UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Basic Logic

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Homework E and F

Exercise E (3 points).

Let us mathematically define a notion of a tree (as discussed by van Dalen on pp. 12-13). Let S be the set of finite binary sequences, i.e., $\langle 0,1,1,0,1\rangle \in S$. The empty sequence is denoted by $\langle\,\rangle$. For s and t, we say that s **is an initial segment of** t if s is at most as long as t and s and t agree on their common part (i.e., $\langle 0,1,0,0,1\rangle$ is an initial segment of $\langle 0,1,0,0,1,0\rangle$, but not of $\langle 0,1,0,1,1,1\rangle$). Every sequence is an initial segment of itself. We say that s is a **proper initial segment of** t if s is an initial segment of t and $s \neq t$.

A **tree** T is a subset of S that is closed under taking initial segments, i.e., if $t \in T$, and s is an initial segment of t, then $s \in T$ as well. If T is a tree, then $s \in T$ is called a **terminal node** (or "leaf") of T if there is no $t \in T$ such that s is a proper initial segment of t.

Let At be the set of atoms and Conn be the set of connectives. Given a tree T, we call a function $\ell : T \to \mathrm{At} \cup \mathrm{Conn}$ a **labelling for** T if it has the following property:

- if $t \in T$ is a terminal node, then $\ell(t) \in \mathrm{At}$,
- if $t \in T$ is not a terminal node, then $\ell(t) \in \mathrm{Conn}$.

We call $\langle T, \ell\rangle$ a **labelled tree** if T is a tree and ℓ is a labelling for T. Let LT be the set of all labelled trees.

Give a proper definition of the parsing tree (as indicated on p. 11 of van Dalen's book) using the above precise mathematical notation.

Exercise F (4 points)

Let us define the ab**-language**. Its words consist of particular sequences of the letters a and b, given by the following definition:

- The empty sequence $\langle\,\rangle$ is a word.
- If w is a word, then awb is a word.
- If w is a word, then aw is a word.
- If w is a word, then bwa is a word.

In other words, the set of **words** is the smallest set closed on the mentioned operations. Precisely formulate an induction principle for the ab-language (in analogy to Theorem 1.1.3 in the book; 1 point) and prove the following statement: "Every word has at least as many as as it has bs." (3 points)