

Basic Logic

2007/2008; 1st Semester

Total (19 points)

O. (6 points)

(a) Show that $\{\rightarrow, \neg\}$ is a functionally complete set of connectives, assuming that $\{\vee, \neg\}$ is a functionally complete set of connectives.

(b) (*) Show that $\{\leftrightarrow, \neg\}$ is not a functionally complete set of connectives.

Hint: Consider the number of times a formula with these connectives is true in a truth table. A complete proof is probably excessive.

P. (3 points)

Give a conjunctive normal form (CNF) and a disjunctive normal form (DNF) of

$(p \wedge \neg r) \rightarrow (q \wedge \neg r)$.

Q. (10 points)

Show

(a) $\vdash \varphi \rightarrow (\psi \rightarrow \varphi \wedge \psi)$,

(b) $\vdash \varphi \wedge (\psi \wedge \sigma) \rightarrow (\varphi \wedge \psi) \wedge \sigma$,

(c) $\vdash \varphi \rightarrow \varphi \wedge \varphi$,

(d) $\vdash ((\varphi \rightarrow \psi) \wedge (\varphi \rightarrow \sigma)) \leftrightarrow (\varphi \rightarrow \psi \wedge \sigma)$,

(e) $\vdash (\varphi \rightarrow \varphi \wedge \psi) \leftrightarrow (\varphi \rightarrow \psi)$.