Additional exercise week 20-21

1. Let A be a pseudodifferential operator of order m, m even, with a symbol that is given by an expansion

$$a(x,\xi) = \sum_{k=0}^{\infty} a_{m-k}(x,\xi).$$

as described in Strichartz section 8.4 (a classical symbol). We assume that the top order symbol a_m is real and satisfies

$$a_m(x,\xi) \geq C_1|\xi|^m \qquad \text{for } |\xi| \geq 1.$$

Show that there is a pseudodifferential operator B, with classical symbol $b(x,\xi) = \sum_{k=0}^{\infty} b_{m/2-k}(x,\xi)$ such that

$$B^2 = A + R,$$

where R is an operator with C^∞ kernel. Hint: Study the construction of the parametrix in Strichartz page 189-190 or in Alinhac and Gerard setion 5.4.