Logical analysis of time: from events to time

- main question for now: how do humans arrive at the concept of a time line, a linear order of instants?
- preliminary hypothesis: time line is constructed from events
- what *is* an event cognitively speaking? (Zacks & Tversky)
- assuming time is primary, one can say: 'a segment of time at a given location that is conceived by an observer to have a beginning and an end'
- but this definition is not available if events are primary; we will therefore *axiomatize* events
- an important analogy: object :: space = event :: time
- hypothesis: events stored in memory according to the same principles as objects
- most important similarity: *granularity* and *hierarchical structure* of event description

Event perception and description

- examples: hearing a tone, a wink, stopping a penalty kick, singing an aria, a marriage ceremony, a solar eclipse, a stock market crash, World War II, a tank battle, an individual act of bravery, a memorial service, writing a book about World War II, ...
- linguistic observation: events can be denoted by nouns (often derived from verbs) and by gerunds ('*–ing*')
- how are these denotations established?
- an important analogy: object :: space = event :: time
- important ingredient in object recognition is the sudden change in the intensity gradient which marks the boundary between environment and object
- one may look for an analogue in event perception/construction

How do people individuate events?

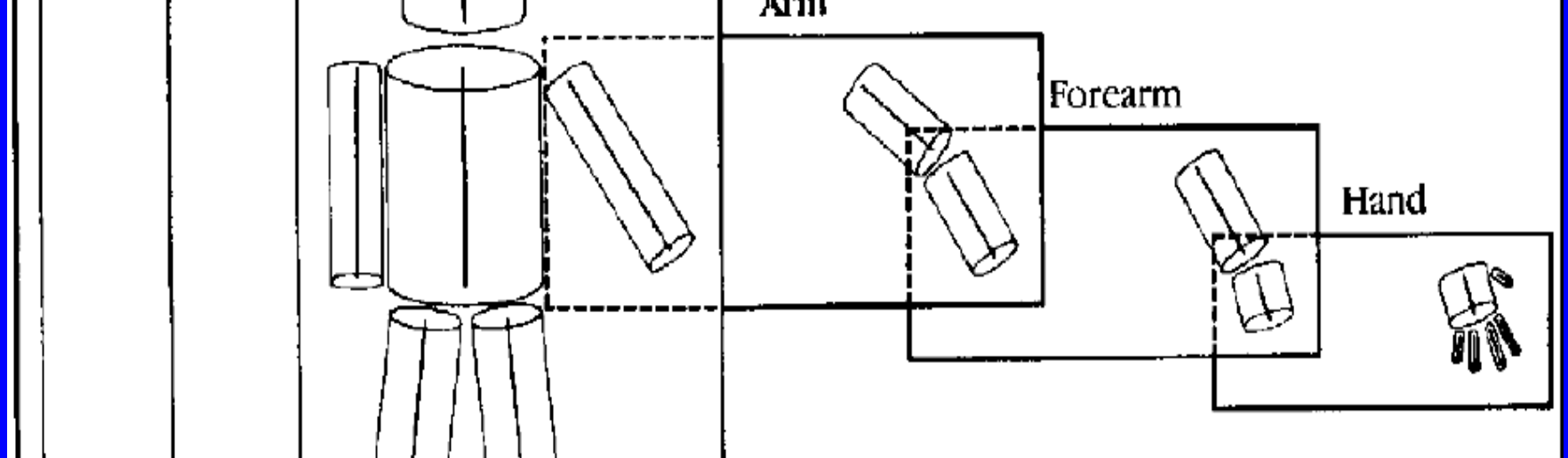
Event boundaries are determined by *changes*; Zacks & Tversky distinguish

1. a change in the 'sphere' of the behaviour between verbal, social and intellectual
2. a change in the predominant part of the body
3. a change in the physical direction of the behaviour
4. a change in the object of the behaviour
5. a change in the behaviour setting
6. a change in the tempo of the activity

Granularity of events

The smallest psychologically reified events, on the order of a few seconds, may be defined primarily in terms of *simple physical changes*. For example, think of a person grasping another's hand, the hands going up, going down, releasing. Longer events, lasting from about 10s to 30s, may be defined in relation to some straightforward intentional act: the events described above, on the time scale indicated, form a handshake. From a few minutes to a few hours, events seem to be characterized by *plots* (i.e. the goals and plans of their participants) or by socially conventional form of activity. Perhaps the handshake was part of signing a treaty. On time scales that are long enough, it may be that events are characterized *thematically*. In this example, perhaps the treaty signing was part of an event called a 'peace process'. *In general, it seems that as the time scale increases, events become less physically characterized and more defined by the goals, plans, intentions and traits of their participants*.

Granularity of objects


Human
Arm
Forearm
Hand

Granularity of events

In general, it seems that as the time scale increases, events become less physically characterized and more defined by the goals, plans, intentions and traits of their participants.

- this will have consequences for the semantics of natural language:
- compare the meanings of 'cough', 'jump' (can roughly be characterized by physical changes) with those of 'persuade', 'build a house' (involve goals, e.g. mental state or physical state)

Logical analysis of time: from events to instants

Possible views on the relation between events and instants

- an event is *composed* of instants (as a set is determined by its elements); this notion of event is *extensional*
- or instants mark the *boundaries* of events, which can themselves be 'empty', if nothing changes while the event lasts; this notion of event is *intensional* – two events may be both empty yet different
- in addition, there might also be cognitively different *kinds* of events, or as we will call them later: representations of events (e.g. perfective and imperfective) – another form of intensionality
- one may also have complex relations between events such as causation
- for a start we choose the simplest logical setup: a one-sorted language with variables over events, and relations P for 'precedes' and O for 'overlap', working in classical logic
- (this is the event-ontology in Kamp/Reyle, *From discourse to logic*)

Logical analysis of time: linear time from an event structure

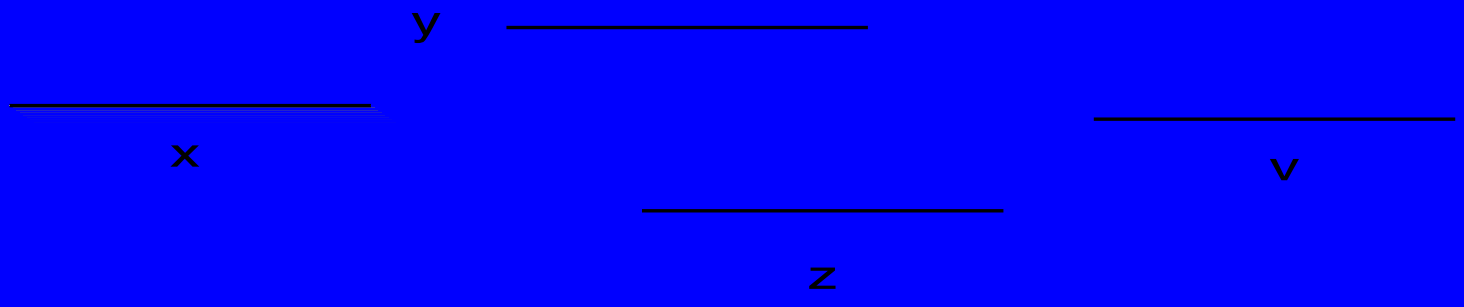
Language: variables over events, binary predicates P for 'precedes' and O for 'overlap' ($\sim$ 'simultaneous with', but not necessarily transitive).

Let E be a set of events. An *event structure* (E, P, O) satisfies the *Walker axioms* (all variables are assumed to be universally quantified):

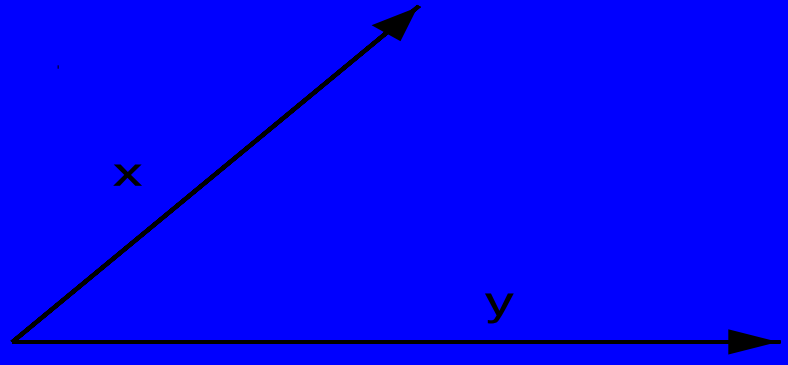
1. $P(x,y) \rightarrow \neg P(y,x)$ (i.e. $\forall x \forall y (P(x,y) \rightarrow \neg P(y,x))$)
2. $P(x,y) \wedge P(y,z) \rightarrow P(x,z)$
3. $O(x,x)$
4. $O(x,y) \rightarrow O(y,x)$
5. $P(x,y) \rightarrow \neg O(x,y)$
6. $P(x,y) \wedge O(y,z) \wedge P(z,v) \rightarrow P(x,v)$
7. $P(x,y) \vee O(x,y) \vee P(y,x)$ (linearity)

These axioms aim at carving out a linear structure from a possibly much more complex set of events.

P(x,y) & O(y,z) & P(z,v) implies P(x,v)



P(x,y) or O(x,y) or P(y,x)



This configuration is forbidden.

Representations of events on a time line

- we will construct an extensional representation of events (due to Russell 1910; also Kamp 1979), as well as an intensional representation (due to Walker 1948)
- in the *extensional* representation, an event corresponds to set of points ('instants') which forms a closed interval $[i, j]$ (i.e. $i \leq j$ and $i < k < j \Rightarrow k \in [i, j]$)
- if a corresponds to $[i, j]$ and b to $[k, l]$ and $P(a, b)$, then $j < k$
- instants are fairly nondescript
- in the *intensional* representation, an event corresponds to a 'formal open interval' (i, j), where $i < j$, which may or may not contain instants (but Leibniz said empty time is impossible?!)
- if a corresponds to (i, j) and b to (k, l) and $P(a, b)$, then $j \leq k$
- instants are special: they correspond to changes (cf. Zacks & Tversky)

Russell instants

A *Russell instant* is a maximal set of pairwise overlapping events. More formally: i is a Russell instant if

1. $d, e \in i$ implies $O(d, e)$
2. $d \notin i$ implies there exists $e \in i$ such that $\neg O(d, e)$

(The root of this definition can be found in Leibniz: an instant is a set of 'fully compatible' events.)

Theorem 1 *Given an event structure (E, P, O), the collection of Russell instants I can be linearly ordered.*

PROOFSKETCH. For $i.j \in I$, put $i < j$ iff there are $d \in i$, $e \in j$ such that $P(d, e)$. EXERCISE. Complete the proof, i.e. show that $<$ is a linear ordering:

1. $i < j \;\Rightarrow\; \neg(j < i)$
2. $i < j \wedge j < k \;\Rightarrow\; i < k$
3. $i \neq j \;\Rightarrow\; i < j \vee j < i$

□

Correspondence between events and intervals

Theorem 2 *For each $e \in E$, the set $\{i \in I \mid e \in i\}$ is a nonempty interval in $(I, <)$, i.e. of the form $(-\infty, m)$, $[n, \infty)$ or $[n, m]$ for $n, m \in I$.*

PROOFSKETCH. Show that for $i, j \in \{i \in I \mid e \in i\}$ and $k \in I$ such that $i < k < j$, $k \in \{i \in I \mid e \in i\}$.

EXERCISE. Complete the proof. □

An unsatisfactory feature of this construction from a psychological point of view: this concept of 'instant' is not very good model of the 'psychological present' – undirected and discrete.

The ‘specious present’

[T]he practically cognized present [i.e. the specious present] is no knife-edge, but a saddle-back, with a certain breadth of its own on which we sit perched, and from which we look in two directions of time. The unit of composition of our perception of time is a *duration*, with a bow and a stern, as it were–a rearward- and a forward-looking end. It is only as parts of this *duration-block* that the relation of *succession* of one end to the other is perceived. We do not first feel one end and then feel the other after it, and from the perception of the succession infer an interval of time between, but we seem to feel the interval of time as a whole, with its two ends embedded in it. The experience is from the outset a synthetic datum, not a simple one; and to sensible perception its elements are inseparable, although attention looking back may easily decompose the experience, and distinguish its beginning from its end. (William James, Principles of Psychology, p. 574)

Walker instants as a better model of the present

Assume the same 7 axioms for events. A *Walker instant* of an event structure (E, P, O) is a triple $(\hat{P}, \hat{C}, \hat{F})$ such that

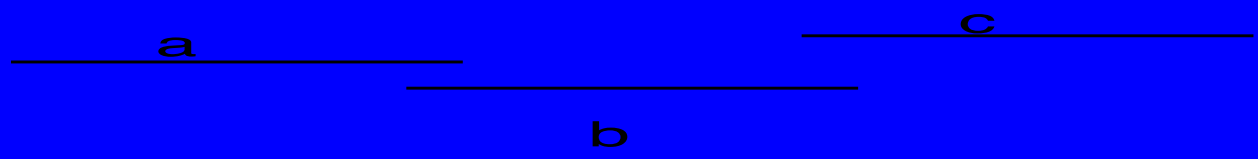
1. $\hat{P} \cup \hat{C} \cup \hat{F} = E$
2. $\hat{P}, \hat{F}$ are non-empty
3. $a \in \hat{P},\ b \in \hat{F}$ implies $P(a, b)$
4. if $c \in \hat{C}$, there exist $a \in \hat{P}$, $b \in \hat{F}$ such that $O(a, c),\ O(b, c)$.

- like James' 'specious present', a Walker instant is directed: $\hat{P}$ is the *realized*, $\hat{C}$ is the *current*, $\hat{F}$ is the *not-yet-realized*
- Walker instants reflect *continuous* transition between present and past, future: see the following picture


a in P
b in F
c in C

Intuitive meaning of Walker instants

- if all events overlap, there is exactly one Russell instant, but no Walker instant
- Walker instants mark change:
- consider the events $\{a, b, c\}$ where $P(a, c)$, $O(a, b)$, $O(b, c)$
- two Russell instants, one Walker instant:
  - Russell instants $\{a, b\}, \{b, c\}$
  - Walker instant $\hat{P} = \{a\}, \hat{C} = \{b\}, \hat{F} = \{c\}$; can be viewed as a 'cut' between a and c


a
c
b

Correlating events and Walker instants

EXERCISE. Let $E = \{a, b, c, d, e\}$ with

$$P(a,c), P(a,e), P(a,d), P(b,e), P(c,e); O(a,b), O(b,c), O(b,d), O(d,e)$$

Two instants i_1, i_2 can be defined (which?). We can make *open* intervals correspond to events according to the following principle: if i is a cut between x, y with $P(x,y)$, then x (y) corresponds to an open interval whose right (left) endpoint is i. If there is no cut to the left (right) of x, its left (right) endpoint is $-\infty$ (∞). Show that

- a corresponds to $(-\infty, i_1)$
- c corresponds to (i_1, i_2)
- e corresponds to (i_2, ∞)
- b corresponds to $(-\infty, i_2)$
- d corresponds to (i_1, ∞)

First three intervals do not contain instants – empty time? measurable?

Compare: Russell instants for the same situation

EXERCISE. Again let

$$E = \{a, b, c, d, e\}$$

with

$$P(a,c), P(a,e), P(a,d), P(b,e), P(c,e); O(a,b), O(b,c), O(b,d), O(d,e)$$

Show that the Russell instants are:

$$1 = \{a, b\}, 2 = \{b, c\}, 3 = \{b, d\}, 4 = \{d, e\},$$

show that these form a linear order, and that we have the following correspondence of events to intervals:

$$a = [1],\ c = [2],\ e = [4],\ b = [1,3],\ d = [3,4]$$

A refinement: taking the future into account?

(possible essay topic)

- if the event structure is finite, there will be a latest event (maximal in the sense of P), which is in the future but overlaps with the current (if there is one)
- it seems reasonable to assume any human event structure is finite – the latest event then plays the role of a 'generic', unknown future event
- the event structure will grow by adding events on the right, subject to axioms 1 – 7
- but a different approach seems possible, in which also anticipated (or even only contemplated) events are incorporated
- these cannot be ordered linearly – what are the proper axioms (and the proper logic) for this case?

Extending the vocabulary

The pictures show the desirability of e.g. the predicates $Begin(c, d)$ ('c begins before b') and $End(c, d)$ ('c ends before b'). These predicates can be axiomatized by means of an extension of 1 – 7, or defined explicitly by

$$Begin(c, d) \leftrightarrow \exists b(P(b, d) \wedge \neg P(b, c))$$

and

$$End(c, d) \leftrightarrow \exists b(P(c, b) \wedge \neg P(d, b)).$$

Temporal expressions like 'suddenly' require a predicate such as $Abut$, defined by

$$Abut(a, b)$$

$$\leftrightarrow$$

$$P(a, b) \wedge \neg\exists c(P(c, b) \wedge End(a, c)) \wedge \neg\exists d(P(a, d) \wedge Begin(d, b)).$$

The past determines everything

Lemma 1 *Every instant is completely determined by its past. That is, (a) if* (E, P, O) *is an event structure, and* $(\hat{P}, \hat{C}, \hat{F})$ *an instant, then*

1. $\hat{P}$ *is a nonempty proper subset of* E

2. if $\exists a \in \hat{P}\ \neg End(a, d)$ *then* $d \in \hat{P}$

3. if for all $a \in \hat{P}$*:* $End(a, d)$*, then* $\exists b \forall a \in \hat{P}\ (P(a, b) \wedge O(d, b))$.

(b) Conversely, if the set $\hat{P} \subseteq E$ *satisfies the preceding three conditions, then an instant* $(\hat{P}, \hat{C}, \hat{F})$ *is defined by putting*

$$\hat{F} \ = \ \{b \in E \mid \forall a \in \hat{P}\ P(a, b)\}$$

and

$$\hat{C} \ = \ \{c \in E \mid \exists b \in \hat{P}\ O(b, c) \wedge \forall a \in \hat{P}\ End(a, c)\}.$$

This instant is the only one whose past equals $\hat{P}$.

Walker instants generate a linear order

Let $\mathcal{I}(E)$ be the set of Walker instants derived from (E, P, O).

Definition 1 *Put* $(\hat{P}, \hat{C}, \hat{F}) < (\hat{P}', \hat{C}', \hat{F}')$ *if* $\hat{P}$ *is properly contained in* $\hat{P}'$.

Lemma 2 $(\mathcal{I}(E), <)$ *is a linear order.*

Proofsketch. Use Lemma 1. □

Walker instants may generate a continuous linear order

(Russell instants don't).

Definition 2 *Let (E, P, O) an event structure, and $a, c, d \in E$. One says that the pair (c, d)* splits *a if $P(c, d)$ and both $O(c, a)$ and $O(d, a)$. (E, P, O) is* dense *if for all $a, b \in E$ with $O(a, b)$ there are $c, d \in E$ such that (c, d) splits a and b. A subset $S \subseteq E$ is* dense *if for all $a, b \in E$ there are $c, d \in S$ such that (c, d) splits a and b.*

Comment: this is a qualitative version of the infinite divisibility of time.

From events to continuous time

Theorem 3 *If (E, P, O) is an event structure, then $(\mathcal{I}(E), <)$ is order–isomorphic to the reals if (E, P, O) is non-empty and dense, and has a countable dense subset.*

Observe that the theorem establishes only topological equivalence with the real line; there is as yet no metric. On this topic, consult G.J. Whitrow, *The natural philosophy of time.*

Granularity: from finite to infinite event structures

- we may assume all event structures are finite,
- but they are capable of twofold growth:
- as future becomes past, new order-maximal events are added
- and existing events may be split by 'having a closer look'
- example of granularity

  (1) I woke up this morning, feeling refreshed

  (2) I woke up at 6.02 am, feeling refreshed

- a suitable limit construction (Thomason 1989) then yields an unbounded and dense event structure

Outlook

- so far we have axiomatized events using *temporal* predicates only: 'precedes', 'overlaps', 'abuts', ...
- this addressed what some feel is a philosophical (and also a semantic) problem: 'moments of time don't exist, only events or intervals'
- therefore semantics of tense ('past', 'future') must involve quantification over events (intervals), not moments
- but the situation is vastly more complicated, because notions of goal and causation have been left out of the picture
- we will see that tense (and aspect) involve these as well
- we must therefore look for richer theories of events