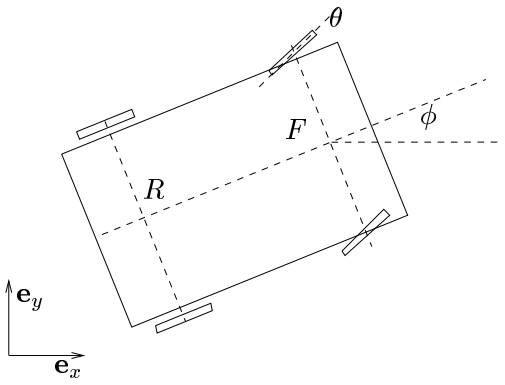
Car representation for motion planning

If you want to control a car, or plan paths for it, you need to represent it conveniently. This representation needs to specify the position and orientation of the car (the car *configuration*), and also be able to specify the basic actions (the car *control*).

(1) How many parameters are required?

(2) We can choose those parameters in various ways. For instance, to denote the position of the car, you can pick any point of it for reference. And to denote its direction, you can take any axis as reference axis. One of these choices affects subsequent calculations more than the other. Which, reference point or reference direction?



(3) There are two reference points that should come to mind for representation of the car position: a point F on the middle of the front axis, and a point R on the middle of the rear axis. Why are other points less attractive?

(4) Discuss the relative advantages of representation by F or by R. (Hint: consider effect of backward/forward motions on the reference point; and consider the accuracy of the motion representation.)

(5) With these considerations, choose one as your reference point P. The motion of that reference point depends on the control parameters of the car. How many are there, and what are they?

(6) Fix these control parameters at certain values, and give the trajectory of the reference point, i.e. the reference point as a function of time t. (Hint: what do you expect the answer to be?)

The expressions become complicated if you do them in general coordinates, so you had better do them for a standard situation (reference point at position 0, orientation of car 0) and then transform them to the general situation afterwards. Use homogeneous coordinate transformations (most handy!) and/or straightforward geometry (most insightful)! There is one dimension of the car that plays an important role in the precise shape of the trajectories. Which, and how?

(7) Along the trajectory of P, the orientation of the car changes as well. Give the expression.

(8) Taken together, the trajectory of position and orientation of the car describes an elementary motion in terms of its configuration parameters. It is therefore a curve in *configuration space*. Sketch this curve, or plot it using a mathematics program such as Maple, Matlab or Mathematica.

(9) Suppose we want to do path planning using an A^*-like algorithm (*"planning is search"*). In order to do so, we need to find a state graph, of which the nodes are states of the car, and the connections possible motions. How is this related to what we have done before?

(10) So you need to digitize configuration space. What considerations are important in choosing the resolution? Consider that you have to be able to represent both states and transitions sufficiently accurately to make the results of A^* meaningful for the control of a real car.

(11) Now that you realize all this, would you still choose the same point (F or R) to represent the car's position?

(12) If you want to plan paths that resemble those of a human driving a car, you have to choose an appropriate cost function for the transitions in the state graph. Pick one, and reason why you have taken it like that. (Hint: Consider the kinds of maneuvers that you would like to be the result of the A^*-like planning process in different circumstances.)