On Causal Explanations of Quantum Correlations

Robert Spekkens
Perimeter Institute

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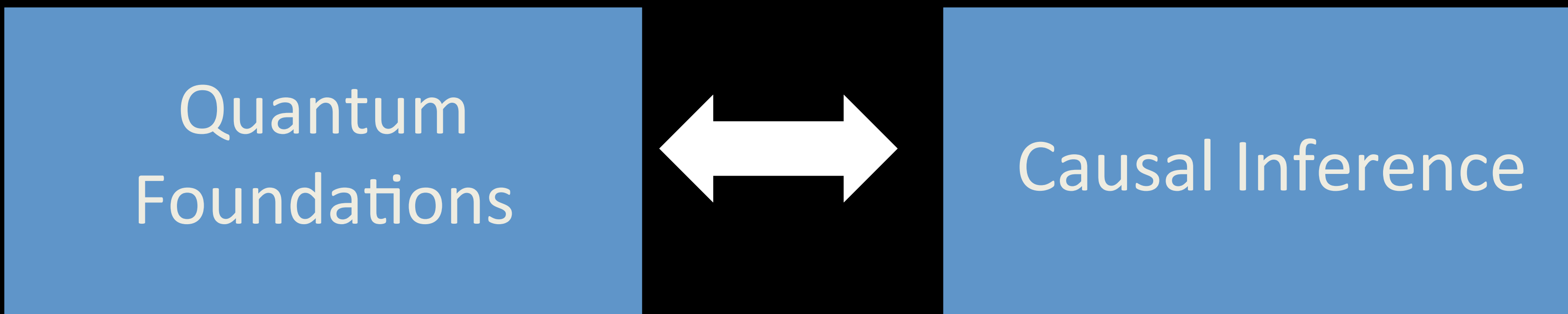
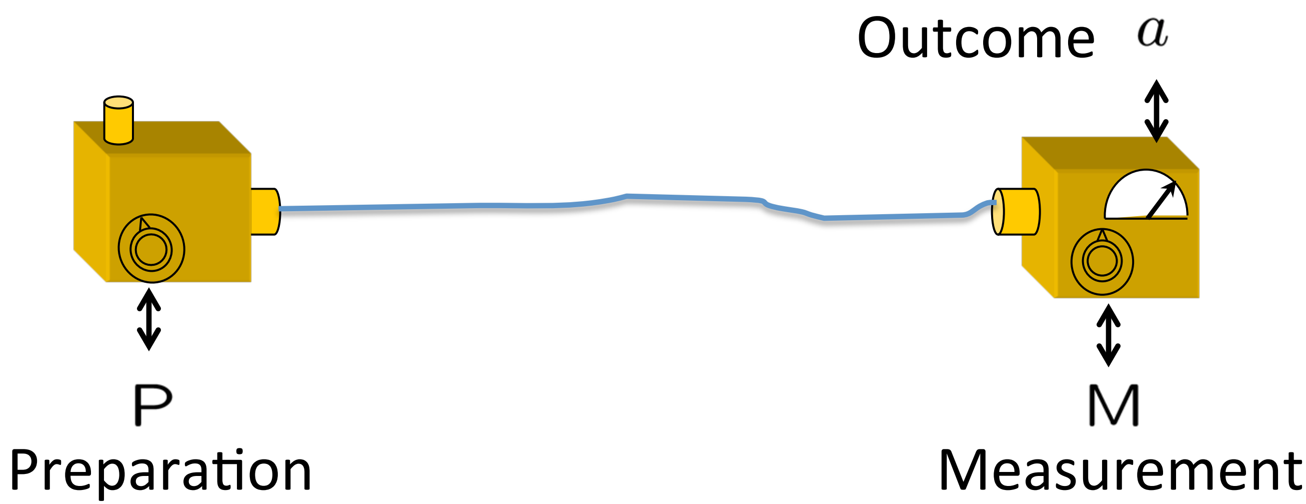
Quebec City

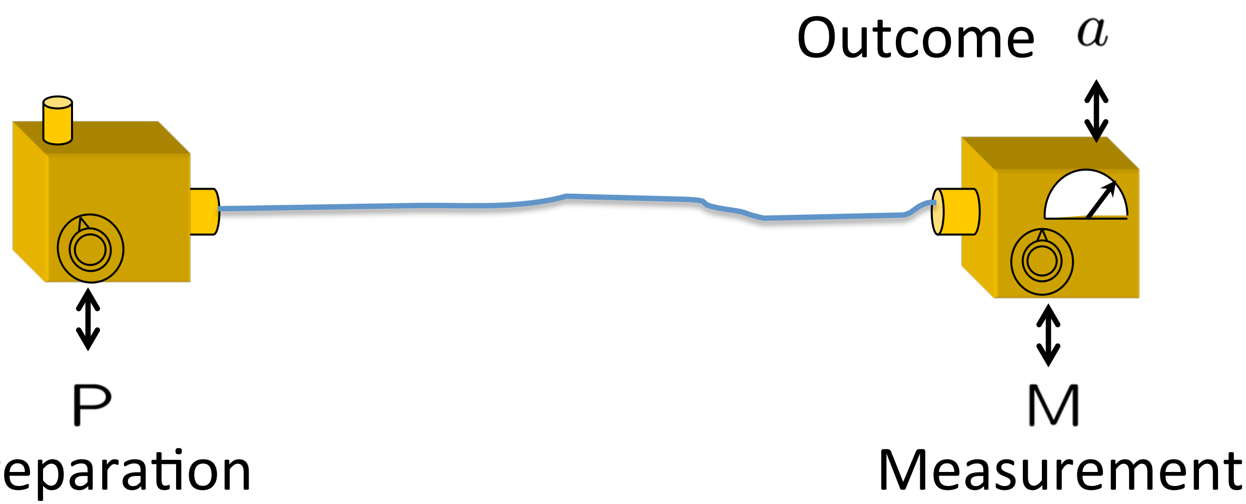

Quantum Foundations
Causal Inference

Operational characterization of Quantum Theory



$p(a|\mathsf{P},\mathsf{M}) \equiv$ The probability of outcome *a* given measurement M and preparation P

Operational characterization of Quantum Theory



Vector in Hilbert space

$$|\psi\rangle \in \mathcal{H}$$

Hermitian operator

$$\hat{A}$$

Eigenvectors $\{|a\rangle\}$

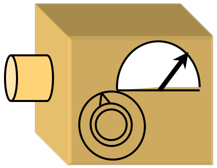
$$p(a|P, M) = |\langle\psi|a\rangle|^2$$

Limit on joint measurability

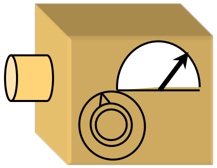
A set of Hermitian operators can only be jointly measured if they commute relative to the matrix commutator.

$[\hat{Q}, \hat{P}] \neq 0$

measure $\hat{Q}$

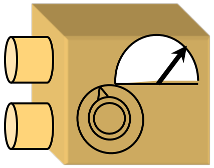


measure $\hat{P}$



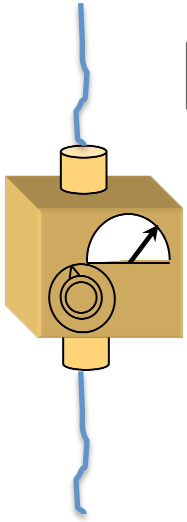
$[\hat{Q}_A - \hat{Q}_B, \hat{P}_A + \hat{P}_B] = 0$

measure $\hat{Q}_A - \hat{Q}_B$ and $\hat{P}_A + \hat{P}_B$



Collapse rule

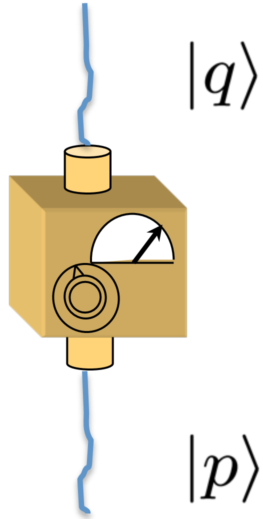
Measure $\hat{Q}$ find q



$|q\rangle$

Collapse rule

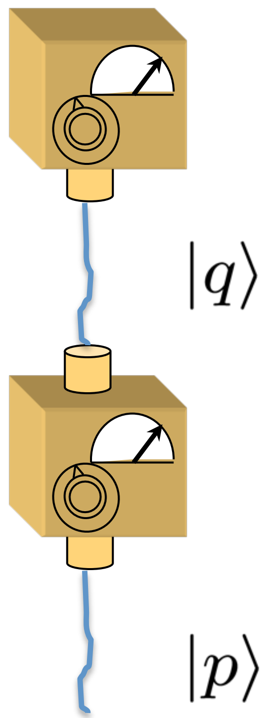
Measure $\hat{Q}$ find q



Prepare $\hat{P}$ with value p

Collapse rule

Measure $\hat{P}$ find p'



Measure $\hat{Q}$ find q

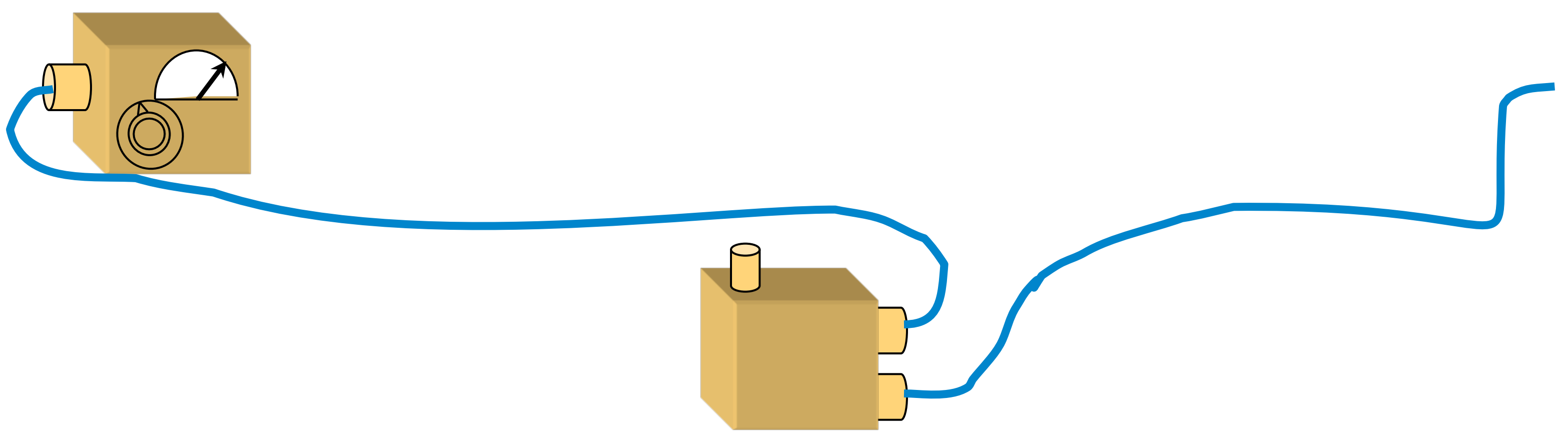
Prepare $\hat{P}$ with value p

"Objective Randomness!"

Einstein-Podolsky-Rosen experiment

Measure $\hat{Q}_A$ find q

collapses to $|q\rangle$

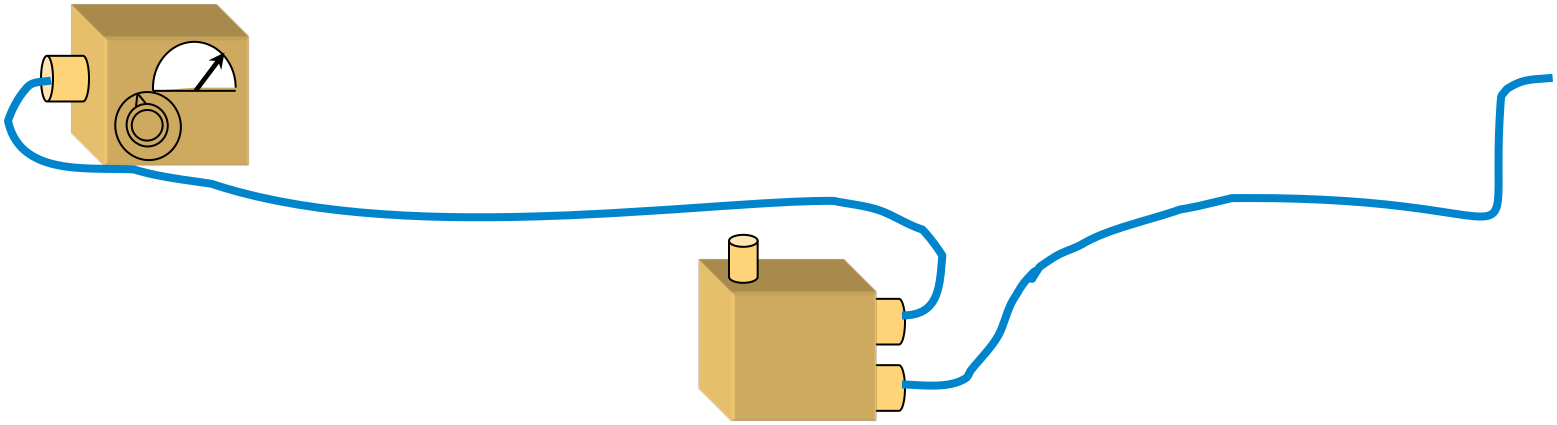


$$|\psi_{\mathrm{EPR}}\rangle$$

$$(\hat{Q}_B - \hat{Q}_A)|\psi_{\mathrm{EPR}}\rangle_{AB} = 0$$

$$(\hat{P}_B + \hat{P}_A)|\psi_{\mathrm{EPR}}\rangle_{AB} = 0$$

Einstein-Podolsky-Rosen experiment



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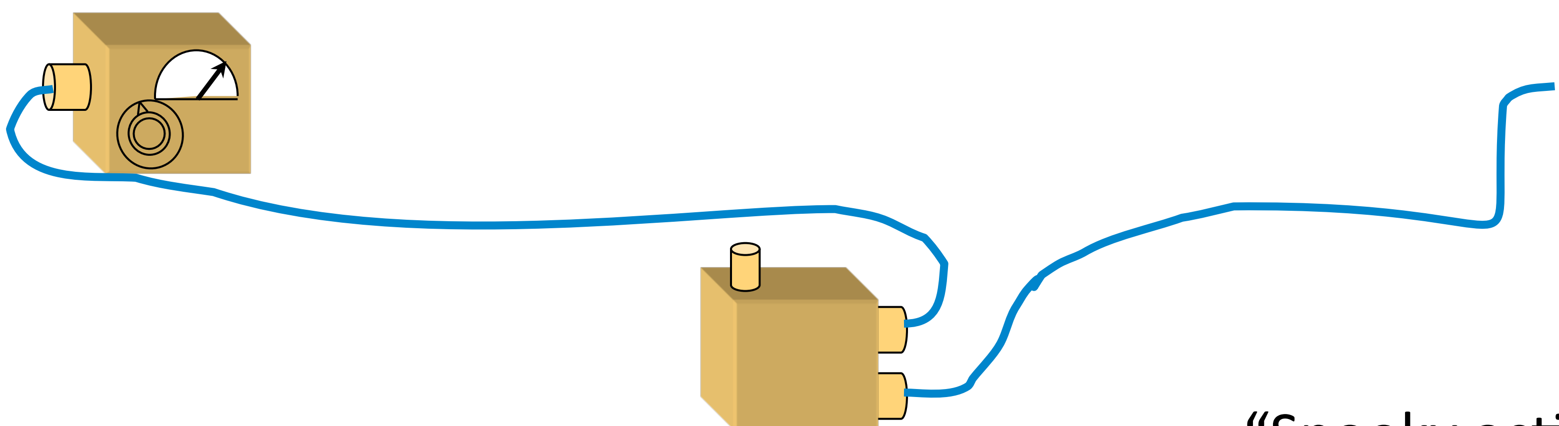
Einstein-Podolsky-Rosen experiment

Measure $\hat{P}_A$ find p

collapses to

$| -p\rangle$



$|\psi_{\mathrm{EPR}}\rangle$

$(\hat{Q}_B - \hat{Q}_A)|\psi_{\mathrm{EPR}}\rangle_{AB} = 0$

$(\hat{P}_B + \hat{P}_A)|\psi_{\mathrm{EPR}}\rangle_{AB} = 0$

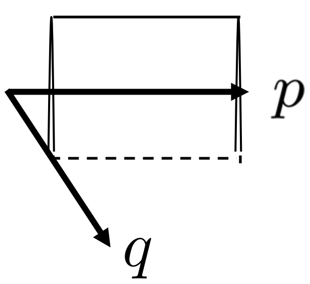
"Spooky action at a distance"

Statistical theory of classical mechanics with an epistemic restriction

A set of variables can only be **jointly known** if they commute relative to the Poisson bracket.

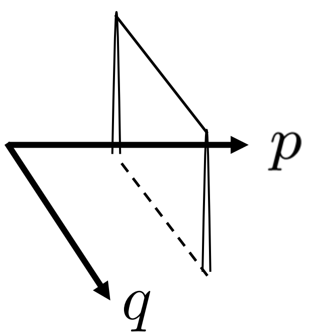
know Q

$$P(q,p) \propto \delta(q-a)$$



know P

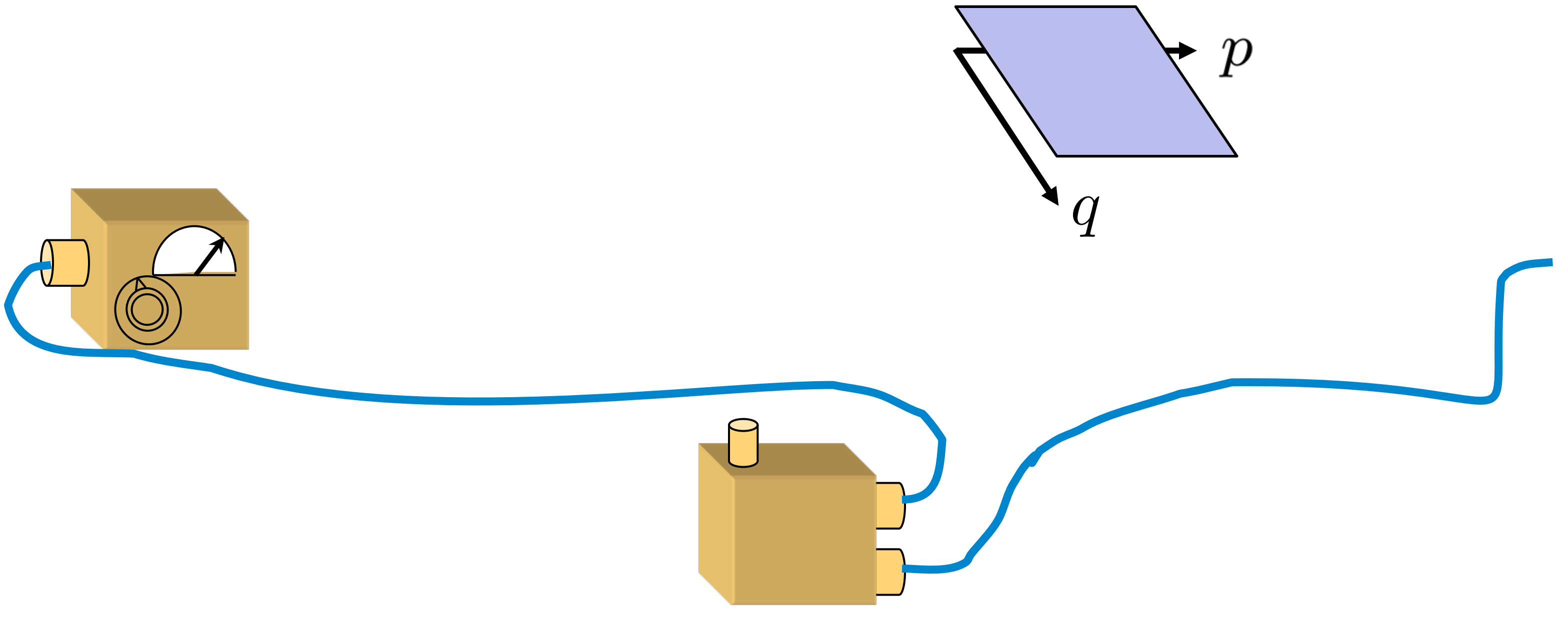
$$P(q,p) \propto \delta(p-b)$$



know $Q_A - Q_B$ and $P_A + P_B$

$$P_{\mathrm{EPR}}(q_A, p_A, q_B, p_B) \propto \delta(q_A - q_B)\delta(p_A + p_B)$$

Einstein-Podolsky-Rosen experiment



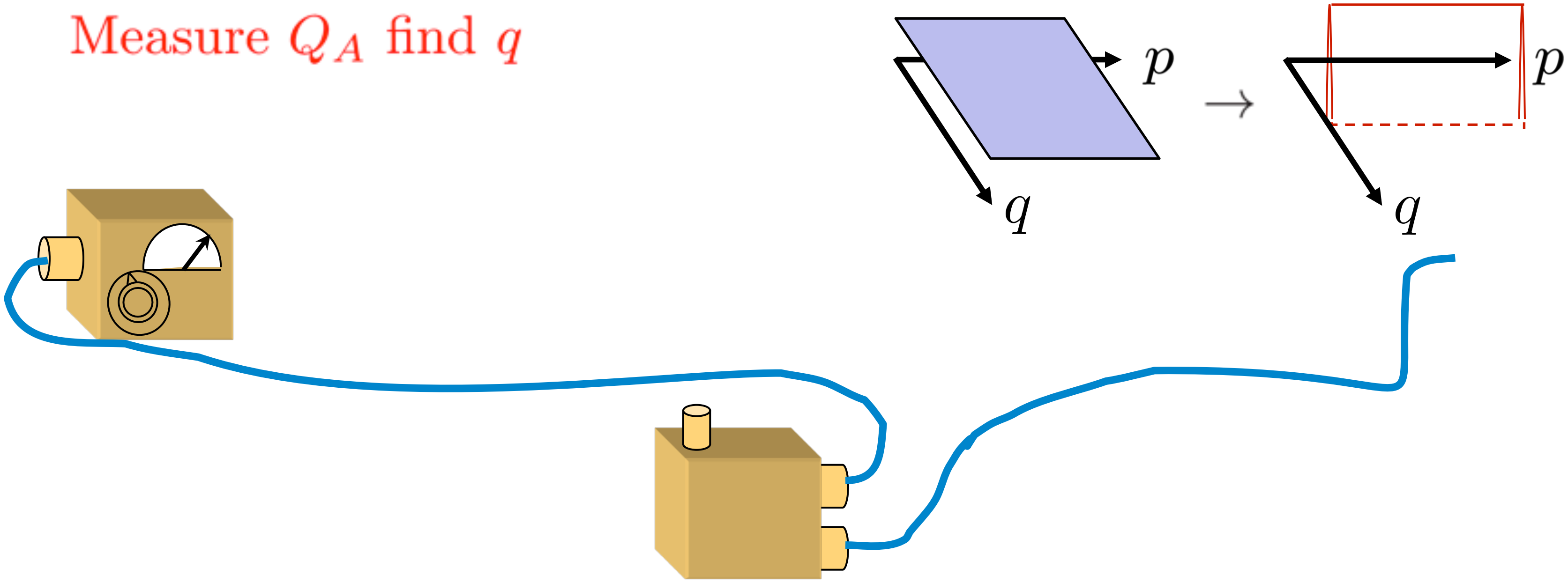
$$P_{\mathrm{EPR}}(q_A, p_A, q_B, p_B) \propto \delta(q_A - q_B)\delta(p_A + p_B)$$

$$Q_B - Q_A = 0$$

$$P_B + P_A = 0$$

Einstein-Podolsky-Rosen experiment

Measure Q_A find q

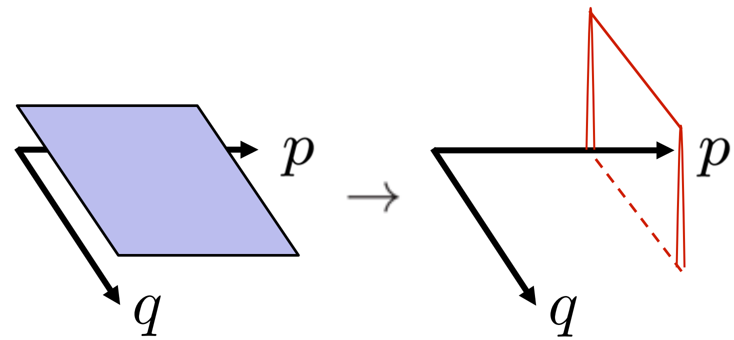


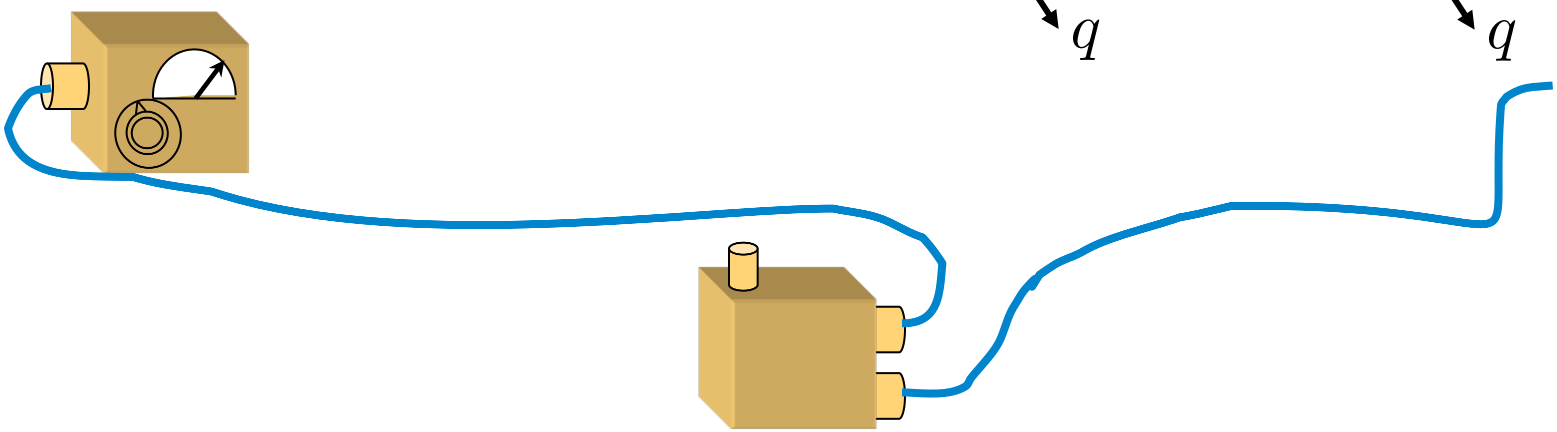
$$P_{\mathrm{EPR}}(q_A, p_A, q_B, p_B) \propto \delta(q_A - q_B)\delta(p_A + p_B)$$

$$Q_B - Q_A = 0$$
$$P_B + P_A = 0$$

Einstein-Podolsky-Rosen experiment

Measure P_A find p



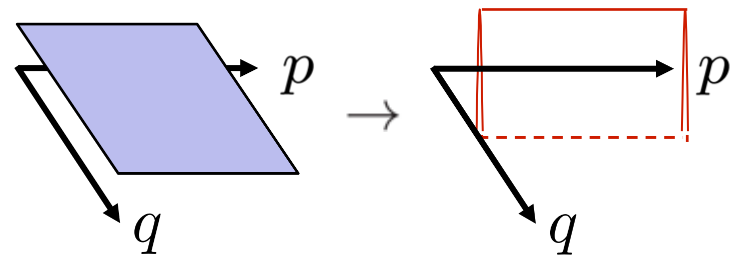


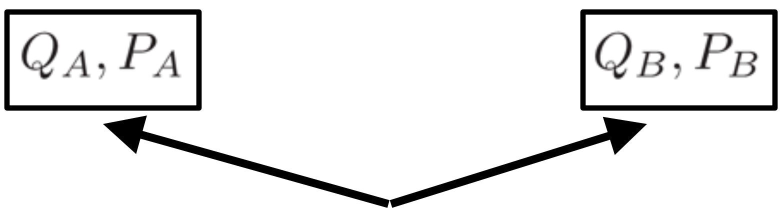
$$P_{\text{EPR}}(q_A, p_A, q_B, p_B) \propto \delta(q_A - q_B)\delta(p_A + p_B)$$

$$Q_B - Q_A = 0$$
$$P_B + P_A = 0$$

Einstein-Podolsky-Rosen experiment

Measure Q_A find q



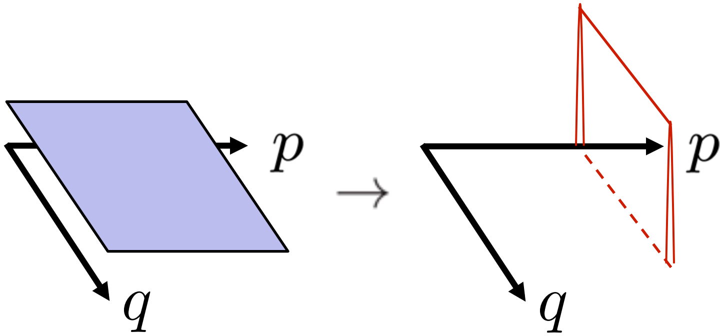


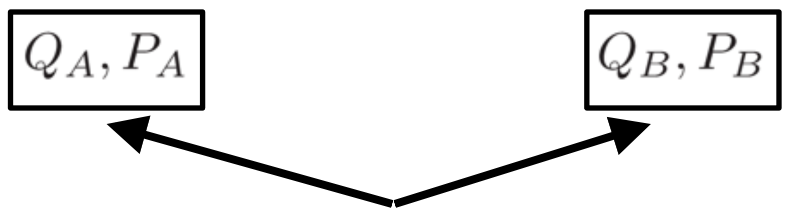
$$P_{\mathrm{EPR}}(q_A, p_A, q_B, p_B) \propto \delta(q_A - q_B)\delta(p_A + p_B)$$

$$Q_B - Q_A = 0$$
$$P_B + P_A = 0$$

Einstein-Podolsky-Rosen experiment

Measure P_A find p





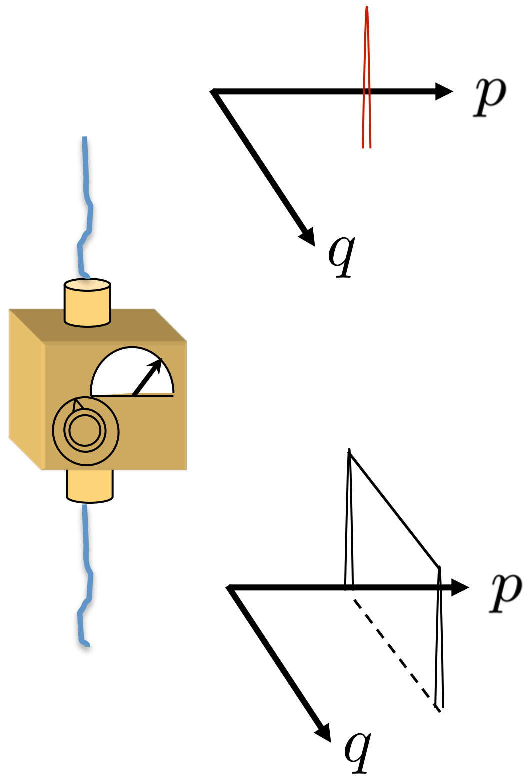
$$P_{\mathrm{EPR}}(q_A, p_A, q_B, p_B) \propto \delta(q_A - q_B)\delta(p_A + p_B)$$

$$Q_B - Q_A = 0$$

$$P_B + P_A = 0$$

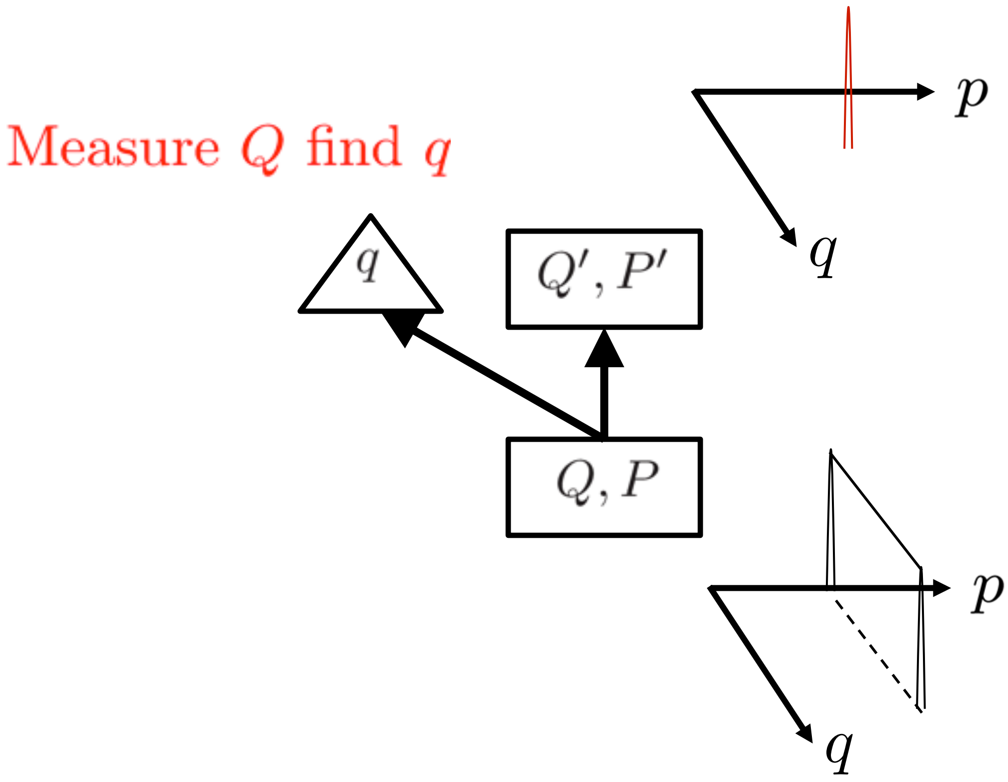
Collapse Rule

Measure Q find q



But this would violate the epistemic restriction!

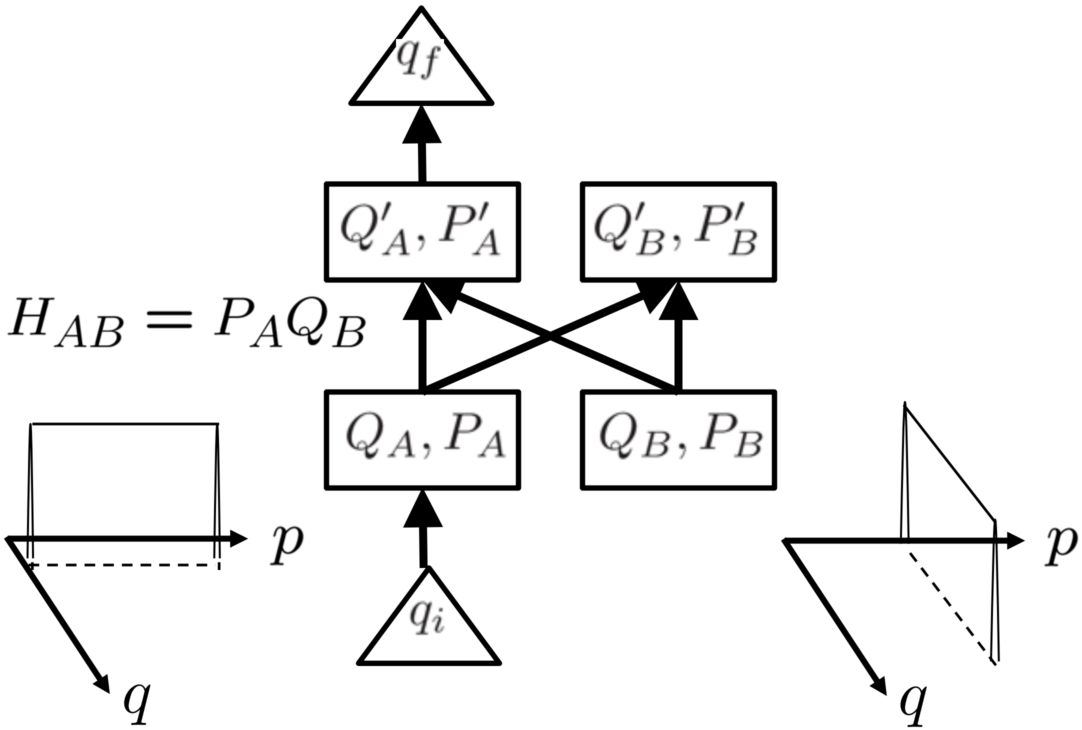
Collapse Rule



But this would violate the epistemic restriction!

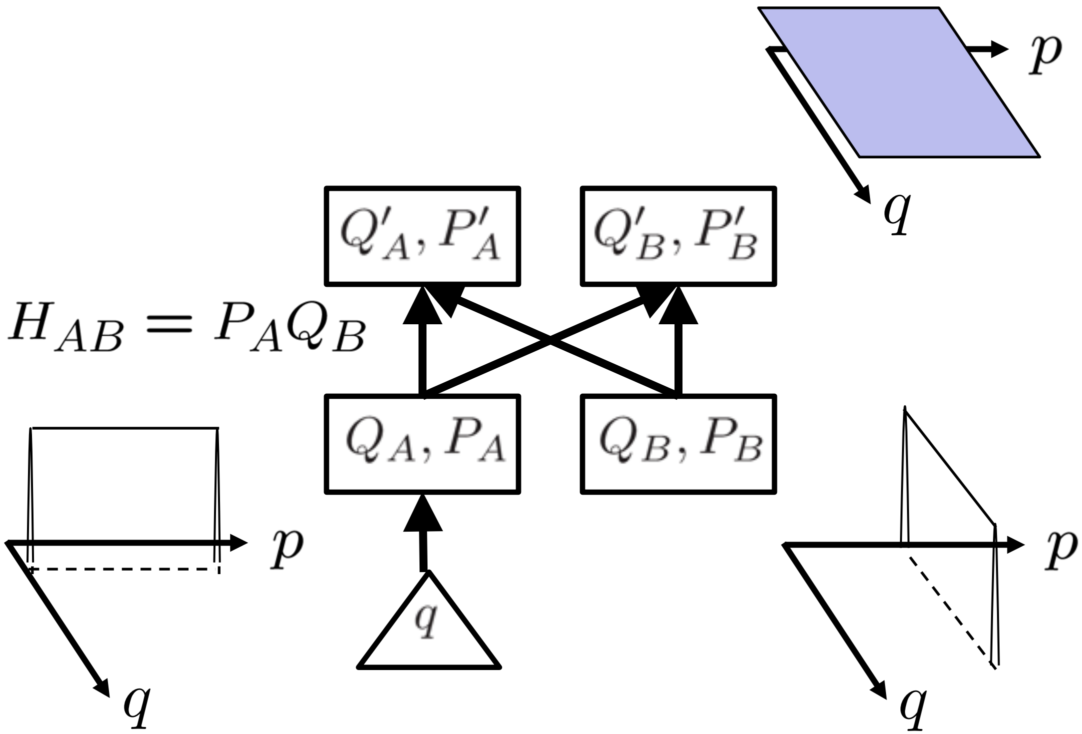
Collapse Rule

Measure Q'_A find q_f



Prepare Q_A with value q_i

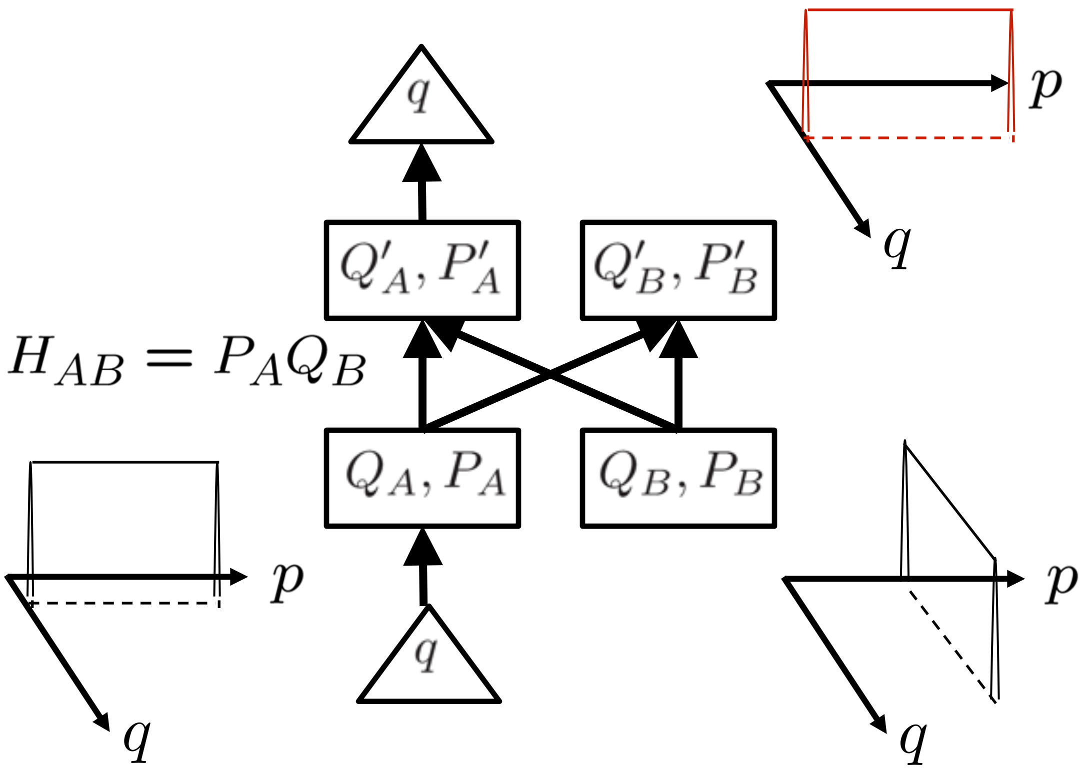
Collapse Rule



Prepare Q_A with value q_i

Collapse Rule

Measure Q'_A find q_f

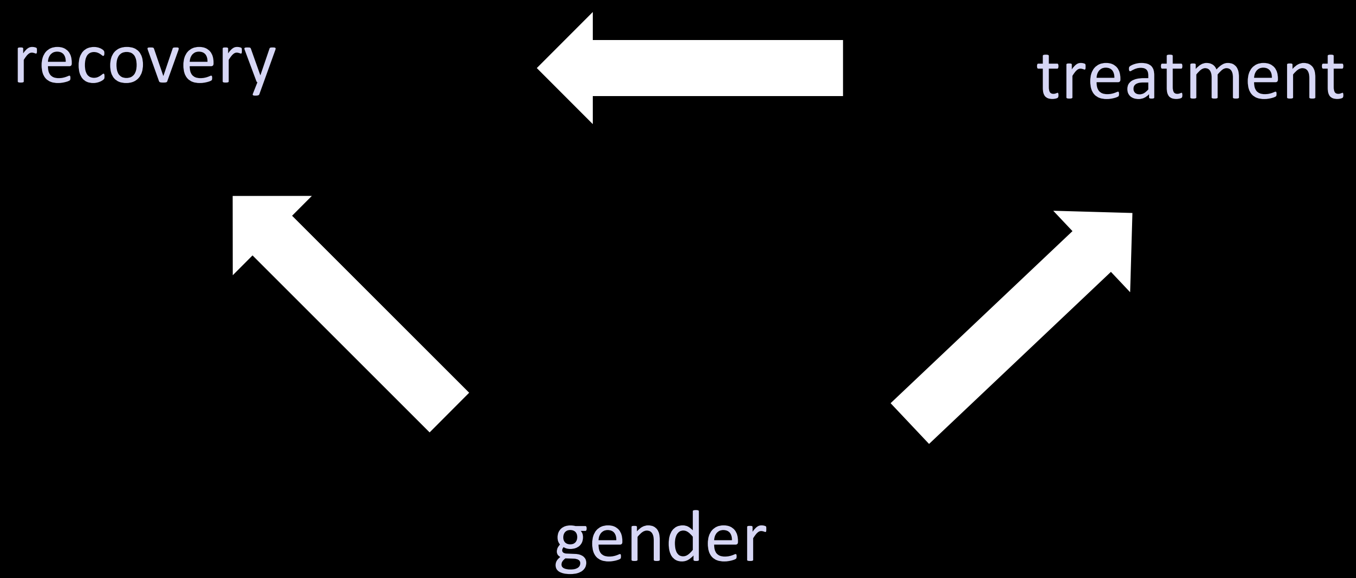


Prepare Q_A with value q_i

“But our present quantum mechanical formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature --- all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble.”

E.T. Jaynes

Simpson's Paradox



P(recovery | do (treatment)) ≠ P(recovery | observe (treatment))

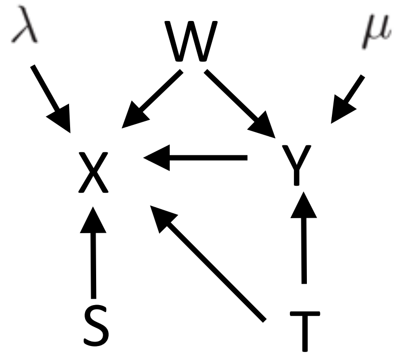
Influence inference

Brief review of causal inference algorithms

J. Pearl, Causality: Models, Reasoning and Inference
P. Spirtes, C. Glymour, R. Scheines, Causation, Prediction and Search

Functional causal model

Causal Structure

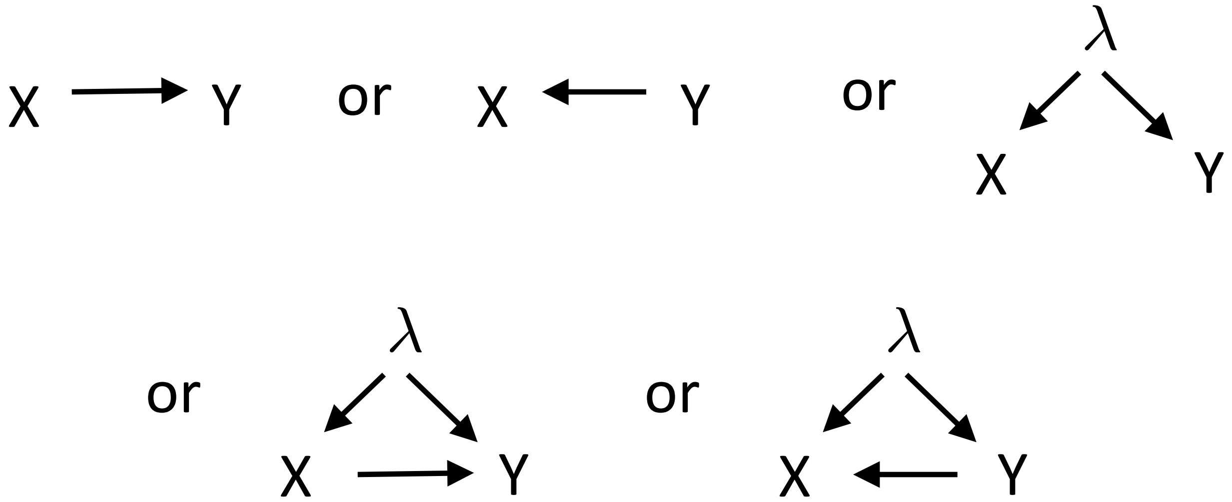


Parameters

$$P(W)$$
$$P(S)$$
$$P(T)$$
$$P(\lambda)$$
$$P(\mu)$$
$$X = f(S, T, W, Y, \lambda)$$
$$Y = g(T, W, \mu)$$

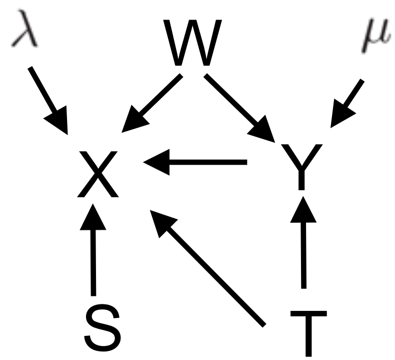
Reichenbach’s principle

If X and Y are dependent, then



Functional causal model

Causal Structure



Parameters

$P(W)$

$P(S)$

$P(T)$

$P(\lambda)$

$P(\mu)$

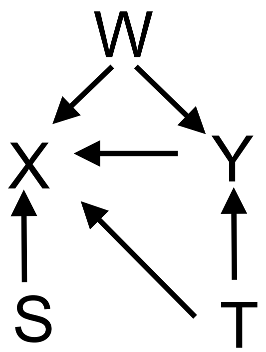
$X = f(S, T, W, Y, \lambda)$

$Y = g(T, W, \mu)$

- Parentless variables are independently distributed

Causal model

Causal Structure



Parameters

$P(W)$

$P(S)$

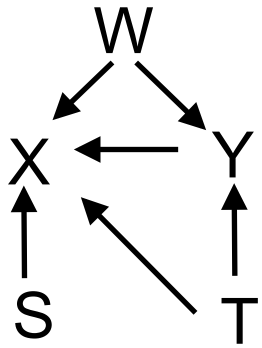
$P(T)$

$P(X|S, T, W, Y)$

$P(Y|T, W)$

Causal model

Causal Structure



Parameters

$P(W)$

$P(S)$

$P(T)$

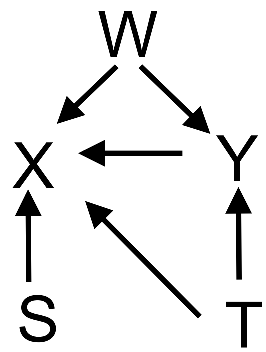
$P(X|S,T,W,Y)$

$P(Y|T,W)$

$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

Causal model

Causal Structure



Parameters

$P(W)$

$P(S)$

$P(T)$

$P(X|S,T,W,Y)$

$P(Y|T,W)$

$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

Causal inference algorithms seek to solve the inverse problem

Inferring facts about the causal structure from the conditional independences

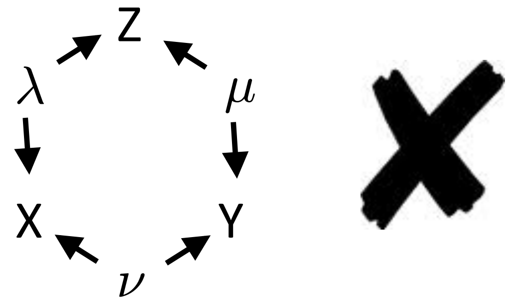
Faithfulness (No fine-tuning)

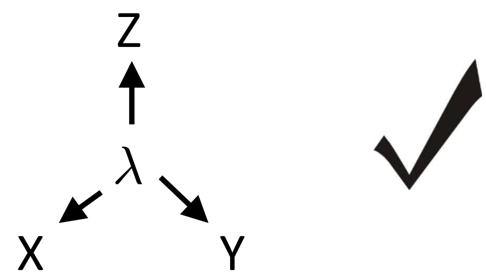
A causal model of an observed distribution is fine-tuned if the conditional independences in the distribution only hold for a set of measure zero of the values of the parameters in the model

Inferring facts about the causal structure from the strength of correlations

Strength of Correlations

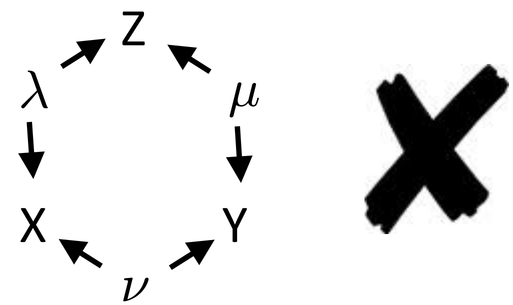
$$P(X,Y,Z) = \tfrac{1}{2}[000] + \tfrac{1}{2}[111]$$

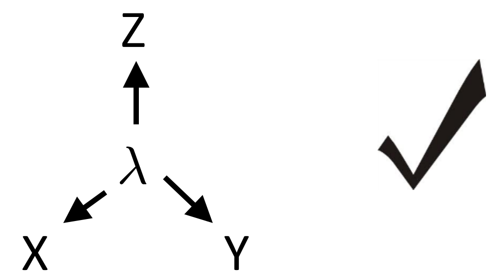




Strength of Correlations

$$P(X,Y,Z) = (1-\epsilon)\left(\tfrac{1}{2}[000] + \tfrac{1}{2}[111]\right) + \epsilon(\text{other})$$

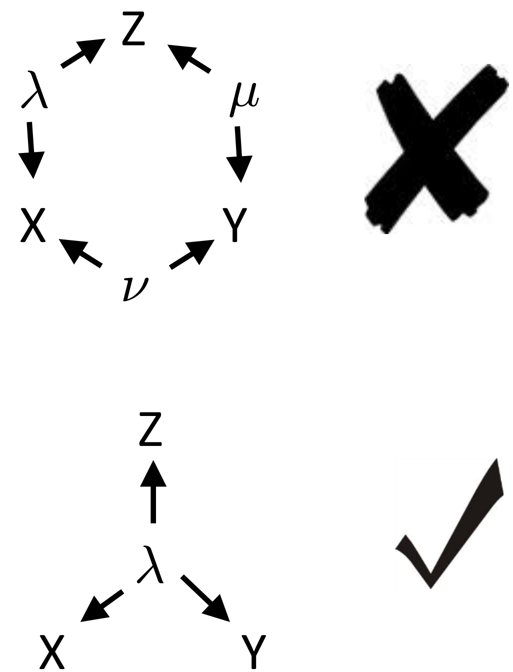




Janzing and Beth, IJQI 4, 347 (2006)
Steudel and Ay, arXiv:1010:5720
Fritz, New J. Phys. 14, 103001 (2012)
Branciard, Rosset, Gisin, Pironio, PRA 85, 3 (2012)

Strength of Correlations

$$P(X,Y,Z) = \frac{1}{3}[001] + \frac{1}{3}[010] + \frac{1}{3}[100]$$



Joint work with Matt Pusey, Tobias Fritz, and Wah Loon Keng

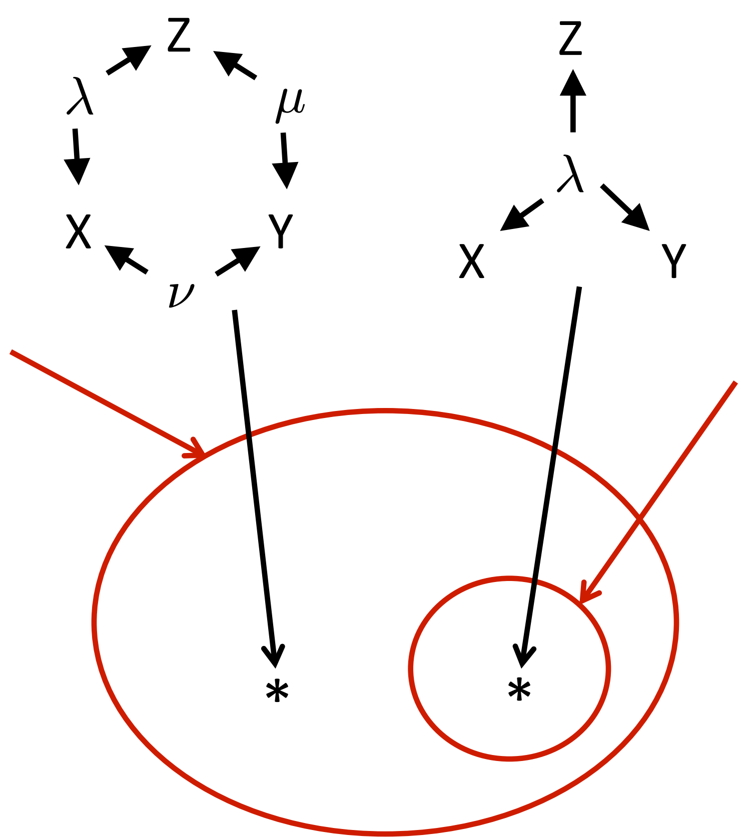
A deficiency of many causal inference algorithms

Certain versions of Occam's razor lead to incorrect causal conclusions

E.g. T. S. Verma, Technical Report R-191, Univ. of California (1993).

$$P(X,Y,Z) = \tfrac{1}{3}[001] + \tfrac{1}{3}[010] + \tfrac{1}{3}[100]$$

Set of CI relations among X, Y, Z is the empty set



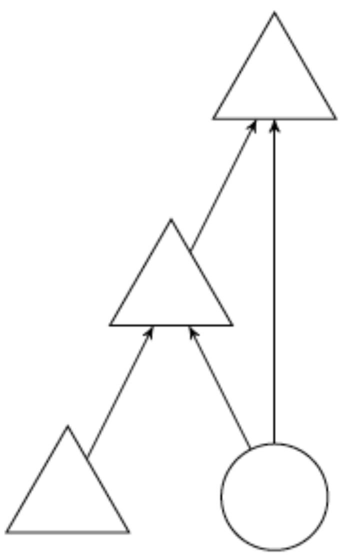
Set of faithful causal models for the given set of CI relations on observed variables

Set of faithful causal models for the given probability distribution over the observed variables

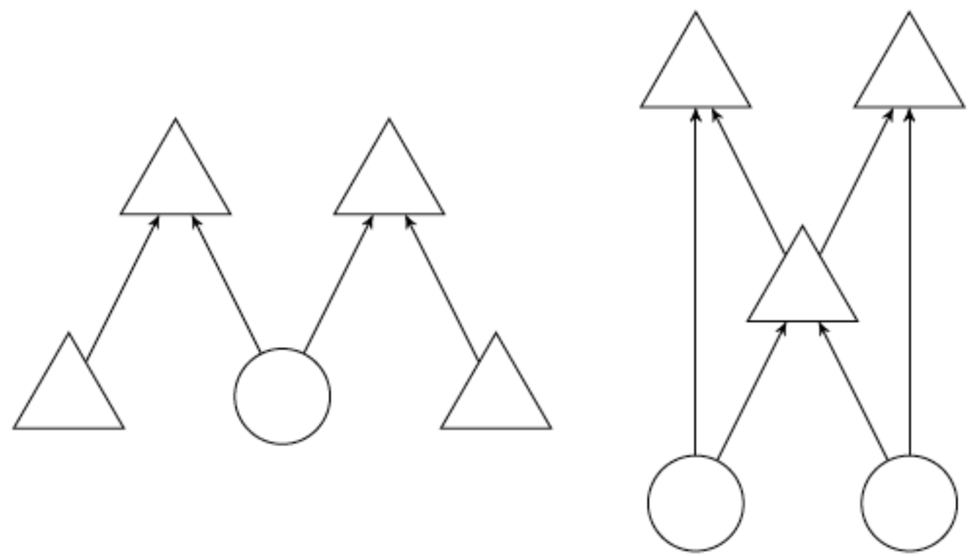
What are the causal structures for which CI relations do not capture all the constraints on the observed distribution?

A sufficient condition was found in:
Henson, Lal, Pusey, arXiv:1405.2572

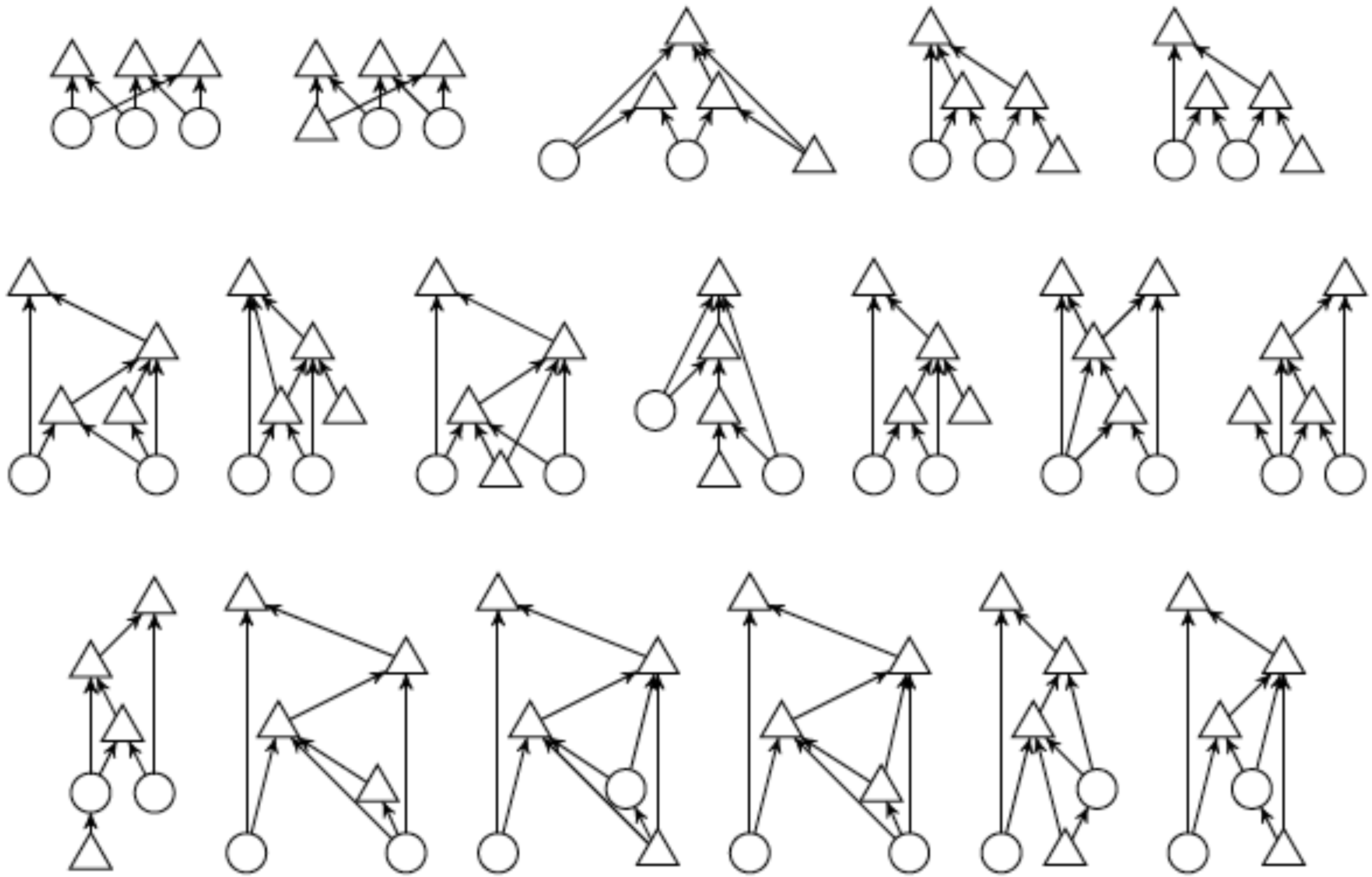
4 nodes



5 nodes

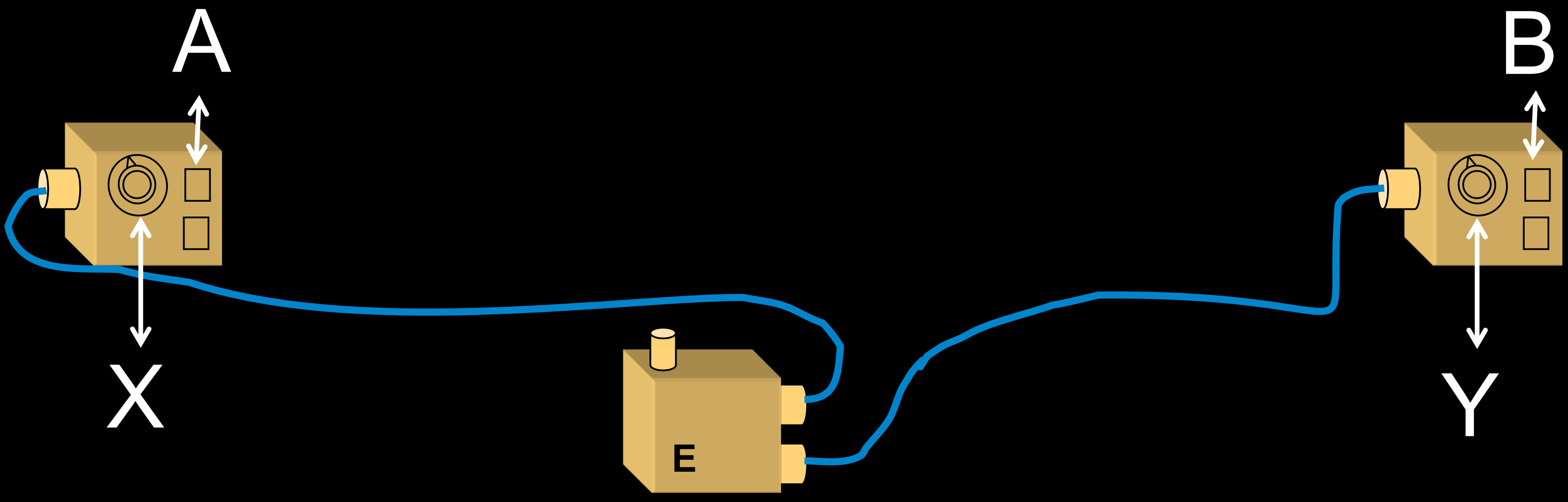


6 nodes

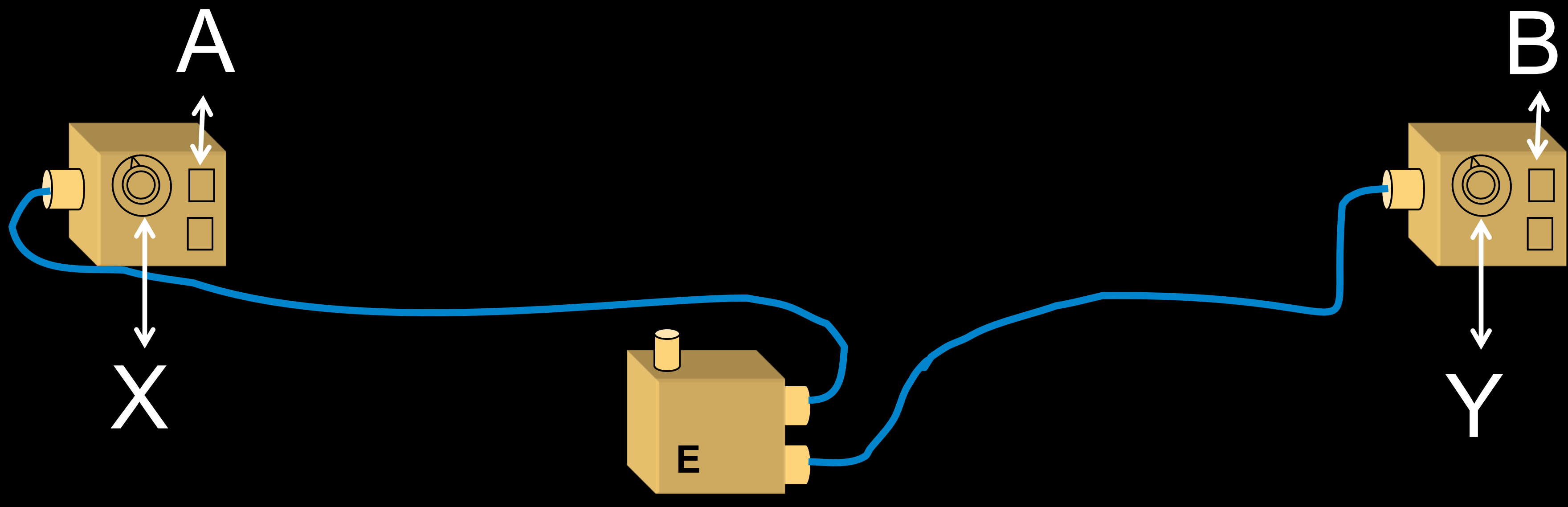


Can we find a causal explanation of quantum correlations?

Chris Wood and RWS, arXiv:1208.4119

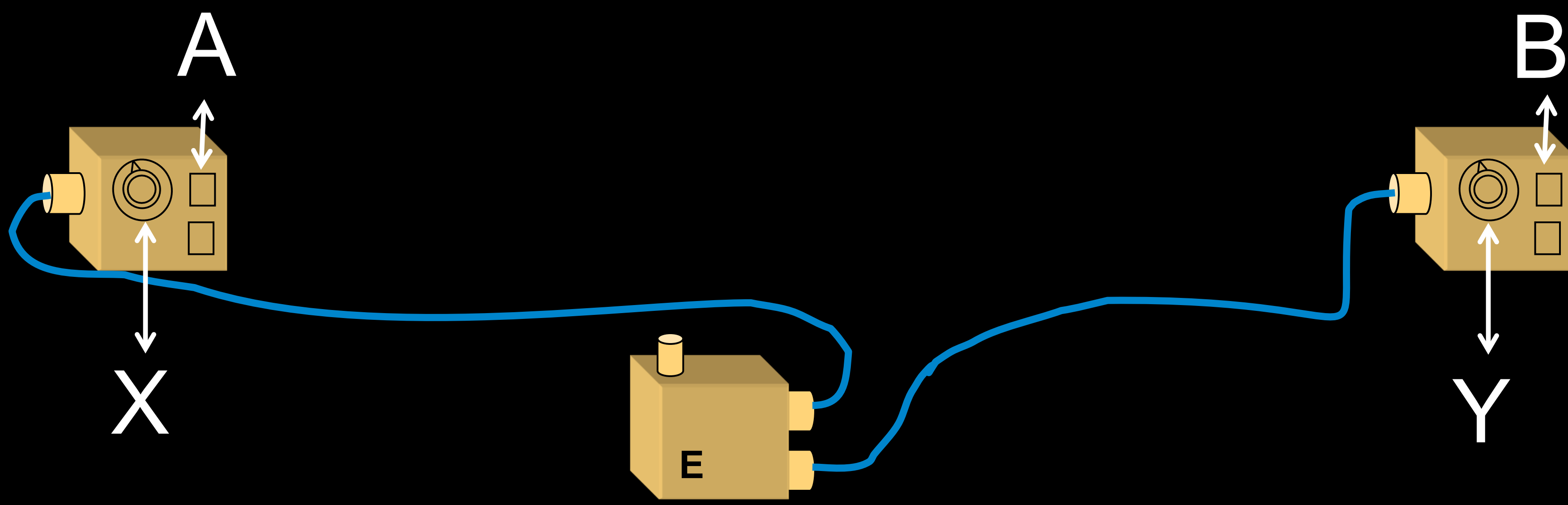


What P(A,B,X,Y) is observed?



$$P(X,Y)$$
$$= (\tfrac{1}{2}[0] + \tfrac{1}{2}[1])(\tfrac{1}{2}[0] + \tfrac{1}{2}[1])$$

$$P(A,B|X,Y)$$
$$= \tfrac{1}{2}[00] + \tfrac{1}{2}[11] \quad \text{if } XY = 0$$
$$= \tfrac{1}{2}[01] + \tfrac{1}{2}[10] \quad \text{if } XY = 1$$



$$P(X,Y)$$

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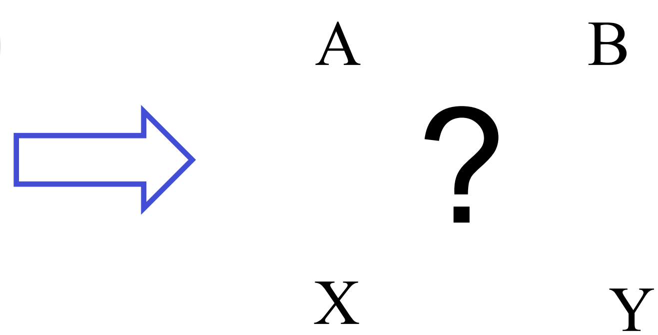

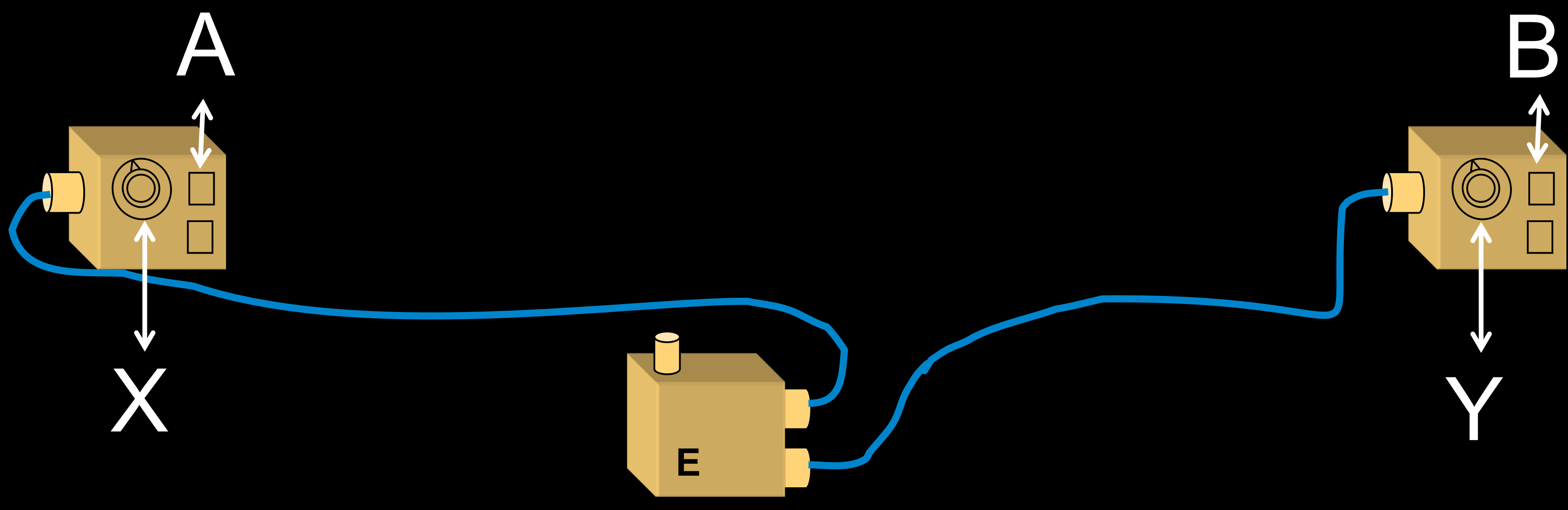

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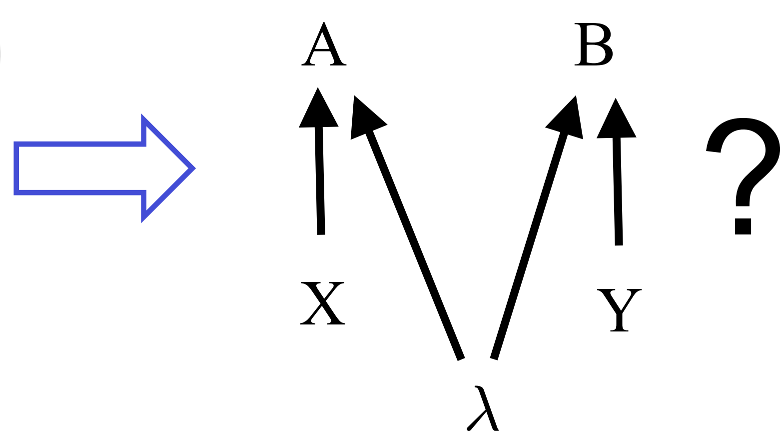
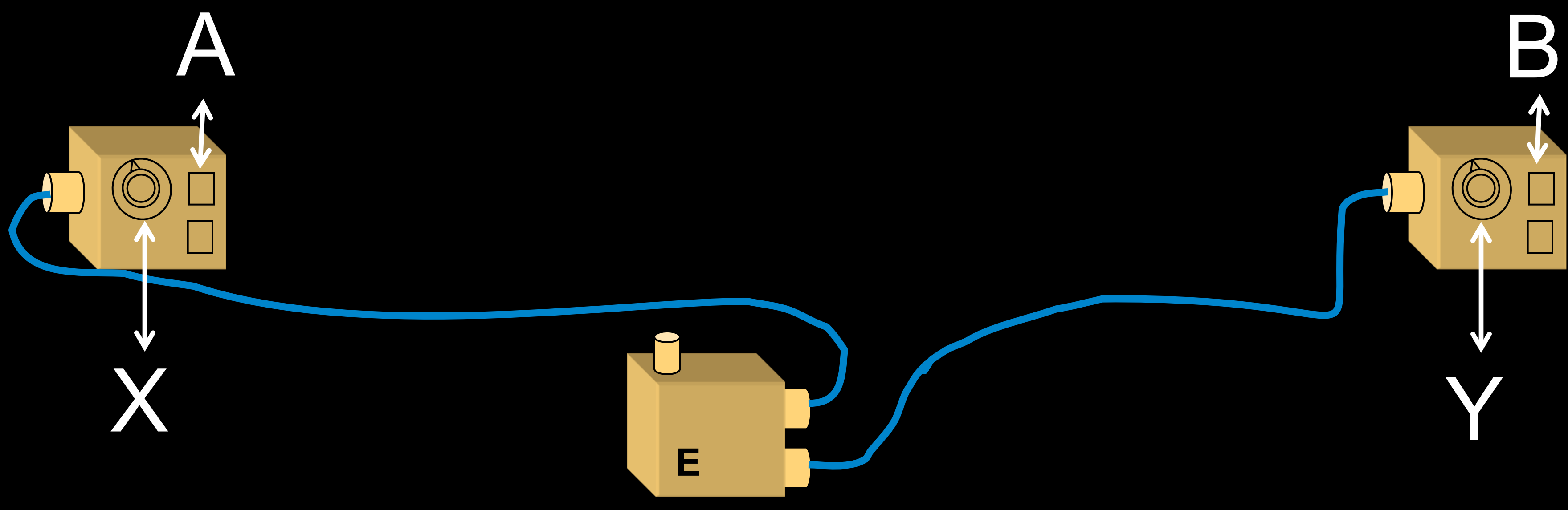
$$= \left(\tfrac{1}{2}[0] + \tfrac{1}{2}[1]\right)\left(\tfrac{1}{2}[0] + \tfrac{1}{2}[1]\right)$$

$$P(A,B|X,Y)$$

$$= \tfrac{1}{2}[00] + \tfrac{1}{2}[11] \quad \text{if } XY = 0$$

$$= \tfrac{1}{2}[01] + \tfrac{1}{2}[10] \quad \text{if } XY = 1$$





$P(X,Y)$

$$= (\tfrac{1}{2}[0] + \tfrac{1}{2}[1])(\tfrac{1}{2}[0] + \tfrac{1}{2}[1])$$

$P(A,B|X,Y)$

$$= \tfrac{1}{2}[00] + \tfrac{1}{2}[11] \quad \text{if } XY = 0$$

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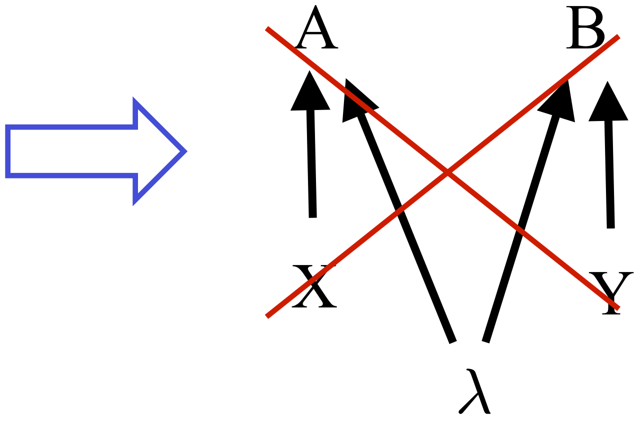

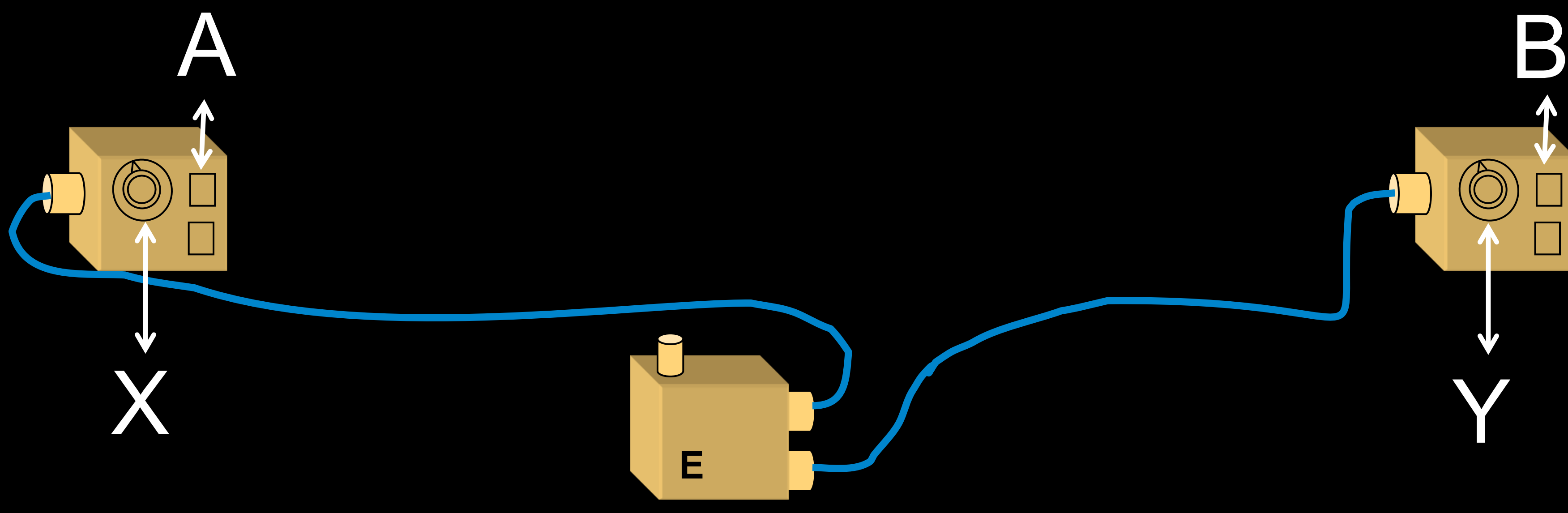

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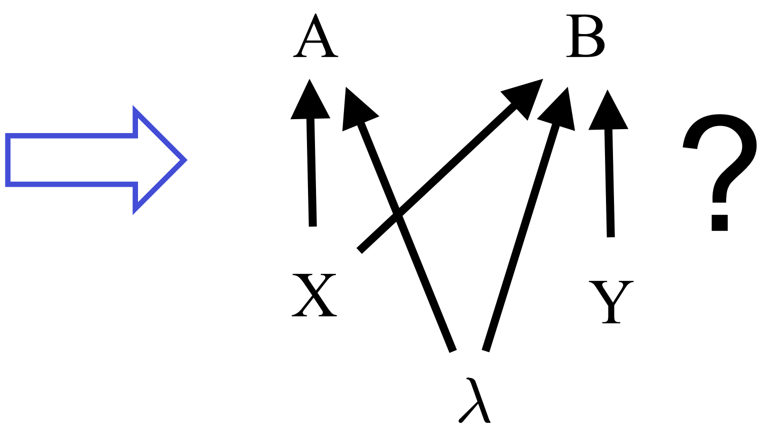

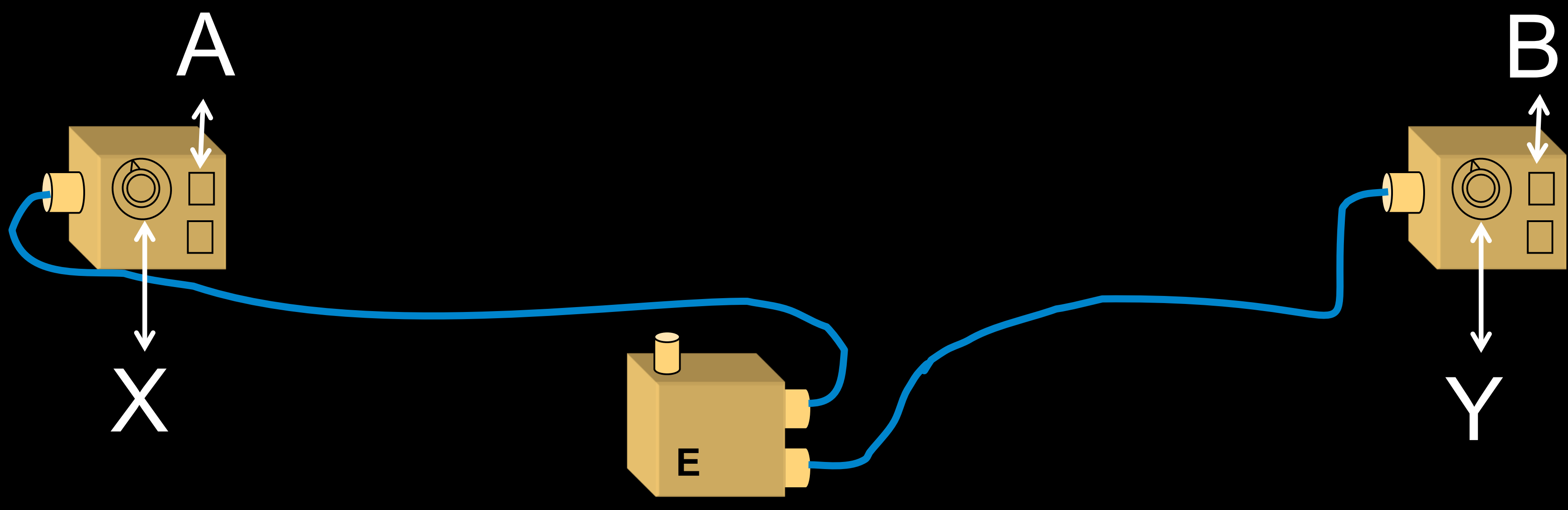

$P(X,Y)$

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$P(A, \ldots$

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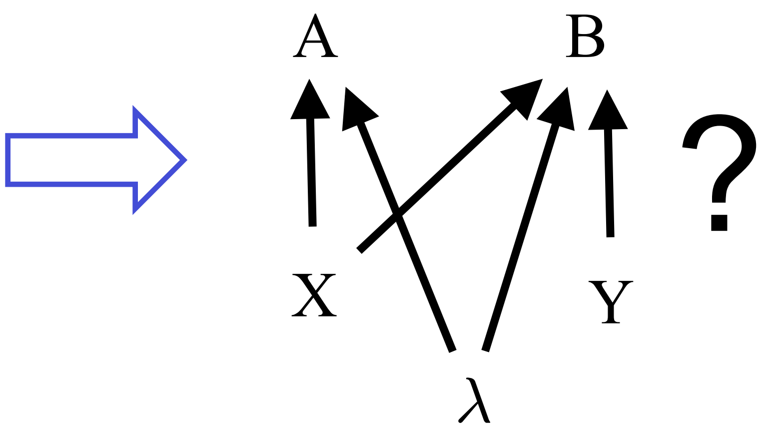
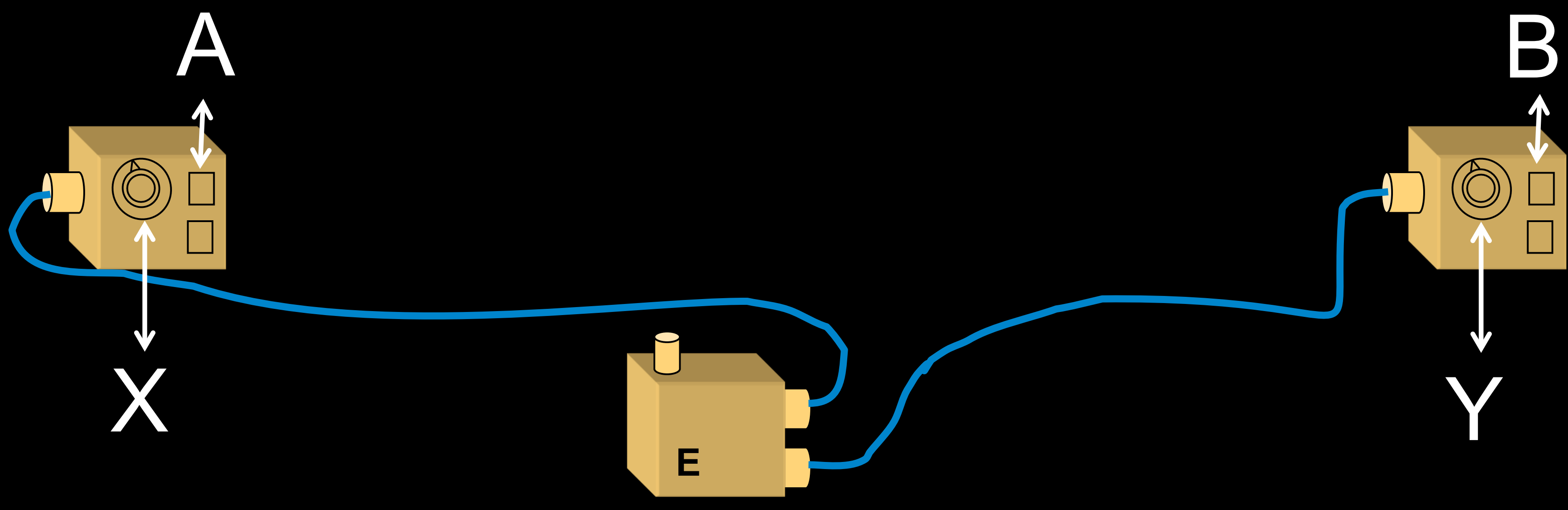
$= \frac{1}{2}[01] + \frac{1}{2}[10] \quad \text{if } XY = 1$

$X \perp Y$

$B \perp X | Y$

$A \perp Y | X$





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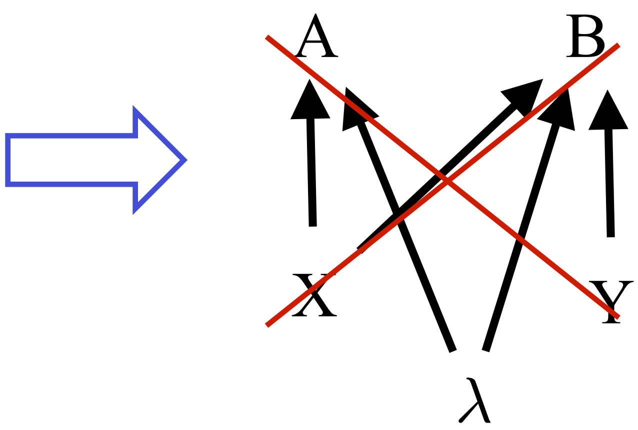
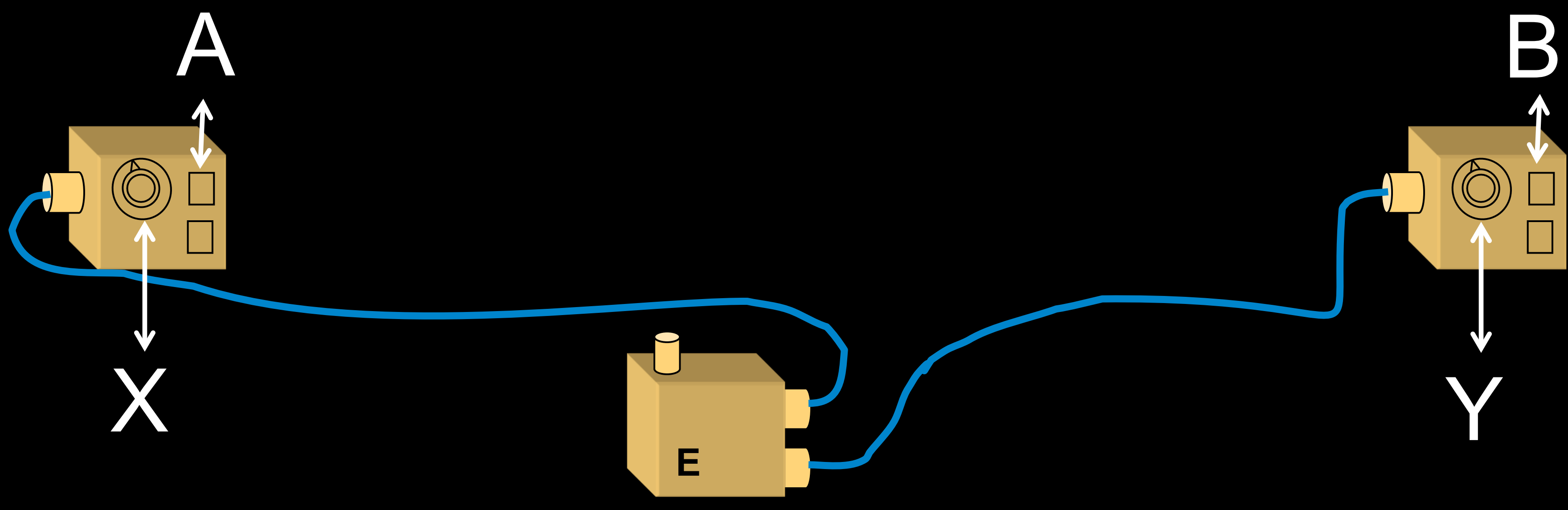
$= \frac{1}{2}[01] + \frac{1}{2}[10] \quad \text{if } XY = 1$





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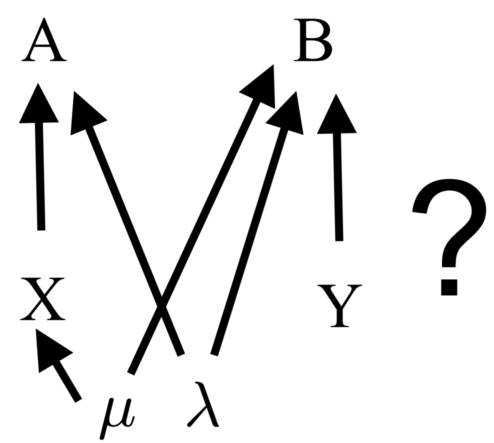

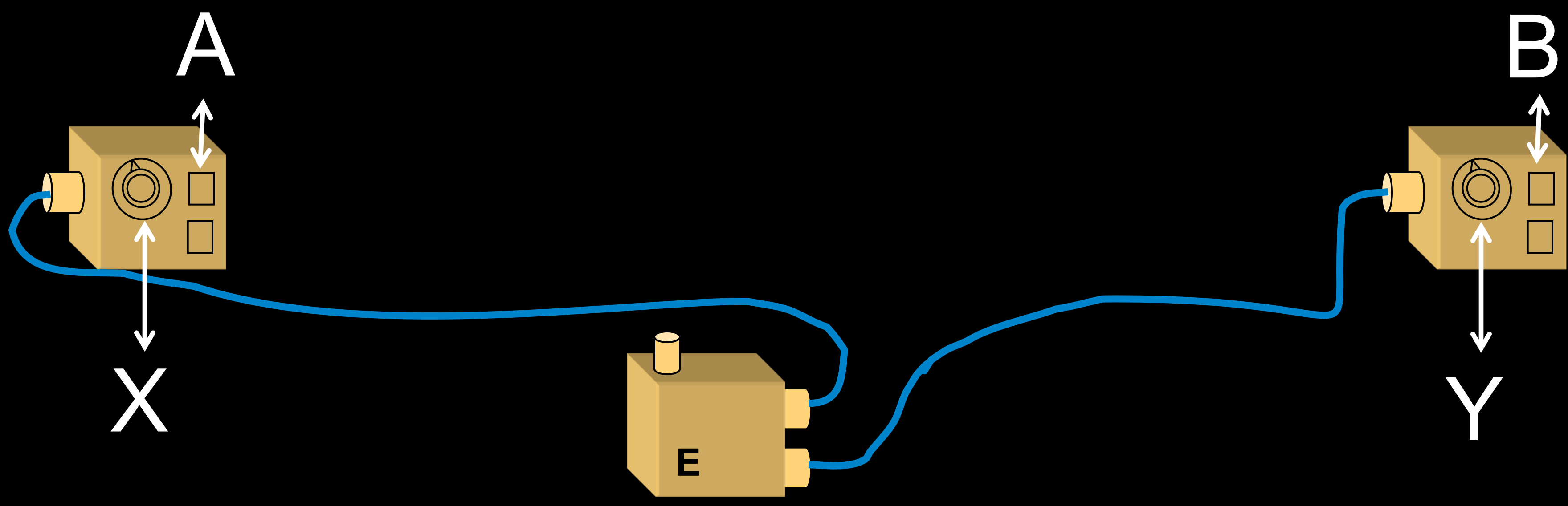

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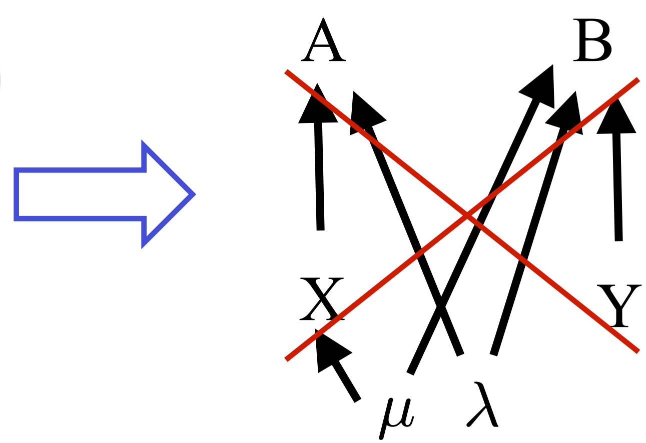

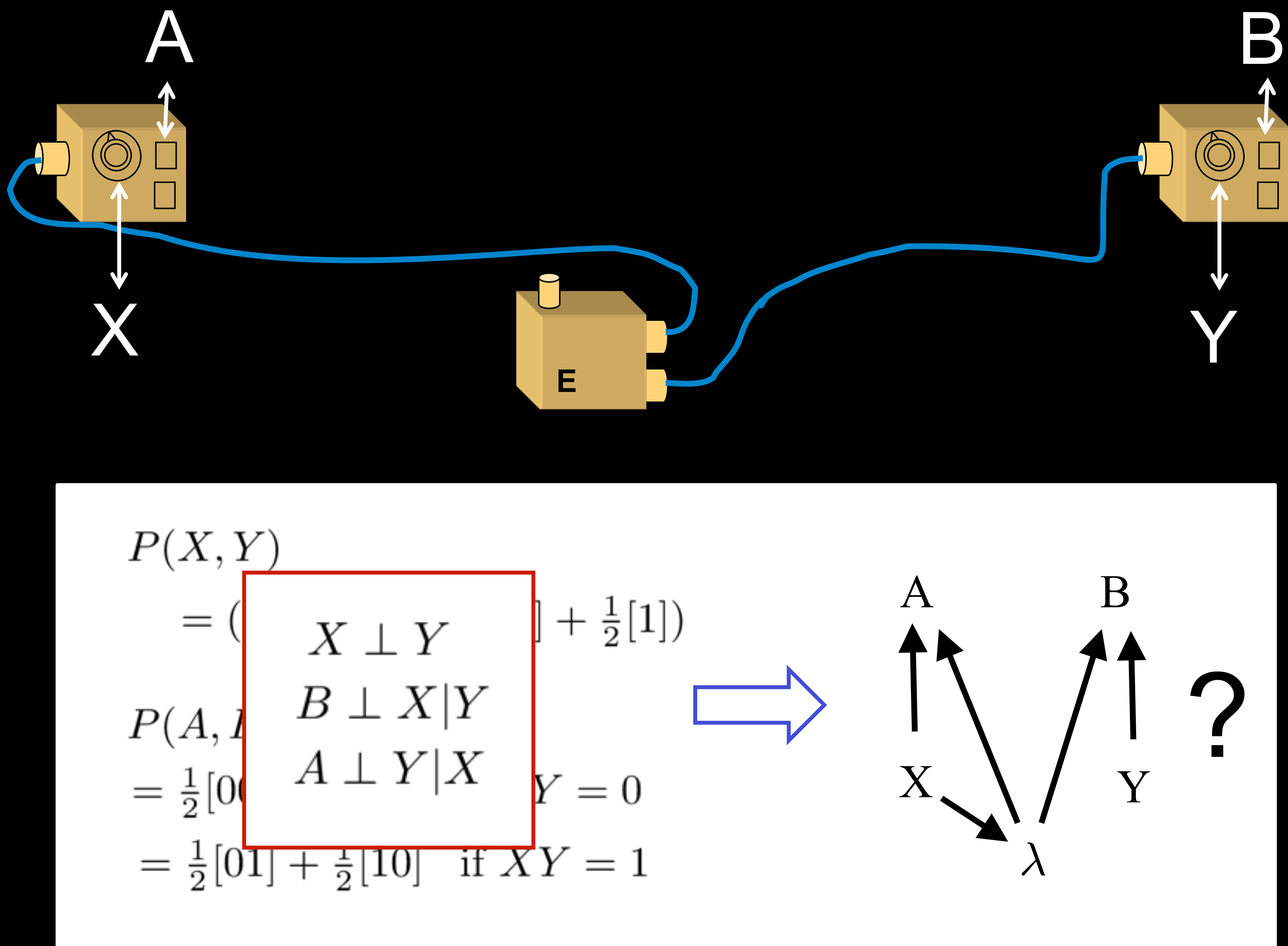
A
B
X
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P(X, Y)
= ( ] + ½[1])
X ⊥ Y
B ⊥ X|Y
A ⊥ Y|X
P(A,
= ½[0 Y = 0
= ½[01] + ½[10] if XY = 1
A
B
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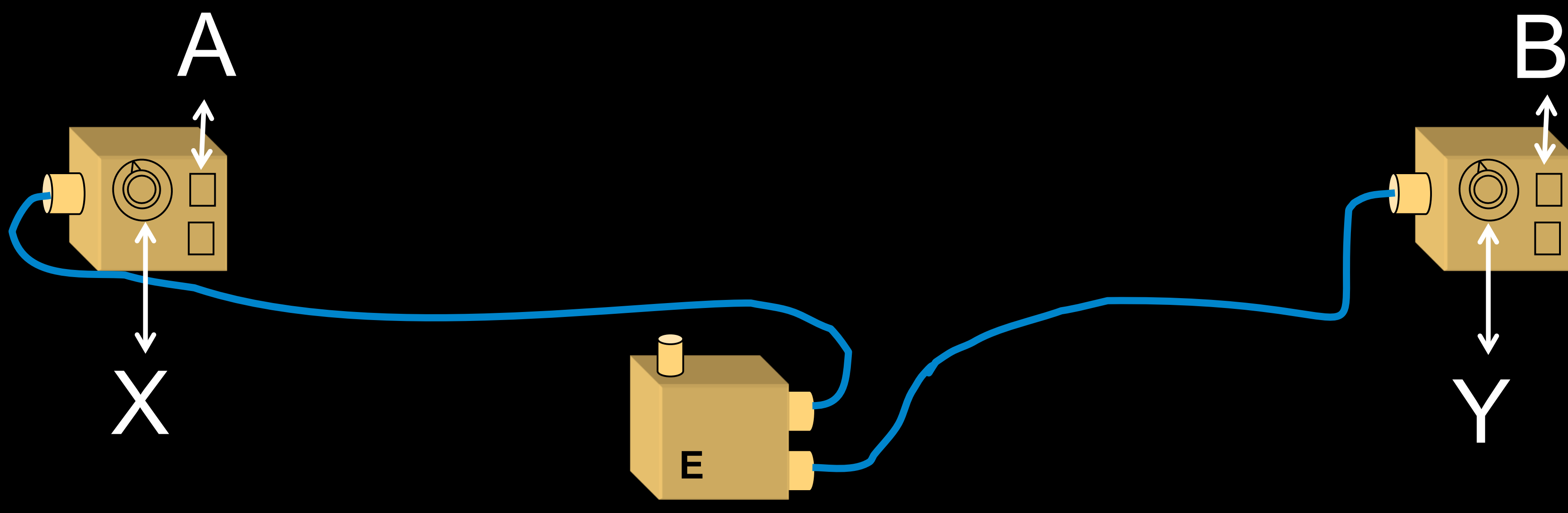
Y
λ
?



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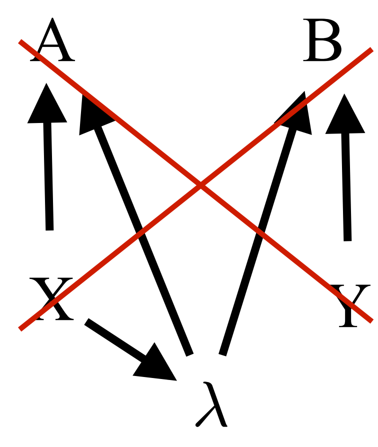

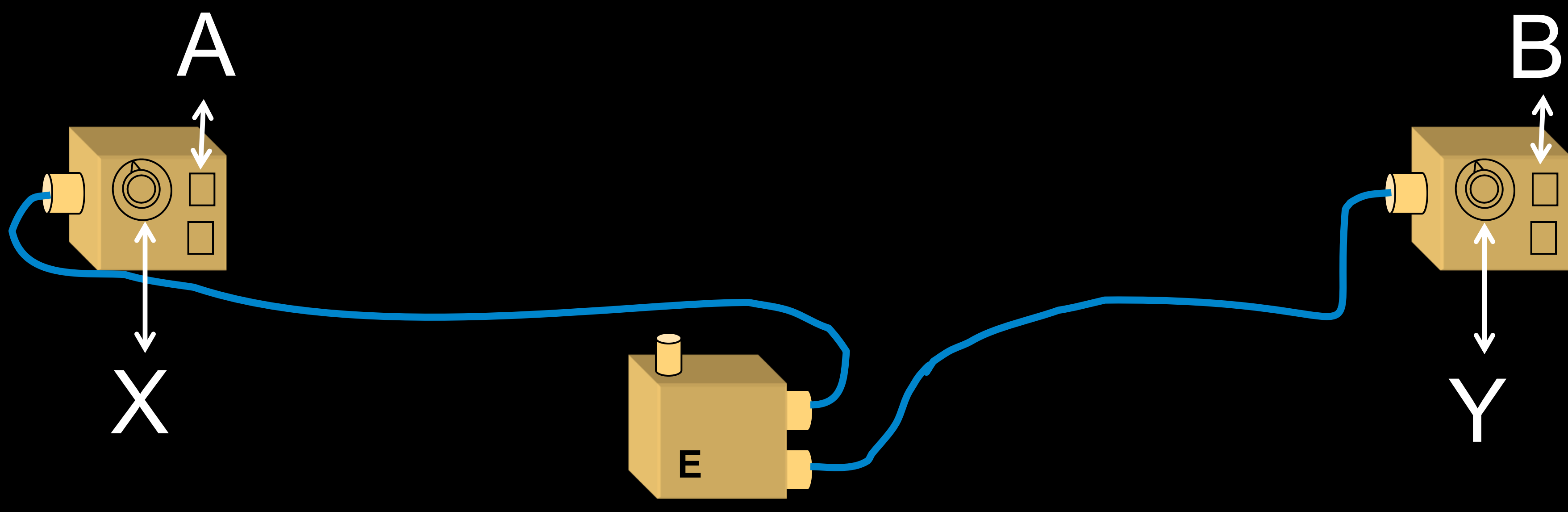

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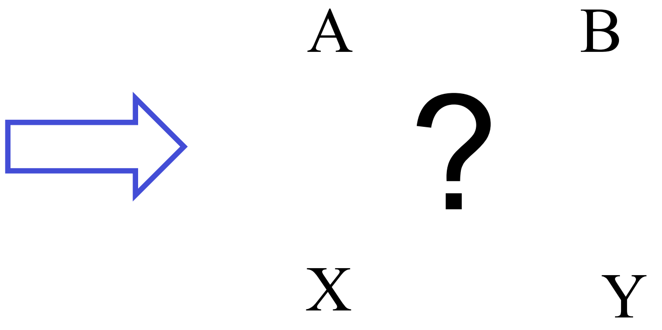

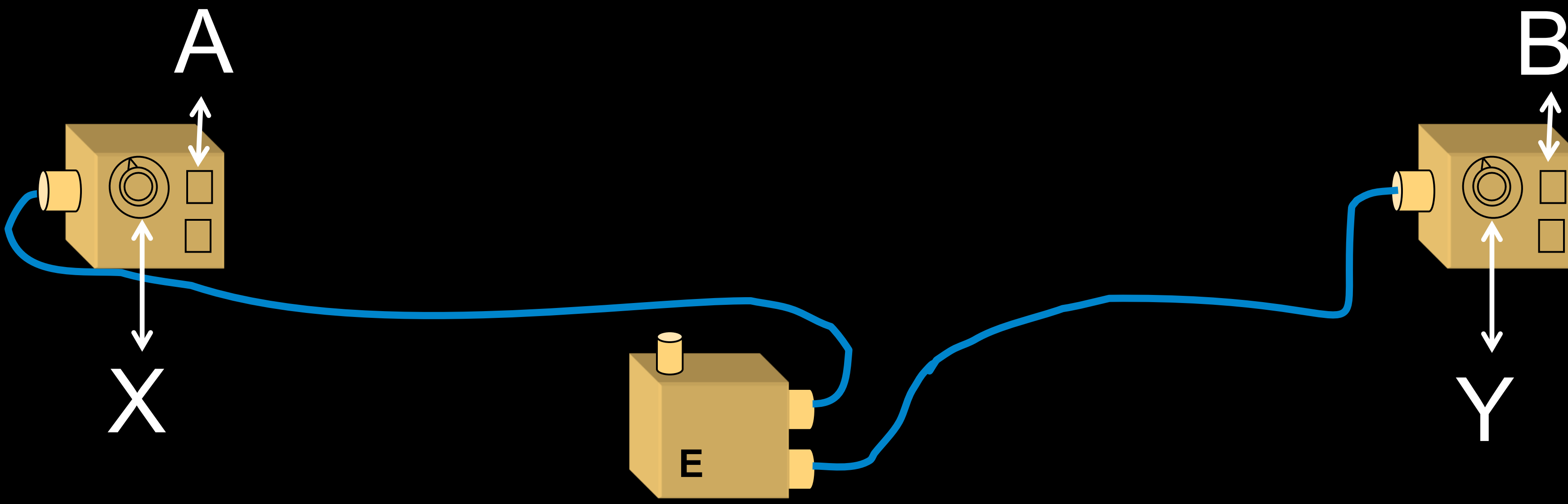

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$= ($ … $] + \frac{1}{2}[1])$

$P(A, I$ …

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$= \frac{1}{2}[01] + \frac{1}{2}[10] \quad \text{if } XY = 1$

$X \perp Y$

$B \perp X | Y$

$A \perp Y | X$

Nothing works!

- Reichenbach's principle

- No fine-tuning

⇩

Contradiction with

$$P(X,Y) = (\tfrac{1}{2}[0] + \tfrac{1}{2}[1])(\tfrac{1}{2}[0] + \tfrac{1}{2}[1])$$

$$P(A,B|X,Y) = \tfrac{1}{2}[00] + \tfrac{1}{2}[11] \quad \text{if } XY = 0$$
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$$X \perp Y$$
$$B \perp X|Y$$
$$A \perp Y|X$$

- Reichenbach’s principle
- No fine-tuning

$$\Downarrow$$

Contradiction with quantum theory and experiment

- Reichenbach’s principle

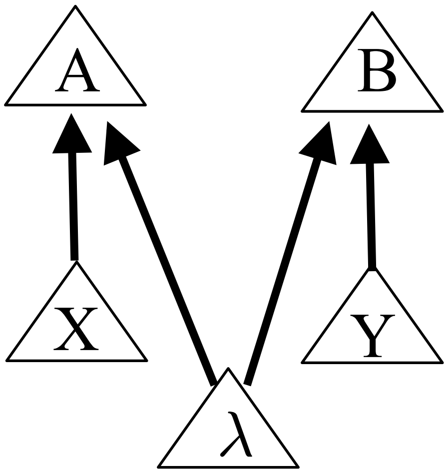
- No fine-tuning

- In the causal model, unobserved nodes are described by classical variables and our knowledge of these is described by classical probability theory

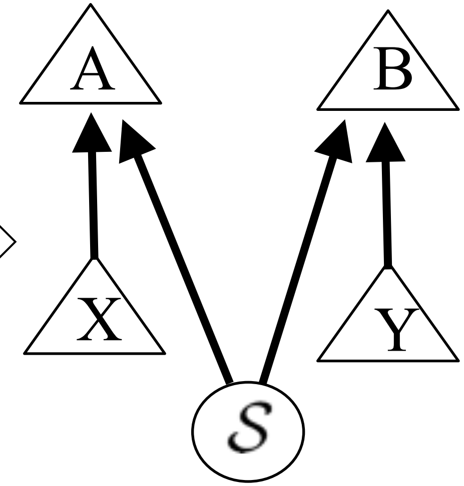
⇓

Contradiction with quantum theory and experiment

Quantum Causal Models



$P(X)$
$P(Y)$
$P(\lambda)$
$P(A|\lambda, X)$
$P(B|\lambda, Y)$



ρ_X
ρ_Y
$\rho_{\mathcal{S}}$
$\rho_{A|X\mathcal{S}}$
$\rho_{B|Y\mathcal{S}}$

$$P(A,B|X,Y) = \sum_\lambda P(A|\lambda, X) P(B|\lambda, Y) P(\lambda)$$

$$\rho_{AB|XY} = \mathrm{Tr}_{\mathcal{S}}(\rho_{A|X\mathcal{S}}\rho_{B|Y\mathcal{S}}\rho_{\mathcal{S}})$$

Cannot reproduce the quantum correlations

Can reproduce the quantum correlations

See: Leifer and RWS,
Phys. Rev. A 88, 052130 (2013)

Quantum conditional independence

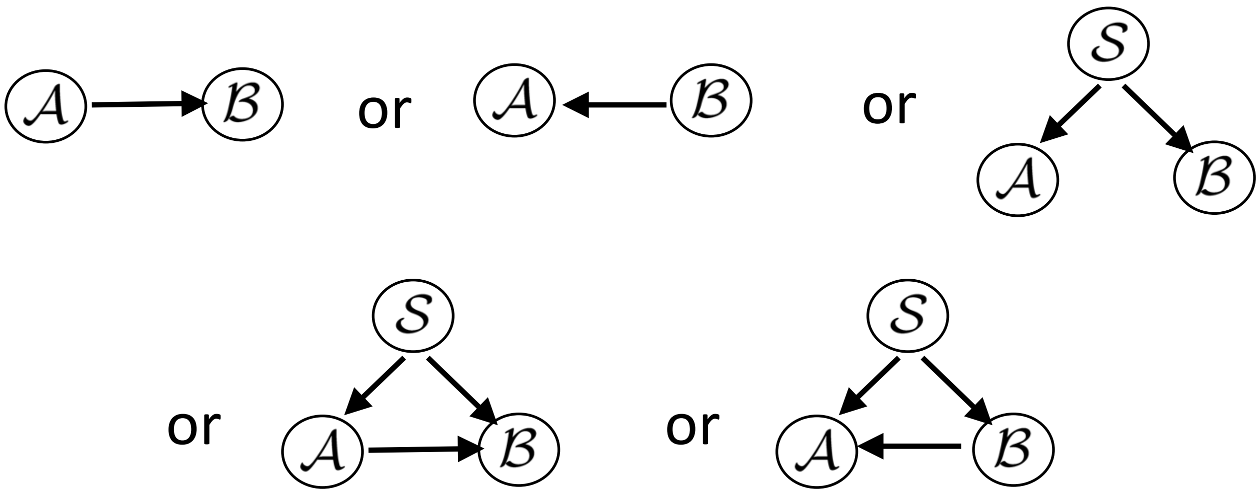
$$\rho_{\mathcal{A}|\mathcal{B}\mathcal{C}} = \rho_{\mathcal{A}|\mathcal{C}}$$
$$\rho_{\mathcal{B}|\mathcal{A}\mathcal{C}} = \rho_{\mathcal{B}|\mathcal{C}}$$
$$\rho_{\mathcal{A}\mathcal{B}|\mathcal{C}} = \rho_{\mathcal{A}|\mathcal{C}}\rho_{\mathcal{B}|\mathcal{C}}$$

Denote this $(\mathcal{A} \perp \mathcal{B}|\mathcal{C})$

Actually, it is only this simple in special cases!

Modified Reichenbach's principle

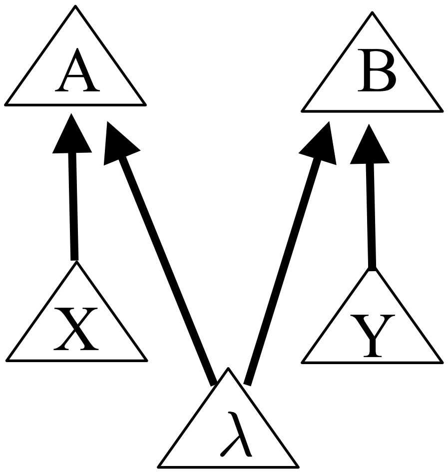
If $\mathcal{A}$ and $\mathcal{B}$ are quantum dependent, then



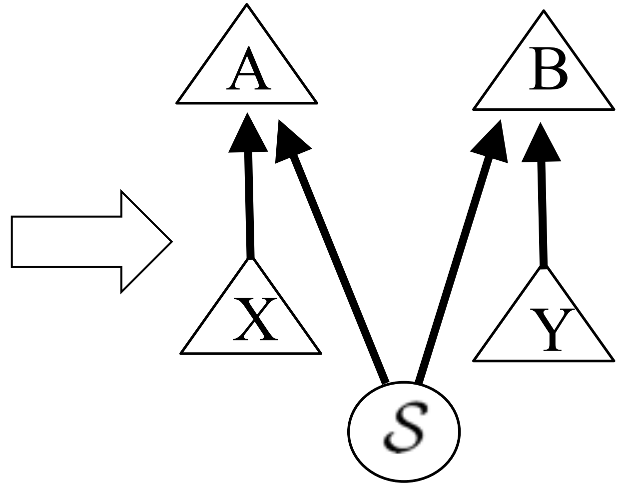
Modified Faithfulness (No fine-tuning)

A quantum causal model underlying an observed quantum state is unfaithful if the quantum conditional independences in the observed quantum state only hold for a set of measure zero of the values of the parameters in the model.

Quantum Causal Models



$P(X)$
$P(Y)$
$P(\lambda)$
$P(A|\lambda, X)$
$P(B|\lambda, Y)$



ρ_X
ρ_Y
$\rho_{\mathcal{S}}$
$\rho_{A|X\mathcal{S}}$
$\rho_{B|Y\mathcal{S}}$

$$P(A, B|X, Y) = \sum_\lambda P(A|\lambda, X)P(B|\lambda, Y)P(\lambda)$$

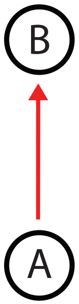
$$\rho_{AB|XY} = \mathrm{Tr}_{\mathcal{S}}(\rho_{A|X\mathcal{S}}\rho_{B|Y\mathcal{S}}\rho_{\mathcal{S}})$$

A Quantum Advantage for Causal Inference

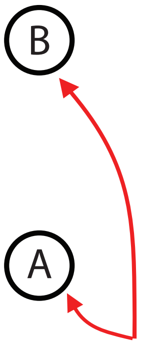
Theory collaborators: Katja Ried, Dominik Janzing
Expt'l collaborators: Megan Agnew, Lydia Vermeyden, Kevin Resch
arXiv: 1406.5036

Classical causal inference

Direct cause



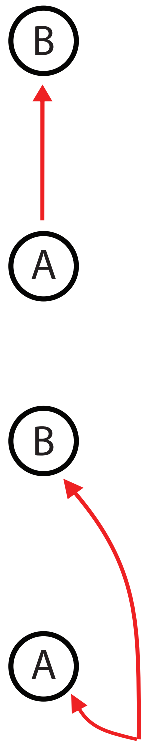
Common cause



Passive observation of A
→ No information about
causal structure

Classical causal inference

Direct cause



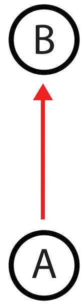
Common cause

Passive observation of A
→ No information about causal structure

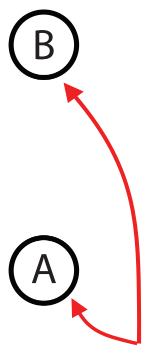
Intervention on A
→ Complete solution of causal inference problem

Quantum causal inference

Direct cause



Common cause

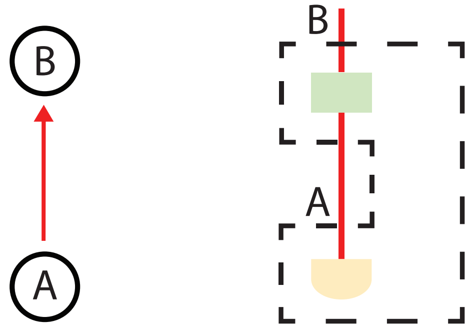


Passive observation of A
→ Still information about causal structure

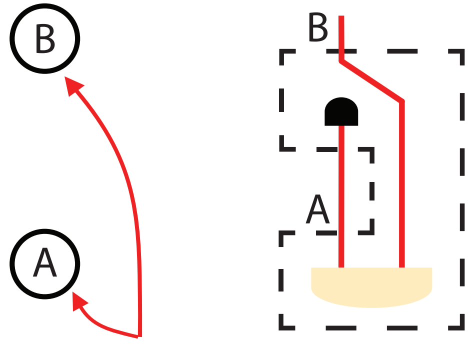
Intervention on A
→ Complete solution of causal inference problem

Quantum causal inference

Direct cause

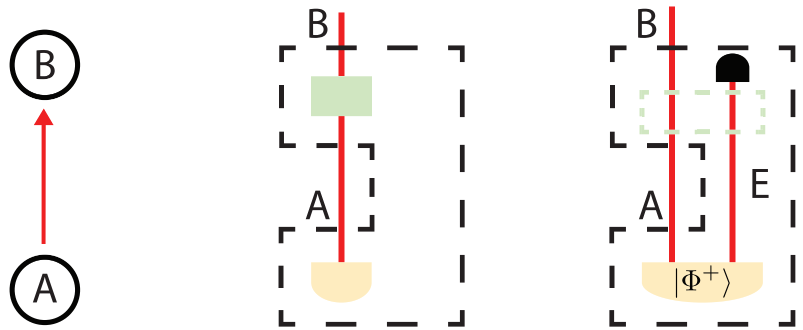


Common cause

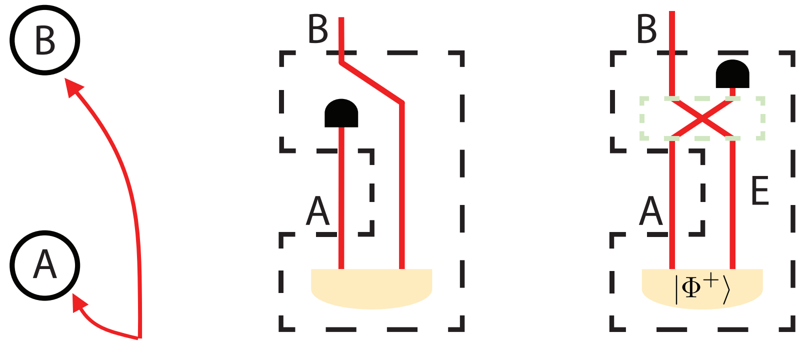


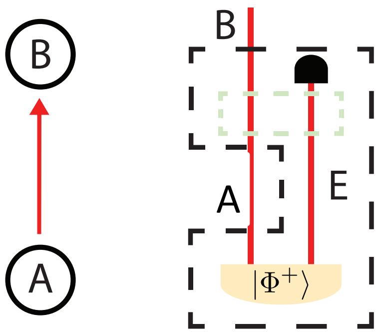
Quantum causal inference

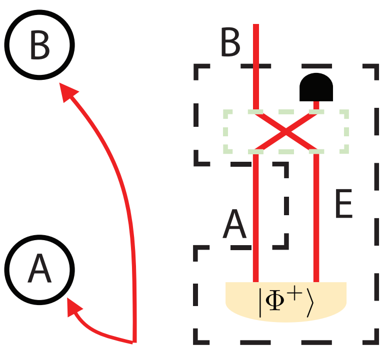
Direct cause



Common cause




B
A
B
A
E
$|\Phi^+\rangle$


B
A
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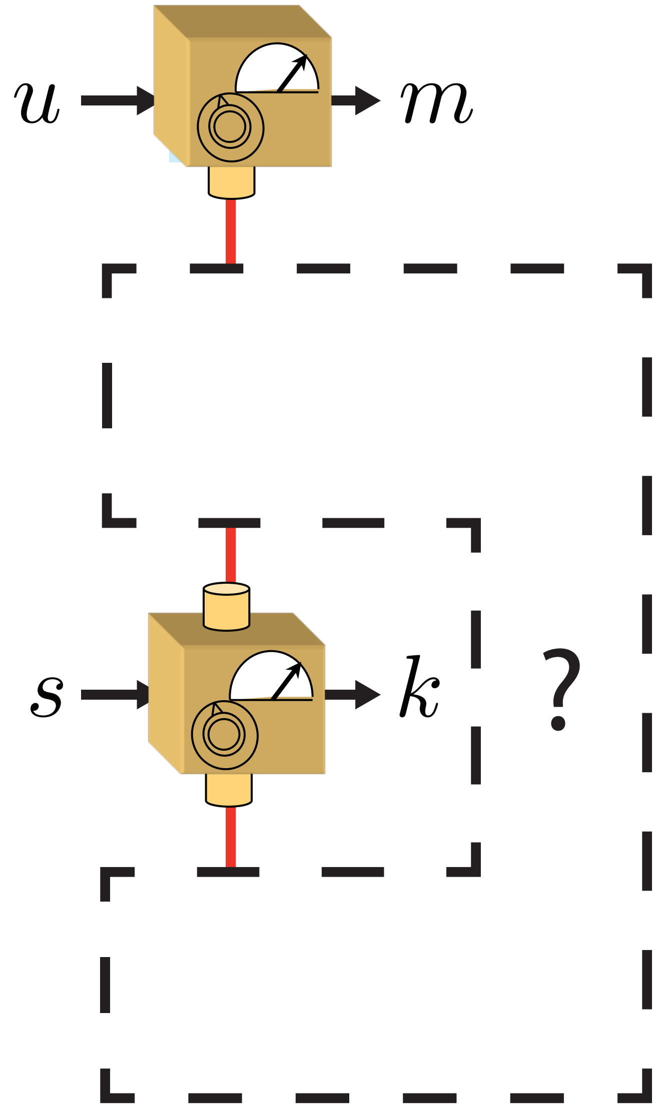
$|\Phi^+\rangle$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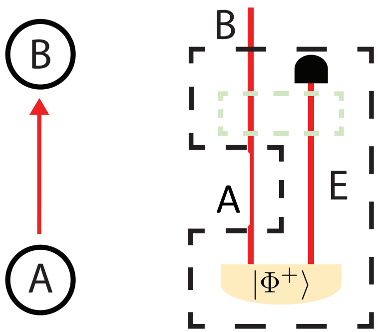
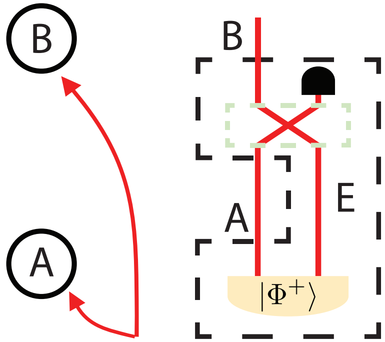
Experimental set-up

prob. (1-p)

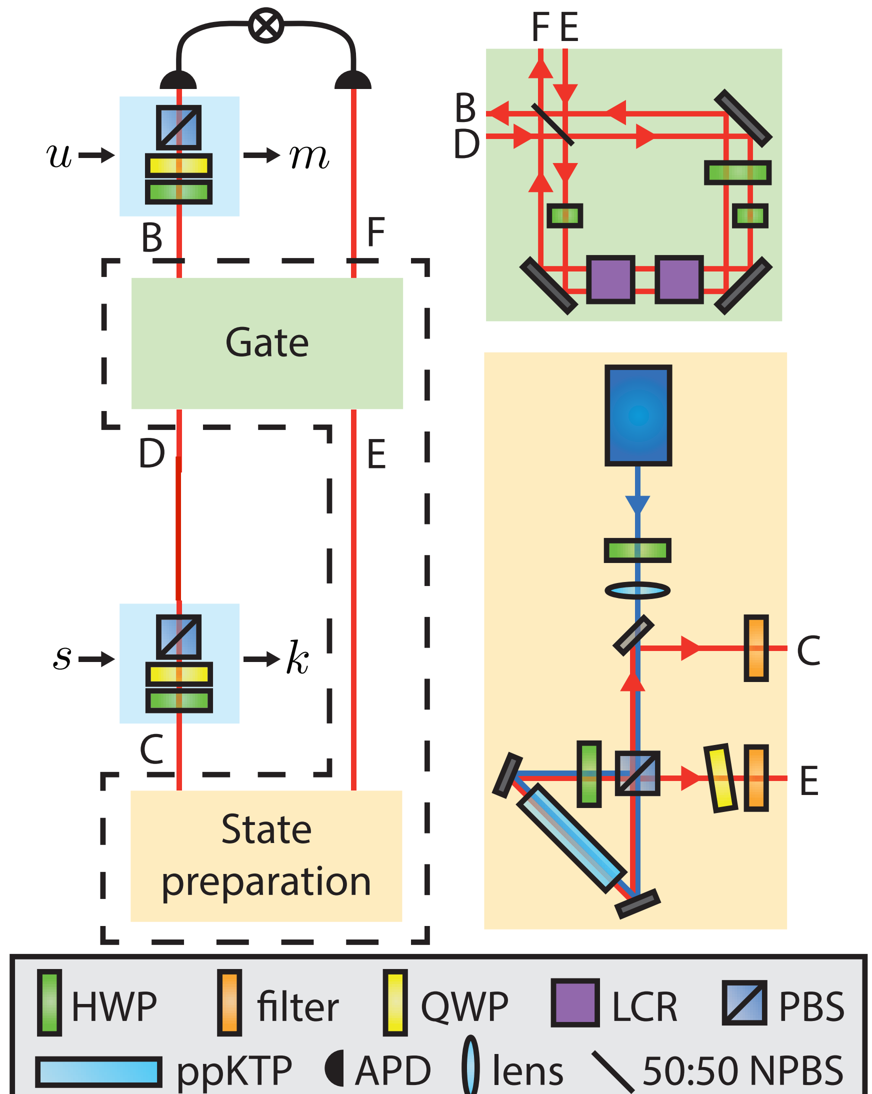


prob. p



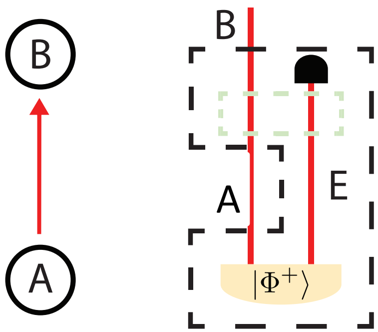
$$p \in [0, 1]$$

a controlled parameter

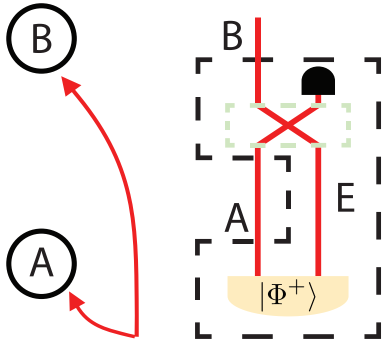


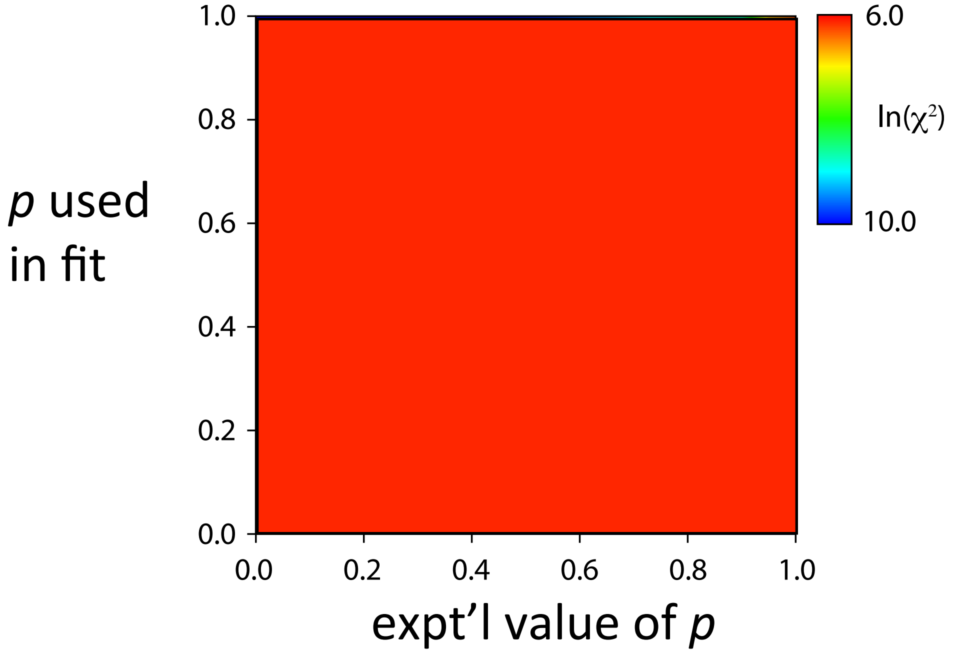
What we would see classically

prob. (1-p)

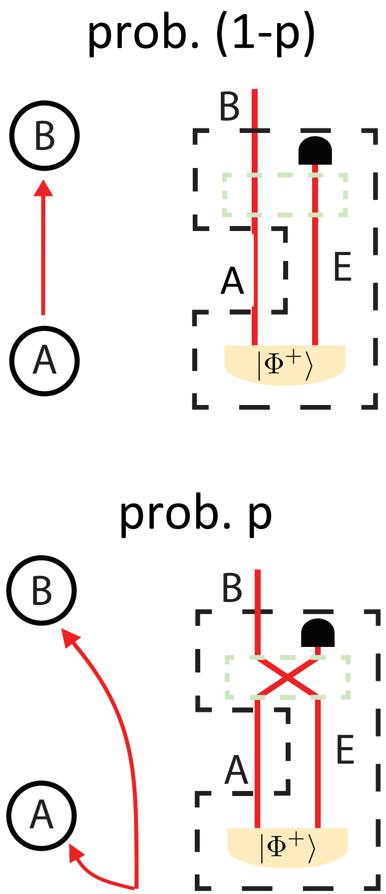


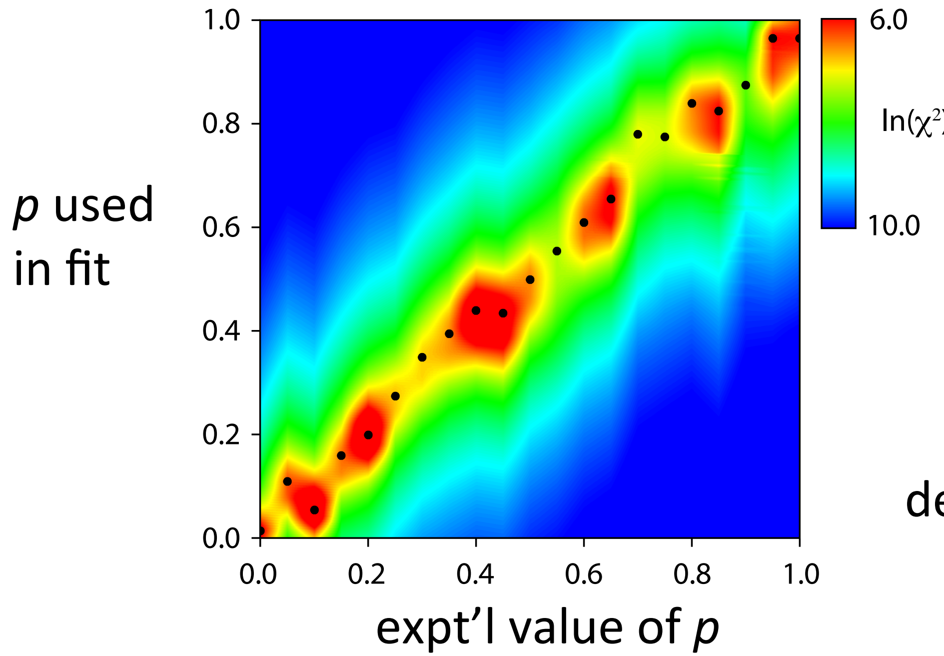
prob. p





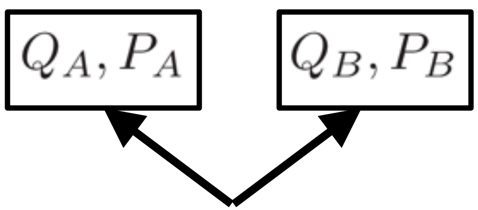
Experimental Results





rms deviation 0.032

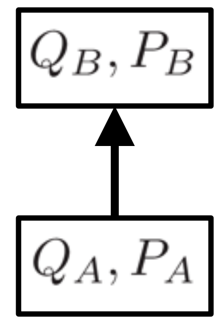
A sketch of the origin of the quantum advantage



$$P_{\mathrm{EPR}}(q_A, p_A, q_B, p_B) \propto \delta(q_A - q_B)\delta(p_A + p_B)$$

$$Q_B - Q_A = 0$$
$$P_B + P_A = 0$$

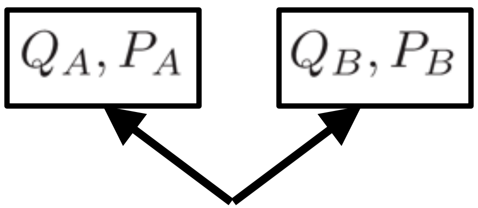
$$P_{\mathrm{id}}(q_B, p_B | q_A, p_A)$$
$$\propto \delta(q_A - q_B)\delta(p_A - p_B)$$



$$Q_B = Q_A$$
$$P_B = P_A$$

A sketch of the origin of the quantum advantage

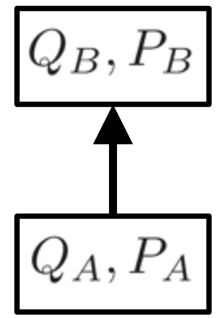
$$P_{\mathrm{EPR}}(q_B, p_B | q_A, p_A)$$
$$\propto \delta(q_A - q_B)\delta(p_A + p_B)$$



$$P_{\mathrm{id}}(q_B, p_B, q_A, p_A) \;\; \propto \delta(q_A - q_B)\delta(p_A - p_B)$$

$$Q_B - Q_A = 0$$
$$P_B - P_A = 0$$

Not allowed!



$$Q_B = Q_A$$
$$P_B = -P_A$$

Not allowed!

Conclusions

- The framework of causal inference provides a very elegant formulation of Bell's theorem
- Quantum causal models are a promising avenue for achieving a causal explanation of quantum correlations
- Tools developed in the community working on Bell's theorem are likely to be useful for improving causal inference algorithms
- Quantum theory provides an advantage for causal inference in certain contexts

References

C. Wood and RWS, *The lesson of causal discovery algorithms for quantum correlations: Causal explanations of Bell-inequality violations require fine-tuning,* arXiv:1208.4119

M. Leifer and RWS, *Towards a formulation of quantum theory as a causally neutral theory of Bayesian inference,* Phys. Rev. A 88, 052130 (2013)

Katja Ried, Megan Agnew, Lydia Vermeyden, RWS, Kevin Resch, *Inferring causal structure: a quantum advantage,* arXiv: 1406.5036

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