Modelling Human Growth

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We have used the recent Dutch growth study to create learning material for pupils in the second stage of pre-university education (age 15-18 yr.) to carry out practical investigation tasks. We shall present the learning material and discuss the classroom experiences.

Introduction

Body growth of children is often used in mathematics textbooks as an example for discussing processes of change, statistics, and discrete and dynamical models of growth. However, the real world context is in many exercises only used for 'dressing up' a mathematical problem or as an 'ideal' illustration of a mathematical concept. In Figure 1, we show the real diagrams of mean height and increase of mean height for Dutch boys together with a textbook example of a boy called Hank. How credible is Hank's growth curve?

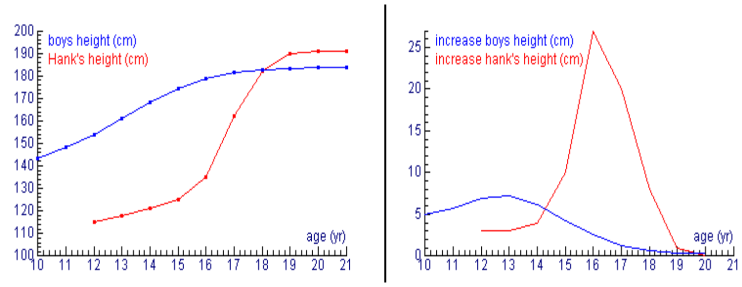

Figure 1. Real and Artificial Growth Curves.

One may argue that real data are too unmanageable or distracting for successful introduction of a mathematical subject. But this argument does not hold anymore when it comes to applications of mathematics by pupils in real contexts, assuming that the pupils have convenient tools to collect, to process, and to analyse real data. In our research we examine the possible contribution of ICT and context situations to the realisation of challenging mathematical investigation tasks for pupils. It provides input for the ongoing development of the learning environment Coach [1] and learning materials for pupils that stimulate and enable them to carry out mathematical investigation tasks at a rather high level. Software and pupils' activities are tested in practice.

The Learning Material

The learning material is designed for pupils who are in their first year of the second stage of pre-university education (age 15-16 yr.), who have no experience with practical investigation tasks, and who have not worked with Coach before. Our main objectives are to let the pupils

- work with real data and with diagrams that are actually used in health care;
- experience how much useful information can actually be obtained from diagrams;
- see that the change of a quantity is often as important and interesting as the quantity itself;

- practice ICT-skills;
- carry out practical work in which they can apply much of their mathematical knowledge.

The learning material is based on medical literature, especially on documents that describe the growth data and the results of the 4th Dutch survey [2]. English versions of the assignments can be downloaded from `www.science.uva.nl/~heck/research/growth/` Estimated study load is 6 to 8 hours. First, pupils carry out the following three assignments:

1. The Dutch Growth Study of 1997. A quiz gives the pupils an idea of how much they already know about body growth and puberty. We use a newspaper article to introduce the 4th nation-wide growth study and to illustrate the relevance of growth data.

2. Mean Height Growth. Pupils get familiar with the main tools of Coach for studying human growth. They learn how to make data plots and increase diagrams of height in relation to age, and they learn how to interpret these diagrams in the context of child growth.

3. Secular Height Growth in 1980-1997. Pupils use the data from the Dutch surveys of 1980 and 1997 to study the changes in mean height for Dutch children in this period.

Finally, pupils select one of the following subjects to investigate by themselves:

A. Growth Charts of Native Dutch Children. Pupils learn how growth charts are made, what they mean, and how they are used. They also compare their own data with their peer group.

B. Mathematical Model of Height Growth for Girls. Pupils compare the mean height growth for girls with Turner syndrome with the growth for healthy girls and they make a simple mathematical model of the height growth for girls until puberty.

C. Mean Weight Growth. Pupils investigate the mean weight growth for healthy Dutch boys and girls in relation to age. They search for points in common and for differences, and they make a simple mathematical model of weight-for-age for children until puberty.

D. Mathematical Weight-for-Height Model. Pupils investigate the mean weight growth for Dutch children in relation to height and make a mathematical model of weight-for-stature.

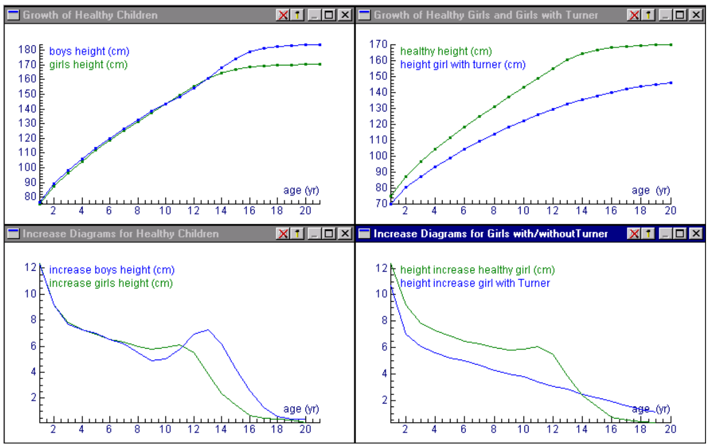

Figure 2. Mean Height and Increase in Mean Height for Dutch Children.

The main role of ICT in the activities is to visualise, process, and analyse real data. On the left-hand side of Figure 2 are the diagrams of mean height growth for Dutch children in relation to their age and the corresponding increase diagrams. Two things catch the eye: the growth spurt during puberty and the fact that this occurs for boys later than for girls, and with higher intensity. More differences in body growth between boys and girls can be read off.

The diagrams on the right-hand side of Figure 2 are about mean height growth of healthy girls and girls with Turner syndrome. They reveal two symptoms of Turner syndrome: slow growth and no pubertal growth spurt. For girls with Turner syndrome the growth rate is after 3 years almost a linear function of age. This means that the graph of the mean height for girls with Turner syndrome is a parabola. Actually, the quadratic model describes it up to a millimetre! Who still dares to say that working with real data in mathematics lessons is too troublesome? In the assignments, pupils do similar experimental modelling of growth curves.

The Classroom Experiment

The experiment took place in a class of 26 pupils working in pairs. In the first week, work took mainly place in the computer lab during the regular mathematics lessons. In the second week, the pupils could make use of the computer facilities at school to continue all by themselves. They had to hand in the report of their work (written with a text editor), a questionnaire, and a diskette with their Coach activities and results. Together with the classroom observations and video recordings, these materials give us an impression of what the pupils actually do and of how they experience the work. Below, we list our main findings; for more details we refer to the extended article on the ICTMT5 CD-rom.

The strongest impression makes the good quality of the pupils' work in general. For example, Linda and Joni describe the difference in height and height increase between girls with Turner syndrome and healthy girls as follows: "The difference in height for girls with and without Turner syndrome is not so big until the age of two. You can also see that they skip puberty more or less. Where a healthy girl starts growing faster during puberty and you can see a peak in the diagram, this is not the case for girls with Turner syndrome. They grow as it were constant." Clearly, these pupils make good sense of the diagrams.

The abilities of pupils to create and interpret graphs has been focus of numerous research studies ([3] and references therein). What we observe in our group of pupils is that they usually have no problems with interpretation tasks up to intermediate level. However, we notice that they are not used to think of change of a quantity as an interesting quantity itself.

Pupils get quickly familiar with Coach. They like the easy way to create graphs from tables and to copy results into their report. Some pupils report difficulties with getting the graphs exactly in the way they want them: choosing unique names for quantities and labelling axes are the biggest problems. Coach has a function-fit tool that enables pupils to work with various regression models. We see that they feel free to try any function type in the tool. But, when asked a simple function-fit, most pupils interpret this as a fit with a straight line.

All pupils indicate that they like it that they may choose the last assignment. Some admit that they choose the task that they guess will be easiest, others (mostly girls) give more personal reasons. For example, Linda and Joni write: "Turner syndrome looks to us an interesting subject. You have to compute something about girls and then it is nice to see if it matches with yourself." None of the pupils find the assignments easy, but their biggest complaint is that time was too short. Some pupils prefer the normal mathematics lessons in which the teacher is always around to help them; others enjoy the freedom in this kind of activities or like the subject. We quote Inge and Annemieke: "It was fun to do; something different from the textbook and an interesting subject.” In comparison with boys, girls seem to be more interested in the subject of body growth, they pay more attention to the report, and they perform in general better. In this practical work, weak pupils also have a chance to do better.

Further investigation: the ICP-model

One can hardly expect that pupils discover a mathematical model for height growth in relation to age by themselves. But it is already nice if they can validate a proposed mathematical model. A fine model to test is the infancy-childhood-puberty (ICP)-model [4]. This model breaks down growth mathematically into three partly superimposed components:

1. *Infancy* (0-3 years). Restricted growth, in which the growth rate is a linear function with respect to height. It is represented by the modified exponential curve:

   $H_1 = a_1 - b_1 e^{-c_1 t}$, where the symbol t stands for age.

2. *Childhood* (from 3 years of age to the onset of puberty). A simple quadratic function fits growth during this period very well:

   $$H_2 = a_2 t^2 + b_2 t + c_2$$

3. *Puberty.* The contribution of the pubertal growth spurt to the final height can be modelled using a logistic function:

   $$H_3 = \frac{a_3}{1 + e^{c_3 - b_3 t}}$$

Here, a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3 are positive parameters, which must be estimated from the growth data. The mean height for each age is given by $H = H_1 + H_2 + H_3$.

One can use the following curve fitting procedure. First, begin searching for a parabola that on the one hand fits well the height between 3 and 10 years of age, and that on the other hand reaches its maximum at the age of 20 years, when height growth usually stops. This curve fitting process is supported in Coach by a manual function fit. After subtraction of the extrapolated values of the childhood component from the observed values during the periods before and after this phase, two additional components are extracted and modelled. The results for Dutch boys and girls are:

Boys: $H_1 = 76.4 - 19.4 e^{-1.56t}$ $\quad H_2 = -0.235t^2 + 9.5t - 4.7$ $\quad H_3 = \dfrac{16.1}{1 + e^{16.4 - 1.2t}}$

Girls: $H_1 = 74.3 - 18.7 e^{-1.65t}$ $\quad H_2 = -0.256t^2 + 9.8t - 4.8$ $\quad H_3 = \dfrac{8.6}{1 + e^{12.4 - 1.1t}}$

The difference between computed and measured values of mean height for Dutch boys turn out to be everywhere less than 0.5 centimetres! This is a beautiful result for a formula that is completely built up from mathematical models that are studied at school. The same model can be successfully applied to weight in relation to age.

References

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