1. *Magnetic susceptibility of electron gas*

a. Why is

$$\chi = \frac{N_e}{4}\frac{(g\mu_B)^2}{k_B T}$$

(ie, number of electrons times the χ for a single spin-$\frac{1}{2}$) *not* a correct result for an electron gas?

b. Give a derivation of the correct expression

$$\chi = \mathcal{D}(\epsilon_F)(\tfrac{1}{2}g\mu_B)^2 \ .$$

2. *Specific heat*

Compute the specific heat of a single $s = \frac{1}{2}$ spin in an external magnetic field $\vec{B}$, as a function of temperature. Sketch the curve $C(T)$ and estimate the location of the maximum. The characteristic peak is called the Schottky anomaly.

3. *Conductivity*

Show that the expression (2.14) for the conductivity of an electron gas reduces to the Drude formula (2.13) when the relaxation time is independent of ϵ and T.

4. *Hybridization*

Assume that we have a $d = 1$ spin-less electron gas with a density of states which is constant and equal to ρ for $0 < \epsilon < D$ and zero for $\epsilon > D$. At zero temperature there is a Fermi energy ϵ_F, $0 < \epsilon_F < D$. We also have an impurity level ϵ_{imp}, $\epsilon_{\text{imp}} < \epsilon_F$, which can be occupied with zero or one electron.

Now assume that we turn on a hybridization interaction

$$V \sum_{\epsilon} \left(c_i^\dagger c_\epsilon + c_\epsilon^\dagger c_i \right) .$$

Due to this interaction, the ground state energy of the system will be lowered by an amount ΔE. Compute ΔE in second order perturbation theory (your answer will depend on D).

5. *Perturbation theory for scattering amplitude*

a. Draw the diagrams that represent the contributions to the scattering amplitude $\langle \vec{k}', s', \alpha' | \, T \, | \vec{k}, s, \alpha \rangle$ of chapter 4 in *third* order perturbation theory.

b. For each diagram, label the internal lines and give the appropriate factors f and $(1-f)$ that are needed when summing over internal momenta.

c. Use information from chapter 4 to identify the most divergent contribution to the spin-dependent scattering amplitude in this order. (This problem has an easy solution. You are *not* supposed to work out the formulas that correspond to the diagrams that you drew!)

6. *Higher order in scaling relation*

It can be shown that, through *third* order in j, the scaling relation between d and j reads

$$\frac{dj}{dd} = -\frac{j^2}{d} + \frac{j^3}{2d} \, . \qquad (*)$$

a. show that for $|j| \ll 1$ $(*)$ is equivalent to

$$\delta(\ln(d)) = -\delta j \left(\frac{1}{j^2} + \frac{1}{2j} \right) \, . \qquad (**)$$

b. For scaling according to $(**)$, $d \, e^{-1/j}$ is no longer an invariant. Find an invariant for $(**)$.

c. We know that for the second order scaling relation, $j(d)$ diverges at $d = d_0 e^{-1/j_0}$ when $j_0 > 0$. For both scaling equations $(*)$ and $(**)$, study the behavior of $j(d)$ when d is lowered from d_0, for an initial value $j_0 = j(d_0)$ satisfying $0 < j_0 < 2$. Make a sketch showing the curves $j(d)$ for the three cases (second order, third order $(*)$ and $(**)$) in one figure.

Hint: you may *not* assume that $j \ll 1$ when d gets small; $(*)$ and $(**)$ give different behavior for small d !

d. Which of the results under c. resembles most closely the non-perturbative result obtained by Wilson?