see in the following that for instance James' enigmatic notion of the 'specious present' lends itself quite well to a formalization in these frameworks.

Before we embark on a discussion of the various constructions of time out of events, we must say something about the notion of event, conceived psychologically[5]. So far we have been talking about 'event' rather loosely, roughly as referents of certain natural language expressions. Looking at examples, all of the following seem to fall in the category 'event': hearing a tone, a wink, stopping a penalty kick, singing an area, a marriage ceremony, a solar eclipse, a stock market crash, World War II, a tank battle, an individual act of bravery, a memorial service, writing a book about WWII, ...

Zacks and Tversky offer the following as an archetype of 'event': 'a segment of time at a given location that is conceived by aan observer to have a beginning and an end'. It is clear that few of the above examples fit this characterization, mainly because of imprecise spatial and temporal boundaries (cf. World War I) or because there may be discontinuities (e.g. writing the book about World War II may have been interrupted for several years). Thus, the archetype can at most provide a guideline. Nevertheless, the question remains how people individuate such events.To fix ideas, one may start from the 'equation'

object :: space = event :: time

and apply what is known about object recognition to the time domain.

Thus, one might say that in both cases perception involves establishing boundaries, in the sense of separating figure from ground. Object recognition provides a clue as to how this may be achieved. An important ingredient in object recognition is the sudden change in the intensity gradient which marks the boundary between environment and object. Similarly, one may look for sudden changes to individuate events. The following changes have been proposed

[5]Here we are indebted to the interesting review paper 'Event structure in perception and cognition' by Zacks and Tversky, [19].

1. a change in the 'sphere' of the behaviour between verbal, social and intellectual
2. a change in the predominant part of the body
3. a change in the physical direction of the behaviour
4. a change in the object of the behaviour
5. a change in the behaviour setting
6. a change in the tempo of the activity

These changes may operate at different levels of granularity. Here it is worth quoting Zacks and Tversky in full:

> The smallest psychologically reified events, on the order of a few seconds, may be defined primarily in terms of *simple physical changes*. For example, think of a person grasping another's hand, the hands going up, going down, releasing. Longer events, lasting from about 10s to 30s, may be defined in relation to some straightforward intentional act: the events described above, on the time scale indicated, form a handshake. From a few minutes to a few hours, events seem to be characterized by *plots* (i.e. the goals and plans of their participants) or by socially conventional form of activity. Perhaps the handshake was part of signing a treaty. On time scales that are long enough, it may be that events are characterized *thematically*. In this example, perhaps the treaty signing was part of an event called a 'peace process'. In general, it seems that as the time scale increases, events become less physically characterized and more defined by the goals, plans, intentions and traits of their participants. [19, p. 7]

Events are thus naturally organized in a part-whole hierarchy. In fact, humans have the ability to parse events at different levels of granularity to facilitate processing [19, p. 8]: people tend to divide the stream of behaviour into smaller units when it is unpredictable; providing information about the behaviour's goal leads to larger units. This may reflect the attempt to maintain a coherent understanding of the environment while keeping processing effort low. When a coarse temporal grain is insufficient to achieve this understanding, people shift to a finer grain of encoding. It will noticed that this picture is completely in line with Marr's view of object recognition and of intelligence generally (cf. chapters 5 and 7 of [7]); in fact the same mathematical structures play a role, as will be seen in section 1.3.5 below (compare [16]).

Summarizing: humans apparently have the ability to parse the stream of perceptions into events, and are able to do this at different levels of granularity. This gives an intricate part-whole structure of events. We will now investigate how this structure can lead to the sense of time as we know it.

1.3.1 The Russell-Kamp construction

As is well-known, Newton believed that time was a physical entity in itself: 'Absolute, true and mathematical time, in and of itself, in its own nature flows equably and without relation to anything external ...'. Leibniz believed that time is relative in the sense that it is dependent on the events that occur: no events, no time, and what's more, the structure of time depends on the structure of events. Bertrand Russell, in 'Our knowledge of the external world' ([12], cf. also [13]), was concerned with formalizing the latter point of view, as part of a program to reduce all knowledge of the world to sense data. His construction was later taken up and somewhat modified by Hans Kamp in [5], and it is his version that we shall discuss.

The setup is very simple. The language for talking about event structures has binary predicates P for 'precedes' and O for 'overlap', and nothing else; variables range over events only. Here and in what follows we look only at the temporal dimension of events. Given the informal definition of event given above ('a segment of time at a given location that is conceived by aan observer to have a beginning and an end') it would be more natural to derive both time and space from a event structure; this however leads to formidable technical complications.

The following seven axioms then characterize event structures (all variables are assumed to be universally quantified)

1. $P(x,y) \rightarrow \neg P(y,x)$
2. $P(x,y) \wedge P(y,z) \rightarrow P(x,z)$
3. $O(x,x)$
4. $O(x,y) \rightarrow O(y,x)$
5. $P(x,y) \rightarrow \neg O(x,y)$
6. $P(x,y) \wedge O(y,z) \wedge P(z,v) \rightarrow P(x,v)$

7. $P(x,y) \lor O(x,y) \lor P(y,x)$

The last axiom blatantly forces linearity of time, which is somewhat disappointing, since it seems hard to motivate it independently of linearity. We could simplify the axioms (leaving only 3 and 6) by defining $O(x,y) \leftrightarrow \neg P(x,y) \land \neg P(y,x)$, but this definition has linearity built in.

We will now see how a version of the time line can be derived from event structures. One way (neither Russell's nor Kamp's) of looking at this construction is viewing an instant as the 'specious present' in the sense of James. This might seem contradictory: the 'specious present' has duration, and isn't an instant supposed to have no duration? Yes and no. An instant will be defined as a maximal set of overlapping events. If one intuitively represents an event by a nontrivial interval, and if there are only finitely many events in all, then, still speaking intuitively, an instant will have duration. On the other hand, since we have taken the set of overlapping events to be maximal, instants are different only in so far as they can be separated by events, so that they cannot be further subdivided. These intuitive ideas are formalized in the following

Definition 1 *Let $< E, P, O >$ be a structure which satisfies axioms 1–7.*

1. *An* instant *of $< E, P, O >$ is a maximal subset of pairwise overlapping events, that is, i is an instant of $< E, P, O >$ if*
   (a) $i \subseteq E$
   (b) *$d, e \in i$ implies $O(d, e)$*
   (c) *if $e \in E$ but $e \notin i$ then there exists $d \in i$ such that $\neg O(d, e)$*[6]*.*
2. *Let I be the set of instants so constructed. For $i, j \in I$, put $i < j$ if there are $d \in i$, $e \in j$ such that $P(d, e)$.*

Theorem 1 1. *The structure $< I, <>$ thus constructed is a strict linear ordering.*

2. *For each $e \in E$, the set $\{i \in I \mid e \in i\}$ is a non-empty interval of $< I, <>$.*

Proof. 1. We have to show that $<$ is a linear order. We will do two cases: transitivity and connectedness of $<$. For transitivity, assume $i < j$

[6]**For infinite event structure the axiom of choice is needed to guarantee the existence of such instants.**

and $j < k$. By definition of $<$, there are $a \in i$, $b, b' \in j$ and $c \in k$ such that $P(a, b)$ and $P(b', c)$. Since $b, b' \in j$ we have $O(b, b')$, whence $P(a, c)$. To show connectedness of $<$ we must prove that for instants i, j with $i \neq j$, either $i < j$ or $j < i$. Since i, j are maximal, there is $a \in i - j$, and $b \in j$ such that not $O(a, b)$, whence either $P(a, b)$ or $P(b, a)$.

2. We have to show that $\{i \in I \mid e \in i\}$ is convex, i.e. that for $i < k < j$, where $i, j \in \{i \in I \mid e \in i\}$, also $k \in \{i \in I \mid e \in i\}$. We must show that $e \in k$, i.e. that for all $d \in k$, $O(d, e)$. Suppose for some $d \in k$, $P(d, e)$. By definition of $<$, $k < i$, contradiction. If $P(e, d)$, we would have $j < k$, again a contradiction. □

The intuition behind this construction is that one thinks of time as composed of instants, and hence events must be modelled as particular sets of instants, and conversely, events must be composed of instants. This set theoretic view of time is not the only one possible as we will see below. Furthermore, if the reader now compares this reconstruction of the 'specious present' with James' explication of this notion, she may notice an unsatisfactory feature. James wrote that 'the unit of composition of our perception of time is a *duration* ... with a rearward- and forward-looking end'. The latter feature is absent from the construction of instant thus given. The reader may well wonder whether James' description is at all consistent, but in fact a different construction of instants from events has been proposed by Walker [17], a construction in which instants become directed even though they are indivisible.

1.3.2 Walker's construction

Walker was a physicist interested in the foundations of relativity theory, in particular in the question where the real numbers that physics uses to represent time come from. Many people have thought that the real line is problematic because it introduces a nondenumerable infinity to account for continuity – denumerable infinities like the integers or the rationals would be less problematic in this respect.

From the standpoint of mathematical logic there is much to be said against this view. One may argue for instance the intuitionistic theory of the continuum gives quite a good picture of this structure without invoking higher infinities. Even if one stays with classical logic one may question whether the cardinality of the standard model is the only indicator of ontological complexity. A very different indicator would be the degree to which we can mentally grasp these structures, as evidenced by the possibility of a complete axiomatization. Here the roles are reversed: there is no complete axiomatization of the integers, but there is a complete axiomatization of the structure $< I\!R; <, +, -; 0, 1 >$. Adding functions like sin or exp complicates the picture somewhat, but the resulting structures are still vastly simpler than that of the integers.

Although the preceding excursion gave some reasons to doubt the traditional way of viewing the 'problem of higher infinities', we shall stick here with Walker's own motivation, not least because part of the linguistic literature on instants and events has also been concerned with this question. Walker's intention was thus to construct time as a continuum solely on the basis of (a denumerable infinity of) events and their structure. In a sense, Russell had the same motivation, but in his case it is much harder to see which axioms on events would force the corresponding instant structure to be continuous. We will not give the full proof of Walker's result, but we will highlight a few concepts and theorems that give particular insight into the notion of time.

Definition 2 *An* instant *of an event structure* $< E, P, O >$ *is a triple* (P, C, F) *such that*

1. $P \cup C \cup F \;=\; E$
2. P, F *are non-empty*
3. $a \in P,\; b \in F$ *implies* $P(a, b)$ [7]
4. *if* $c \in C$*, there exist* $a \in P$*,* $b \in F$ *such that* $O(a, c),\; O(b, c)$*.*

The virtue of this definition is that instants become directed: they have a 'rearward-looking end' (the P-part) and a 'forward-looking end', the (F-part). The last condition of the definition gives a kind of continuity: there is no gap between the present and the past, and likewise for the present and the future. If we now compare Walker's instants to Russell's, we see that a Walker instant always occurs in the empty gap between two events; if there are no such gaps, i.e. if all events overlap, then the event structure has no Walker instant. Clearly in this case there is a Russell instant. It will be seen that Walker's construction gives vastly more insight than Russell's into the relations between past, present and future, and in the continuity of time. In fact, one striking advantage was not even noticed by Walker himself: namely that in his setup events need not correspond to sets of points –

[7] P thus stands for 'past', C for 'current' and F for 'future'.

rather, instants serve to separate events, because instants mark change. If nothing happens inside a given event, there will not be an instant inside that event. Walker's construction is thus much closer to intuitonistic conceptions of the continuum. From a linguistic point of view the interesting question is whether indeed only instants marking change occur as denotations. In Reichenbach's analysis [11], the *event time* properly refers to an event or its corresponding interval, but reference time and utterance time seem to refer to instants; do these, and especially the former, always indicate change? We will come back to this question in a later chapter.

For the technical development below we need to dress up event structures a bit.

Definition 3 *The predicate $B(c,d)$ ('c begins before d') is introduced by putting $B(c,d) \leftrightarrow \exists b(P(b,d) \wedge \neg P(b,c))$. Similarly, $E(c,d)$ ('c ends before d') is introduced by $E(c,d) \leftrightarrow \exists b(P(c,b) \wedge \neg P(d,b))$. From now on, and event structure will be a structure of the form $< W; P, O, B, E >$, where W is a set of events, and P, O, B, E are the predicates introduced by the axioms and definitions above.*

The above definition embodies the claim that the relations 'begins before' and 'ends before' can only be asserted on the basis of witnesses. In other words, the definition claims that we have no cognitive primitives for 'begins (ends) before'; the only cognitive primitive is 'precedes'. It is possible to choose a setup where all predicates are primitives; B and E are then connected to P by means of suitable axioms. Section **??** will discuss this briefly. One might wonder why, if B and E are definable in terms of P, these predicates are included in the modified definition of event structure? This is because this automatically gives the right definition of one event structure being a substructure of another, a matter of some importance in section 1.3.5.

Lemma 1 *(Walker [17], as modified by Thomason [14, p. 89]) Every instant is completely determined by its past. That is, (a) if $< W; P, O, B, E >$ is an event structure, and (P, C, F) an instant, then*

1. *P is a nonempty proper subset of W*
2. *if $\exists a \in P \neg E(a,d)$ then $d \in P$*
3. *if for all $a \in P$: $E(a,d)$, then $\exists b \forall a \in P: (P(a,b) \wedge O(d,b))$.*

(b) Conversely, if the set $P \subseteq W$ satisfies the preceding three conditions, then an instant (P, C, F) is defined by putting $F = \{b \in W \mid \forall a \in P: P(a,b)\}$

and $C = \{c \in W \mid \exists b \in P : O(b,c) \wedge \forall a \in P : E(a,c)\}$. *This instant is the only one whose past equals* P.

Before we give the proof, we offer a remark which hopefully dispels some of the mystery of this result (how can the past determine both the current and the future?). The past determines an instant only in the context of a given event structure. Although the three conditions on P only mention P and (implicitly) E, it is precisely the quantification over *all* events in conditions 2 and 3 that does the trick. So this is not a local definition of 'past'. We could not start from just any set P (say downward closed with respect to the precedence relation P), and put $F = \{b \mid \forall a \in PP(a,b)\}$ and $C = W-(P \cup F)$.

Proof of lemma 1. (a) P and F are nonempty; choose $a \in P$, $b \in F$, then $P(a,b)$ whence in particular $a \neq b$. It follows that $P \cap F$ is empty, so that P is a proper subset of W.

Suppose for some a in P, $\neg E(a,d)$ but $d \notin P$, then either $d \in F$ or $d \in C$. In the first case $P(a,d)$ whence also $E(a,d)$, a contradiction. In the second case for some $b \in F$, $O(d,b)$, whence $P(a,b) \wedge \neg P(d,b)$, i.e. $E(a,d)$, again a contradiction.

Now suppose that for all $a \in P$: $E(a,d)$. Since $\neg E(d,d)$, it follows that $d \notin P$ so $d \in F$ or $d \in C$. In the first case for all $a \in P$, $P(a,d) \wedge O(d,d)$ and we are done. In the second case, there is $b \in F$ with $O(d,b)$, for which we also have $\forall a \in PP(a,b)$.

Conversely, suppose P satisfies conditions $1-3$, and define F, C as above. Then F must be nonempty: choose $d \in W - P$, then for all $a \in P$, $E(a,d)$ (otherwise d would have been an element of P), so there must be b satisfying $\forall a \in PP(a,b)$; this b is then in F.

We next show that $C = W-(P \cup F)$. Choose $c \in C$, then by definition $\forall a \in P : E(a,c)$, so that $c \notin P$. Similarly, $\exists b \in P : O(b,c)$ implies that $c \notin F$. Now choose $c \notin C$, then either $\forall b \in P : (P(c,b) \vee P(b,c)$ or $\exists a \in P : \neg E(a,c)$. In the latter case $c \in P$ by definition. In the former case, if for some $b \in P$, $P(c,b)$, then also $\neg E(c,b)$ and $c \in P$. If for all $b \in P$, $P(b,c)$, then $c \in F$.

If $a \in P$ and $b \in F$ then $P(a,b)$ by definition of F. Lastly, we have to show that for $c \in C$, there are $a \in P$, $b \in F$ with $O(a,c) \wedge O(c,b)$. If $c \in C$, then by definition $\exists b \in P : O(b,c) \wedge \forall a \in P : E(a,c)$. The desired result then follows by the third condition on P.

Now let an instant (P, C', F') be given: we show that $C' = C$, $F' = F$, where C, F satisfy the above definition. Choose $d \in F'$, then for all $a \in P$, $P(a,d)$, whence $d \in F$ by definition of F. Suppose there is $d \in F - F'$, then for all $a \in P$, $P(a, d)$. Since $d \notin F'$, $d \in C'$, so that for some $b \in P$, $O(b, d)$, a contradiction. It follows that $C = C'$. □

Let $\mathcal{I}(W)$ be the set of instants. We will now indicate how a sufficiently rich event structure leads to an instant structure which is (order-) isomorphic to the real line. To this end, we first have to show that $\mathcal{I}(W)$ can be turned into a linear order.

Definition 4 *Put* $(P, C, F) < (P', C', F')$ *if* P *is properly contained in* P'.

This definition makes sense in view of lemma 1.

Lemma 2 $< \mathcal{I}(W), <>$ *is a linear order.*

Proof. The only non-trivial part is to show that $P \neq P'$ implies $P \subset P'$ or $P' \subset P$. Suppose there is $d \in P - P'$. Since P' satisfies the conditions of lemma 1, it follows that for all $a \in P'$, $E(a, d)$. Choose $a \in P'$; we show $a \in P$. Since $E(a, d)$ implies $\neg E(d, a)$, there exists $d \in P$ such that $\neg E(d, a)$, whence $a \in P$. □

The next theorem says that $< \mathcal{I}(W), <>$ shares another fundamental property with the reals, namely that every set with an upper bound has a least upper bound, and every set with a lower bound has a greatest lower bound. Such linear orders are called *complete*.

Theorem 2 $< \mathcal{I}(W), <>$ *is complete.*

Proofsketch. Let $X \subseteq \mathcal{I}(W)$ have an upper bound $P* \in \mathcal{I}(W)$, where all instants are identified with their past. Define a past P by $P = \{a \mid \forall b(\exists P' \in X \forall x \in P' P(x, b) \rightarrow P(a, b)\}$ and show that P satisfies the three conditions of lemma 1. □

The order relation on the real line is fully characterized by completeness and the existence of a countable dense subset, where 'dense' means that between any two points of the set there lies a third. Obviously the rational numbers are a countable dense subset, and we have to find an analogue in $< \mathcal{I}(W), <>$. This is in general not possible without imposing a further condition on the event structure.

Definition 5 *Let* $< W; P, O, B, E >$ *an event structure, and* $a, c, d \in W$. *One says that the pair* (c, d) splits a *if* $P(c, d)$ *and both* $O(c, a)$ *and* $O(d, a)$. $< W; P, O, B, E >$ *is* dense *if for all* $a, b \in W$ *with* $O(a, b)$ *there are* $c, d \in W$ *such that* (c, d) *splits* a *and* b. *A subset* $S \subseteq W$ *is* dense *if for all* $a, b \in W$ *there are* $c, d \in S$ *such that* (c, d) *splits* a *and* b.

The next two results will be mentioned without proof, for which see [14].

Lemma 3 *If* $< W; P, O, B, E >$ *is an event structure, then* $< W; P, O, B, E >$ *is non-empty and dense iff* $< \mathcal{I}(W), <>$ *is dense and has no least or greatest element.*

Theorem 3 *If* $< W; P, O, B, E >$ *is an event structure, then* $< \mathcal{I}(W), <>$ *is isomorphic to the reals if* $< W; P, O, B, E >$ *is non-empty and dense, and has a countable dense subset*[8].

Having an event structure $< W; P, O, B, E >$ and the corresponding complete linear order $< \mathcal{I}(W), <>$, we must now determine what in the latter structure corresponds to an event in the former structure. Here the situation is much more complicated than in Russell's case, where an event could be identified with a set of instants, actually an interval. In Walker's case, one might at first think that the proper definition would be: $e \in W$ corresponds to the set of instants (P, C, F) such that $e \in C$. However, if the precedence relation is a itself linear order, all C's will be empty, so this deifnition does not work. Before we give the correct definition, we state a theorem which isolates an important case where the attempted definition does work.

Theorem 4 *If the event structure* $< W; P, O, B, E >$ *is non-empty and dense, and has a countable dense subset, then* $e \in W$ *corresponds to* $\theta_W(e) = \{(P, C, F) \mid e \in C\}$*. That is, the latter set is a non-empty interval, and the relations* P*,* B *and* E *are preserved by the correspondence.*

Proofsketch We only show that $\{(P, C, F) \mid e \in C\}$ is a non-empty interval. If $P \subset P' \subset P''$, and $P, P'' \in \theta_W(e)$, then since (cf. lemma 1) $\exists b \in P: \ O(b, e)$, also $\exists b \in P': \ O(b, e)$. Furthermore, $\forall c \in P'': \ E(c, e)$, whence $\forall c \in P': \ E(c, e)$. Again by lemma 1 it follows that $e \in C'$.

To prove that $\theta_W(e)$ is non-empty, let(c, d) split e. Define $P(e) = \{a \mid P(a, d)\}$, then $P(e)$ satisfies the conditions of lemma 1, so it defines an instant (P, C, F). By definition of C, $e \in C$. □

We now move to the general case. In the absence of density, one cannot guarantee that $\theta_W(e)$ is non-empty. As Thomason rightly remarks in [15], it is a set theoretic prejudice which leads one to expect that an interval

[8] **Of course the resulting structure $< \mathcal{I}(W), <>$ is only *order*-isomorphic to the reals. We have not talked at all about operations such as $+$ and $\times$, which would be necessary for the measurement of time.**

must be composed of, or even contain, points: if a point is a location where something special happens, such as a change of state, there may well be intervals without any points. Density simply forces the existence of many points of change, but it is an idealizing assumption which may not be true for the human conceptualization of time. We will come back to this issue in section 1.3.5 below.

The proper approach is to use *formal open intervals* of a linear order L.

Definition 6 *Let L be a linear order. A* formal open interval *is a pair (x, y) with $x, y \in L \cup \{-\infty, \infty\}$ and $x < y$. Here, we stipulate that $-\infty < u < \infty$ for all $u \in L$. The predicates are defined on the formal open intervals by*

1. $P((x, y), (u, v))$ *iff* $y \leq u$
2. $O((x, y), (u, v))$ *iff* $P((x, y), (u, v))$ *and* $P((u, v), (x, y))$
3. $B((x, y), (u, v))$ *iff* $x < u$
4. $E((x, y), (u, v))$ *iff* $y < v$

A correspondence between events and formal open intervals is then established by

Definition 7 *If $a \in W$, define $\eta_W(a) = (x_a, y_a)$, where x_a is determined by the conditions*

1. *if a is P-minimal, $x_a = -\infty$; and if not $x_a = (P, C, F)$ where*
2. $P = \{c \mid P(c, a)\}$
3. $F = \{c \mid \neg B(c, a)\}$
4. $C = W - (P \cup F)$

Dually, y_a is determined by the conditions

1. *if a is P-maximal, $y_a = \infty$; and if not $y_a = (P, C, F)$ where*
2. $P = \{c \mid \neg E(a, c)\}$
3. $F = \{c \mid P(a, c)\}$
4. $C = W - (P \cup F)$

Lemma 4 1. *x_a, y_a are either instants or points at infinity*

2. $x_a < y_a$

3. η_W *preserves the order relations* P*,* B *and* E

Proof. 1. Suppose that $x_a \neg - \infty$. Then a is not P-minimal, hence $\{c \mid P(c,a)\} = P$ is non-empty. Since $\neg B(a,a)$, F is non-empty as well. If $P(c,a)$ and $\neg B(d,a)$. The latter statement means that $\forall e(P(e,a) \rightarrow P(e,d))$, which combined with $P(c,a)$ implies $P(c,d)$, as desired. Lastly, choose $c \in C$, then by construction $\neg P(c,a) \wedge \exists b(P(b,a) \wedge \neg P(b,c))$. Since $\neg P(c,a)$ and $\neg P(a,c)$ we have $O(a,c)$, where $a \in F$. Choose b satisfying $(P(b,a) \wedge \neg P(b,c))$. This b is in P, and if $P(c,b)$, also $c \in P$; therefore $O(c,b)$ and we are done. The proof for y_a is analogous.

2. This follows from the implication $P(c,a) \rightarrow \neg E(a,c)$, and the fact that a satisfies the consequent, but not the antecedent.

3. This means that $P(a,b) \iff P(\eta_W(a), \eta_W(b))$ and similarly for the other predicates. Suppose $P(a,b)$; we have to show that $y_a < x_b$. The latter statement is equivalent to the implication $\forall c(\neg E(a,c) \rightarrow P(c,b))$. So choose c and assume $\neg P(c,b)$; this gives $\exists b(P(a,b) \wedge \neg P(c,b))$, which is equivalent to $E(a,c)$, as desired. Clearly this argument can also be run in reverse.

Now suppose $B(a,b)$; we have to show that $x_a < x_b$. This statement is equivalent to $\forall c(P(c,a) \rightarrow P(c,b))$. Assume $P(c,a) \wedge \neg P(c,b)$, then $B(b,a)$, a contradiction. Conversely, if $x_a < x_b$, i.e. $\forall c(P(c,a) \rightarrow P(c,b))$ and $\neg B(a,b)$, then $\forall c(P(c,b) \rightarrow P(a,c))$, hence $\forall c(P(c,a) \rightarrow P(a,c))$, a contradiction. The case of E is similar. □

1.3.3 Some linguistic applications

The time line that is used to locate events in, is not physical time, but time as constructed by cognitive operations. In particular it seems reasonable to assume that only those events and instants are available for temporal reference, which been constructed explicitly by some such process as indicated above. Conversely, of course, one may get an idea of the events and instants that have to be constructed by cognition, by looking at tense and aspect in various languages. In this section we will look at a famous example, the relation between Imparfait (Imp) and Passé Simple (PS) in French. (See Kamp [5, 6] and de Swart [2].)

Consider

(5) Il faisait chaud. Jean ôta sa veste et alluma la lampe.

The first sentence has an Imparfait, indicating that a background without explicit bounds. The second sentence twice features the PS, which indicates that (1) the two events mentioned should be placed inside the background provided by the first sentence, (2) in the context of the discourse, these events are punctual in the sense that they cannot be subdivided further, and (3) the events occur in the order indicated.

The PS has additional uses, which should also be taken account of. In the sentences

(6) a. Et la lumière fut.
b. Il chercha.
c. Il écrivit.

the PS has an inchoative use, and refers to an event marking the onset of the state or activity denoted by the untensed verb. It is even possible for the PS to refer to the end of an event: the sentence

(7) Pierre dîna vite.

may have the meaning *Pierre acheva vite de dîner*. Thus the PS may involve reference to an event taken as a whole, and to its left and right endpoints. We will now try to model this simple example in both the Russell-Kamp and the Walker framework, to see which (if any) fits best. We will even simplify the example to

(8) Il faisait chaud. Jean ôta sa veste.

In the Russell-Kamp framework, a first analysis would introduced events a (corresponding to *Il faisait chaud*), and e (corresponding to *Jean ôta sa veste*), such that

(9) $O(a,e)$ (first formalization of(8)). This is clearly not satisfactory, as the representation theorem would yield a linear order consisting of one instant i only, so that both a and e are mapped onto the closed interval $[i,i]$. One may express that e falls inside a by using the defined predicates B and E

(10) $B(a,e) \wedge E(e,a)$ (second formalization of (8))

The representation theorem now gives the linear order $i < j < k$, with a mapped onto $[i,k]$ and e mapped onto $[j,j]$. This is much better: a and e have different representations, and e is pointlike. Still, this seems to interfere with the possibility to have the PS refer to left and right endpoints (assumed to be different). For this, e would have to contain at least two points, which can only happen when there are further events dividing e.

In Walker's framework the outcome is different. The proper formalization would still be (10), but Walker's representation theorem would make a correspond to $(-\infty, \infty)$, and e to the formal open interval (x_e, y_e), where $x_e = (\{b\}, \{a\}, \{e, d\})$, $y_e = (\{b, e\}, \{a\}, \{d\})$. Here, b is a witness for $B(a, e)$, and d is a witness for $E(e, a)$. The 'punctuality' of e is thus expressed by the fact that the formal open interval (x_e, y_e) does not contain any instants. At the same time the representation yields left and right endpoints, thus taking account of the other referential possibilities for the PS. Thus Walker's representation seems to be slightly more versatile than Russell's.

1.3.4 Events, perfective aspect, and punctuality

There is also another sense in which an event can be punctual, namely in cases where the event describes a change of state (including starting and stopping a process). Take the situation described by 'John took off his sweater'. It seems we have three events here:

1. the state in which John is still wearing his sweater: a
2. the state in which John has taken off his sweater: b
3. the action of taking off the sweater: e

These are related as follows: $P(a, b)$, $O(a, e)$, $O(e, b)$, $B(a, e)$ and $E(e, b)$. We need not conceive of a, b as being contiguous: there may be a kind of limbo between the two states. In fact we would not have the means for expressing (non-)contiguity–these will be introduced in the next section.

Walker's representation now gives for the event structure on $\{a, b, e\}$:

1. there is a witness c for $B(a, e)$ and a witness d for $E(e, b)$ (i.e. $\{a, b, e\}; P, O)$ is a substructure of $(\{a, b, c, d, e\}; P, O, B, E)$
2. the linear order L consists of the points $\{-\infty, x, y, z, \infty\}$, where $x = (\{c\}, \{a, e\}, \{b, d\})$, $y = (\{a, c\}, \{e\}, \{b, d\})$ and $z = (\{a, c\}, \{e, b\}, \{d\})$
3. a is represented in L by the interval $(-\infty, y)$, b by (y, ∞) and e by (x, z)[9]

[9] Without the auxiliary witnesses c, d e would have to be identified with $(-\infty, \infty)$, a somewhat counterintuitive representation. Also note that in the representation a and b become contiguous, even though nothing was said about contiguity in the event structure itself.

The event e is no longer 'punctual' in the sense of perfective aspect, because it has internal structure: it contains the point y. On the other hand, as regards a and b only the point y, indicating the change of state, is relevant; and we may as well identify e with y. This will then give a sharp separation between states and processes on the one hand, and events on the other. Our main tool, the event calculus, is built on this sharp separation, and although we will have occasion to modify it somewhat, it is by and large a useful heuristic.

1.3.5 The importance of granularity: Thomason's construction

Walker's work originated in the attempt to justify the use of the real numbers as mathematical representation of physical time. Russell worried how time as used in physical theory can be derived from sense data. Despite their being concerned with physical time, as we have seen both approaches have many insights to offer to cognitive modelling of time. In one respect cognitive modelling of time has to be more subtle, however. One cannot simply assume that we have a dense set of events in memory, to derive from this that cognitive (not just physical) time may be assumed to be continuous. It is much more reasonable to assume that density arises in the limit of adding more and more events, and that, at each stage, memory contains only finitely many events. In cognition, time thus has a granular representation, but we do not assume a fixed bound on the grain size. We may then investigate what would happen if the grain size decreases indefinitely. This is the point of view taken by Thomason's second paper on time and events, [15]. Thomason's treatment is couched in category theoretic terms, but our treatment will stay at the intuitive level, ignoring several fine points.

While the preceding motivation for looking at degrees of granularity concentrates on the gap between finite memory structures and the infinity required for density, semantics itself also furnishes reasons for introducing granularity. To see this, consider again the analysis of the Imparfait and Passé Simple given in section 1.3.3. In a sense, the distinction between perfective and imperfective aspect is one of granularity: perfective aspect (e.g. the Passé Simple) indicates that an event is viewed as a whole, imperfective aspect (the Imparfait) allows the event to have internal structure, for example when other events are located inside it by means of the Passé Simple. We could view the aspectual difference formally by saying that the use of imperfective aspect such as the Imparfait allows the extension of the event structure to another one in which the event corresponding to the Imparfait can be split. By contrast, perfective aspect could be characterized by saying that if e is an event corresponding to, say, Passé Simple, extensions in which e is split are forbidden. This would then explain why in general a sequence of sentences in the Passé Simple is taken to correspond to a set of consecutive events. Such an explanation may seem to be in danger of explaining too much: in a piece of text like

(11) Il pleuvait. Le vent sécoua la porte. Un éclair illumina le ciel.

it is not fixed whether the events are consecutive (and in what order), or perhaps are simultaneous; unlike the default case of the use of the Passé Simple. But here the picture of extensions of event structures may actually be quite useful: if the order of the events is unclear, this simply means that different extensions of the structure $\{a\}$ (corresponding to 'Il pleuvait') are possible. It would be much more clumsy to represent the uncertainty about the ordering in one event structure. We thus view processing discourse as effecting transitions between event structures.

Lastly, in our introduction to section **??** we pointed out that humans naturally organize events in part-whole hierarchies, and are able to switch automatically between different grain sizes. It therefore makes sense to model this process formally.

Having thus motivated the importance of granularity from three different angles, we proceed to the formal details. We are now concerned with a collection of finite event structures, where one event structure may be a substructure of another. A typical case is where we have two event structures $\mathcal{W}_1 = (W_1; P, O, B, E)$ and $\mathcal{W}_2 = (W_2; P, O, B, E)$ such that $W_1 \subseteq W_2$[10] and W_2 arises from W_1 by adding splitting pairs (c, d) for some $a \in W_1$. Another important case is where events are added to $(W_1; P, O, B, E)$ preceding the minimum element in W_1 or succeeding the maximum element. Because event structures are now finite, these elements exist. Both types of addition model a human being's growth of experience through (physical) time. If $\mathcal{W}_1$ is a substructure of $\mathcal{W}_2$, we shall also say that there is an *inclusion mapping* between $\mathcal{W}_1$ and $\mathcal{W}_2$.

Suppose we now apply the Walker construction to $\mathcal{W}_1$ and $\mathcal{W}_2$, yielding linear orders $\mathcal{L}_1$ and $\mathcal{L}_2$, where $\mathcal{L}_i = (L_i, <)$. If $\mathcal{W}_1$ is a substructure of $\mathcal{W}_2$, can we say anything about the relation between $\mathcal{L}_1$ and $\mathcal{L}_2$? At this point it is useful to define mappings between linear orders, as follows

[10] We also require that the interpretation of P in W_1 is the identical to the interpretation of P in W_2, intersected with W_1, and similarly for the other predicates. This is meant by saying that $\mathcal{W}_1$ is a substructure of $\mathcal{W}_2$.

Definition 8 *An* expansion *of a linear order $\mathcal{L}_1$ to a linear order $\mathcal{L}_2$ is a set-valued mapping $g : L_1 \rightarrow L_2$ (i.e. for $x \in L_1$, $g(x) \subseteq L_2$) such that for $x, y \in L_1$: $\forall u \in g(x) \forall v \in g(y)\ u < v$.*

Let us take stock. We have four structures: an event order $\mathcal{W}_1$ which is a substructure of an event order $\mathcal{W}_2$, a linear order $\mathcal{L}_1$ arising as the Walker representation of $\mathcal{W}_1$, and similarly for $\mathcal{L}_2$. We will see shortly that in this case there exists an expansion from $\mathcal{L}_1$ to $\mathcal{L}_2$. This now gives us two routes from $\mathcal{W}_1$ to $\mathcal{L}_2$: $\mathcal{W}_1 \mapsto \mathcal{L}_1 \mapsto \mathcal{L}_2$ and $\mathcal{W}_1 \mapsto \mathcal{W}_2\ \mathcal{L}_2$. These routes had better yield the same result. That this is indeed so is expressed by the last line of

Lemma 5 *Let $f : W_1 \rightarrow W_2$ be an inclusion mapping between $\mathcal{W}_1$ and $\mathcal{W}_2$. Let F be the construction transforming $\mathcal{W}_i$ in $\mathcal{L}_i$. Then there exists an expansion $F(f) = g$ between $F(\mathcal{W}_1)$ and $F(\mathcal{W}_2)$. Furthermore, suppose G is the mapping which transforms a linear order $(L, <)$ into an event structure, by defining formal open intervals and relations thereon as in definition 6. Let G also transform the many-valued mapping g to an inclusion mapping on event structures, as follows: $G(g)((x, y)) = (\max(g(x)), \min(g(y)))$. Then for any event a, $GF(f)((x_a, y_a)) = (x_{f(a)}, y_{f(a)})$ (cf. definition 7).*

Proof. Define g by: if $(P, C, F) \in L_1$,

$$g((P, C, F)) = \{(P', C', F') \in L_2 \mid P' \cap f(W_1) = P,\ C' \cap f(W_1) = C,\ F' \cap f(W_1) = F\},$$

where $f(W_1) = \{f(a) \mid a \in W_1\}$. We have to show that such a (P', C', F') exists. The existence proof is based on lemma 1. Put $P' = \{d \mid \exists a \in P\ \neg E(a, d)\}$, $F' = \{d \mid \forall a \in P\ P(a, d)\}$ and $C' = \{d \mid \forall a \in P\ E(a, d) \wedge \exists a \in P\ \neg P(a, d)\}$. It may then be verified that (P', C', F') satisfies the definition of an instant.

To show that $G(g)$ is an inclusion mapping, one has to show that the range of $G(g)$ applied to $(L_1, <)$ is a substructure of $(L_2, <)$. We do one case: $P((x, y), (u, v))$ iff $y \leq u$ iff $\neg(u < y)$ iff $\exists z \in g(u) \exists w \in g(y)\ w \leq z$ iff $\min(g(y)) \leq \max(g(u))$ iff $P((\max(g(x)), \min(g(y))), (\max(g(u)), \min(g(v)))$.

Lastly, if $g = F(f)$, $GF(f)((x_a, y_a)) = (\max(g(x_a)), \min(g(y_a)))$; we have to show the right hand side equals $(x_{f(a)}, y_{f(a)})$. We prove $x_{f(a)} = \max(g(x_a))$; the argument for y_a is similar. Suppose $x_{f(a)} = (P_{f(a)}, C_{f(a)}, F_{f(a)})$ and $\max(g(x_a)) = (P', C', F')$. By lemma 1, it suffices to show that the pasts of these instants are equal. Let $x_a = (P_a, C_a, F_a)$. First observe that $x_{f(a)}$ is x_a *as computed in* $F(\mathcal{W}_1)$, since f is an inclusion mapping. $\max(g(a))$ is the instant with the largest past extending P_a. Since $x_{f(a)}$ is an instant, its past is automatically included in that of $\max(g(a))$. The converse direction follows because $a \in F_a$, so that for any past P'' in W_2 and any $c \in P'$, $P(c, a)$. □

We proceed to give some examples. Suppose $W_1 = \{a, b\}$, with $P(a, b)$. Its representation is the linear order $\{-\infty, x, \infty\}$, where $x = (\{a\}, \emptyset, \{b\})$. We first extend the set W_1 to $W - 2 = \{a, b, c\}$ with $P(b, c)$, with the representation $\{-\infty, y, z, \infty\}$, where $y = (\{a\}, \emptyset, \{b, c\})$ and $z = (\{a, b\}, \emptyset, \{c\})$. We thus see that the instant x in W_1 has a single 'daughter' y in W_2; z is a new instant. Now consider the extension of W_1 to $W_2 = \{a, b, c\}$ where $P(a, c)$ and $P(c, b)$. Again the representation of the enlarged structure is of the form $\{-\infty, y, z, \infty\}$, but here $y = (\{a\}, \emptyset, \{b, c\})$ and $z = (\{a, c\}, \emptyset, \{b\})$, so that both y and z are daughters of x. (cf. the definition of g in the proof of lemma 5.) This situation requires some comments. In contrast to the previous case it now seems that the instant x is split into y, z. Now at first it seems rather peculiar to split a Walker instant. There is a clear meaning to splitting a Russell instant: this happens when an event in a maximal set of overlapping events is split. A Walker instant represents a change from one event a to another event b, so splitting that instant means interpolating an event c between a and b: upon closer consideration the transition from a to b proceeds via c. One might argue that this should only be possible if a and b are not contiguous. However, the language introduced so far has no means to express that events are, or are not, contiguous. We leave the matter here for the moment, but we shall return to it at the end of the section.

We now give one application of granularity and show how the notion of time as a continuum may arise from an increasing chainof finite event structures. Suppose we have a sequence of event structures $\mathcal{W} - 0 \subseteq \mathcal{W} - 1 \subseteq \mathcal{W} - 2 \subseteq \ldots$, where $\mathcal{W} - n$ is a substructure of $\mathcal{W} - n + 1$. The *direct limit* of the sequence is defined as the event structure $\mathcal{W}$ whose universeis the union $\bigcup_n W_n$, and the interpretation of the predicate P (or $B, \ldots$) is the union of the interpretations of P (or $B, \ldots$) on the structures $\mathcal{W}_n$. This definition of the structure $\mathcal{W}$ is unambiguous because it is constructed from a chain of *sub*structures. For $\mathcal{W}$ so constructed, each $\mathcal{W}_n$ is a substructure of $\mathcal{W}$.It now is easy to construct a countable dense event structure using finite structures only, by making sure that for each $a, b \in W_n$ with $O(a, b)$, there are $c, d \in W_{n+1}$ which split both a and b.If W_n is finite, W_{n+1} can remain finite as well, but in the limit every pair of overlapping events will be split.It follows that $\mathcal{W}$ can be represented by a continuous linear order $\mathcal{L}$ of Walker instants. **While there is an easy to grasp relationship between the event structures $\mathcal{W}$ and the $\mathcal{W}_n$, the relationship between $\mathcal{L}$ and the $\mathcal{L}_n$ (the linear order corresponding to $\mathcal{W}_n$) is not so simple. The best way to think of it is in terms of an *inverse limit*, as follows. Suppose g_n is the expansion from $\mathcal{L}_n$ to $\mathcal{L}_n$.The mapping g_n is set-valued, but it has the property that if $x \neq y$, for all $u \in G_n(x)$ and for all $v \in g_n(y)$, $u \neq v$. It follows that g_n can be inverted to a single-valued mapping h_n defined by $h_n(u) = x$, where x is (unique!) such that $u \in g_n(x)$. The mapping h_n is surjective, but its domain may be a proper subset of L_{n+1}: h_n is *partial.*Note that, since $\mathcal{W}_n$ is a substructure of $\mathcal{W}$, there will exist partial surjective maps from $\pi_n : L \to L_n$, for all n. Some elements of $\mathcal{L}$ may now be described with reference to the $\mathcal{L}_n$, as follows. Let a *thread* be a function ξ from the positive integers to $\bigcup_n L_n$ such that $\xi_n \in L_n$ such that $\xi_n = h_n(\xi_{n+1})$. A thread thus represents one possible way of subdividing an instant *ad infinitum.* For each thread ξ, the event structure $\mathcal{L}$ contains a unique element x such that for all n, $\pi_n(x) = \xi_n$; that this is so, follows from the construction of theorem 2. The event structure $\mathcal{L}$ will however contain many more elements; although the collection of all elements corresponding to threads is uncountable, it is not continuous, so extra elements must be thrown in to close the gaps. **

We now return to the question, how to express that two events are contiguous, so that in an extension no event can be positioned between them. We extend the language with the predicate $A(x, y)$ meaning 'x abuts y from the left'. If one does not consider this predicate to be a cognitive primitive, it must ultimately be definable using P only. This can be done for example as follows (note that B and E can be defined in terms of P)

Definition 9 *$A(a, b)$ iff a is an E-maximal element of $\{c \mid P(b, c)\}$ and b is a B-minimal element of $\{d \mid P(a, d)\}$. (The definition works because in this context event structures are finite.)*

Semantically, the A-predicate is helpful when describing the event structure corresponding to a sentence such as

(12) John pushed the button. Immediately, light flooded the room.

Here, as indicated by 'immediately', the representation should be such that the events are contiguous.

Now if the event structure $\mathcal{W}_1$ is a substructure of $\mathcal{W}_2$ (with respect to all of P, O, B, E, A) one cannot have the situation where $A(a, b)$ is true on $\mathcal{W}_1$, and a new event c is interpolated between a and b in $\mathcal{W}_2$. For the corresponding linear orders, this means that the possible expansions become more restricted, but we will not pursue the matter here.

1.3.6 Axioms for event structures

?? So far we have taken the predicates A, B, E, O to be definable in terms of P, reflecting a belief that the true cognitive primitive is 'precedes', and that there is no automatic encoding of the other predicates. Apparently it is as yet undecided whether that is so. When all of A, B, E, P, O are taken to be primitive, one needs axioms describing their mutual relations. One such set, as given by Thomason [15] is

1. $P(x,y) \ \rightarrow \ \neg P(y,x)$
2. $P(x,y) \ \wedge \ P(y,z) \ \rightarrow \ P(x,z)$
3. $O(x,x)$
4. $O(x,y) \ \rightarrow \ O(y,x)$
5. $P(x,y) \ \rightarrow \ \neg O(x,y)$
6. $P(x,y) \ \wedge \ O(y,z) \ \wedge \ P(z,v) \ \rightarrow \ P(x,v)$
7. $P(x,y) \ \vee \ O(x,y) \ \vee \ P(y,x)$
8. $B(x,y) \ \rightarrow \ \neg B(y,x)$
9. $B(x,y) \ \rightarrow \ \neg B(y,x)$
10. $B(x,y) \ \rightarrow \ B(z,y) \vee B(x,z)$
11. $E(x,y) \ \rightarrow \ E(z,y) \vee E(x,z)$
12. $P(z,y) \wedge \neg P(z,a) \ \rightarrow \ B(x,y)$
13. $P(x,z) \wedge \neg P(y,z) \ \rightarrow \ E(x,y)$
14. $A(x,y) \ \rightarrow \ P(x,y)$
15. $A(x,y) \wedge P(x,z) \ \rightarrow \ \neg B(z,y)$
16. $A(x,y) \wedge P(u,y) \ \rightarrow \ \neg E(x,u)$
17. $\neg E(x,z) \wedge \neg E(z,x) \wedge \neg B(y,u) \wedge \neg B(u,y) \ \rightarrow \ (A(x,y) \leftrightarrow A(u,z))$

Thus, a sufficient condition for $B(d,e)$ is the existence of a witness, but this is no longer a necessary condition; we allow that $B(d,e)$ is a primitive judgement. The formal development as sketched in the preceding sections is not affected by this different interpretation of the predicates.

**This is perhaps the place to mention that Thomason in [15] uses event structures (extended with A) to determine the most general construction of instants from events; most general in the sense that "ways of regarding events as intervals of other instants are nothing more or less than ways of expanding the original instants" [15, p. 48]. Thomason shows that neither Walker's nor Russell's construction is most general in this sense. He gives an example of such a most general construction for the Russell case, where events are thought of as closed sets of instants. The instants themselves are defined using a modifcation of Walker's definition, as follows

Definition 10 *Let* $(W; P, O, B, E, A)$ *be an event structure. A* cut *is a triple* (P, C, F) *of subsets of* W *satisfying*

1. $C \ = \ W - (P \cup F)$
2. $a \in P \wedge b \in F \ \rightarrow \ P(a, b) \wedge \neg A(a, b)$
3. $P(a, b) \ \rightarrow \ a \in P \vee b \in F$
4. $a \in P \wedge b \notin P \ \rightarrow \ E(a, b)$
5. $a \in F \wedge b \notin F \ \rightarrow \ B(b, a)$
6. $C = \emptyset \ \rightarrow \ P \neq \emptyset \wedge F \neq \emptyset$

If the reader compares this definition with Walker's original definition, she will notice that the condition ensuring continuity between present and past, and between present and future, is lacking. Indeed, as the diagram on p. 61 of [15] shows, there are situations where this continuity is absent. We consider this to be a severe defect of definition 11, which is not offset by the resulting possibility of a most general construction of instants from events.**

1.4 Conclusion

We have repeatedly emphasized that time as used in linguistics is time as constructed by cognitive processes. This chapter has been concerned with several cognitive models of time, both formal and informal, and we will now summarize what these models tell us about linguistic representations of time. The following quotation from Block (given in section 1.1.2, but repeated here for convenience) nicely outlines the reasons why from a cognitive point of view, the Walker/Thomason representation is superior to the Russell/Kamp representation

> Perhaps in interaction with human cognitive processes, information relating to the ordering of events from earlier to later gives rise to the common idea that the progression of time may be represented as a line or an arrow. The continuously integrated functioning of perceiving, remembering and anticipating processes apparently produces a relatively automatic awareness of the successive ordering of events. This is a fundamental aspect of all temporal experiences beyond those that merely produce an experience of successiveness with the ability to discriminate temporal order. The primary psychological basis for the encoding of order relationships between events relates to the dynamic characteristics of information processing: in the process of encoding an event, a person remembers related events which preceded it, anticipates future events, or both [1].

This strongly suggests that encodings of the form (P, C, F) are automatic, unlike the Russell/Kamp definition of instant[11]. If that is so, the semantics of tense and aspect taps into the representation of time given by Walker's construction. This then solves a question that has traditionally been worrying semanticists, regarding the mathematical structure of time: should time be modelled by integers, rationals or reals? We questioned whether the notion of ontological complexity implicit in such worries is at all reasonable, but be that as it may, the Walker/Thomason representation theorem shows that time can be taken to have the topological structure of the continuum. This is not yet the same as saying that time can be modelled by the reals, since the latter in addition has a metric; but for most linguistic purposes it suffices to have a continuum with just the relation $<$ defined on it.

A metric may be derived from duration estimates, but we have seen that human judgements of duration are highly variable. Nevertheless, some aspects of duration estimates may be highly relevant to linguistics. In particular, Jones and Boltz' idea (in [4]) that such judgements may proceed via a comparison with default duration values for activities, is suggestive. For if, as they claim, these default values are derived from scenarios, scripts or frames associated to an activity, then these scenarios may be an integral part of the meaning of the linguistics expression(s) referring to the activity. This idea will be exploited below.

[11] Although we lack the space to develop this further, both the data from cognitive psychology and Walker's formal analysis bear a strong resemblance to the analysis given in Husserl's *Vorlesungen zur Phänomenologie des inneren Zeitbewusstseins.*

More generally, throughout this chapter a theme has been that semantics of tense and aspect may be parasitic upon other representations. In particular planning has come to the fore as a reason why humans have a sense of time at all: planning in novel situations requires carefully ordering steps in time, and estimating their duration. If the sense of time derives from planning, it may be worthwhile to explore the possibility that the linguistic representation of time (as tense and aspect) is partly determined by cognitive structures responsible for planning. The next two chapters will outline in detail how this may come about.

Bibliography

[1] R.A. Block. *Cognitive models of psychological time*. Lawrence Erlbaum, 1990.

[2] H. de Swart. Aspect shift and coercion. *Natural language and linguistic theory*, 16:347–385, 1998.

[3] W. James. *The principles of psychology*. Harvard University Press, 1983. Reprint of the 1890 edition.

[4] M.R. Jones and M. Boltz. Dynamic attending and responses to time. *Psychological Review*, 96:459–491, 1989.

[5] H. Kamp. Events, instants and temporal reference. In R. Baeuerle, U. Egli, and A. von Stechow, editors, *Semantics from different points of view*, pages 27–54. Springer Verlag, Berlin, 1979.

[6] H. Kamp and C. Rohrer. Progress report on tense and temporal adverbs in french. Report, Institut fuer maschinelle Sprachverarbeitung, Universitaet Stuttgart.

[7] D. Marr. *Vision: A Computational Investigation into the Human Representation and Processing of Visual Information.* W.H. Freeman, San Fransisco, 1982.

[8] F.T. Melges. Identity and temporal perspective. In R.A. Block, editor, *Cognitive models of psychological time*, pages 37–58. Lawrence Erlbaum, 1990.

[9] J.A. Michon. Implicit and explicit representations of time. In R.A. Block, editor, *Cognitive models of psychological time*, pages 37–58. Lawrence Erlbaum, 1990.

[10] J.A. Michon and J.L. Jackson. Attentional effort and cognitive strategies in the processing of temporal information. In J. Gibbon and L. Allan, editors, *Timing and time perception*, pages 298–321. New York Academy of Sciences, New York, 1984.

[11] H. Reichenbach. *Elements of symbolic logic*. Macmillan, London, 1947.

[12] B. Russell. *Our knowledge of the external world*. Allen and Unwin, London, 1914.

[13] B. Russell. On order in time. *Proc. Camb. Phil. Soc.*, 32:216–228, 1936.

[14] S. K. Thomason. On constructing instants from events. *Journal of Philosophical Logic*, 13:85–96, 1984.

[15] S. K. Thomason. Free construction of time from events. *Journal of Philosophical Logic*, 18:43–67, 1989.

[16] J. van der Does and M. van Lambalgen. A logic of vision. *Linguistics and Philosophy*, 23(1):1–92, 2000.

[17] A. G. Walker. Durées et instants. *Revue Scientifique*, 85:131–134, 1947.

[18] G.J. Whitrow. *The natural philosophy of time*. Clarendon Press, Oxford, second edition, 1980.

[19] J.M. Zacks and B. Tversky. Event structure in perception and cognition. *Psychological Bulletin*, 127(1):3–21, 2001.