A Practical Investigation Task with the Computer at Secondary School: Bridges and Hanging Chains.

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Abstract

Almost everywhere you can come across hanging chains and cables. Examples are necklaces, power lines, and cables that support a bridge surface. Do these cables all hang in the same mathematical shape? The first thought of many a pupil will be: this is a parabola, isn't it? In the computer learning environment Coach you can easily measure this on digital images. It will turn out that the parabolic shape quite often occurs with bridges, but that an ordinary chain does not hang as a parabola. Can this be understood? We shall show that a key idea for solving the problem can be discovered by measuring on digital images and that this can be theoretically explained with basic physics. It also leads to a simple computer model of hanging chains. We shall discuss our learning material and classroom experiences, and in this way present an example of how ICT and context situations can contribute to the realisation of challenging cross-disciplinary investigation tasks.

Introduction

In recent years, the Dutch Ministry of Education has introduced a new concept for education in the upper level of secondary education (called 'Studiehuis' [study house]) which emphasises inquiry skills and self-responsible learning, and it has added new ICT skills to the curriculum [1]. A new examination program has been implemented, in which pupils are required to choose from four fixed combinations of subjects. In this program, the pupils are required to carry out some smaller practical investigation tasks and one rather large, cross-disciplinary research or design assignment. Pupils do this work mostly in the last two years of their secondary education. They usually practise this kind of work one year before in order to get familiar with investigation tasks.

However, the reform is partly obstructed because active students need many tools to do their work and they need time to learn these tools. At the AMSTEL institute we have developed a single versatile, activity-based environment for learning mathematics, science and technology at various pupil levels. It is called Coach. Activities may contain:

- texts with activity explanations and instructions;
- pictures with illustrations of experiments, equipment, and context situation;
- video clips to illustrate phenomena or to make video-based measurements;
- measured data presented as graphs, tables, meters, or digital values;
- models (in graphical or textual mode) to describe and simulate phenomena;
- programs to control devices and to make mathematical computations;
- links to Internet sites as extra resources for students.

In addition, teachers have powerful, easy-to-learn and easy-to-use authoring tools to prepare activities for their pupils. They can select and prepare texts, graphs, video clips, mathematical models, and measurement settings, and they can choose the right level according to age and skills of their pupils.

A general overview of the learning environment has been presented at ICTMT4 [2]. Here, we shall concentrate on the use of the video tool of Coach to do image measurements. We believe that video and image measurement offer great opportunities to study mathematics and science on the basis of real-world situations in challenging activities. However, experiences with such tools come at present mostly from physics education [3-6] and not from the field of mathematics. Advantages of data video and data image, compared to real experiments, are:

- No experimental set-up is required. This saves time, takes away many practical issues that must be dealt with in real experiments, and lowers costs of equipment.
- Processes and objects that are difficult or impossible to measure can still be studied.
- It is not necessary to determine in advance what and how you are going to measure.
- Measurements can be done in an easy and quick way. Data can be verified later on and, if necessary, corrected.

In this paper we shall discuss a classroom experiment in which digital images are a basis for mathematical modelling. The pupils are in their first year of upper secondary education (age 15-16 yr.) and they have chosen the subject combination 'Science and Technology' or 'Science and Health', which prepare for university studies in exact sciences and medical science. The pupils have not carried out mathematical investigation tasks before, nor do they have practical experience with Coach. They have only seen their teachers use it occasionally during science lessons. The main task for the pupils in this classroom experiment is to get familiar with the video tool and to use it to investigate the mathematical shape of bridges and hanging chains. Only the end of the task is a theoretical completion that involves mathematics and physics. They finish with a short investigation of a free-hanging chain, which will not curve like a parabola.

The Learning Material

Our main objectives are to let the pupils:

- work with real data collected from digital images;
- apply mathematical models to investigate shapes;
- practice ICT-skills, in particular use tools to collect data from video clips and images;
- carry out practical work in which they can apply much of their mathematical knowledge.

The English translation of the learning material can be downloaded from the web page `www.science.uva.nl/~heck/research/bridges/` and it consists of four assignments:

1. Bixby Creek Bridge. Pupils get acquainted with the graphical and tabular facilities of Coach and with the tool for curve fitting. They do not yet collect themselves data from the digital image. Instead, the author of the activity has prepared the data.

2. *Zeeburger Bridge*. Pupils analyse the shape of the Zeeburger Bridge in Amsterdam. They record themselves the coordinates of points on one of the arches of the bridge and they derive from these data the height as function of the horizontal distance. In the activity they learn

how to measure on still images with the data video tool of Coach. Instructions are detailed and guide pupils through the example. In Figure 1, you see a screen dump of the activity in which the data have been collected and analysed. We will come back to this later.

3. Five Weights. The parabolic shape of arch bridges in the first two activities is no coincidence. In this activity, the tasks and the pupils' ingenuity will lead them to a mathematical and physical explanation. For this purpose, the pupils investigate a related, but simpler physical model, viz., a weightless chain with objects of equal weight attached at equal horizontal distances (see Figure 3). By measuring slopes and angles, pupils can discover patterns. This helps them to understand that the points of application to which the weights have been attached lie on a parabola.

4. Necklace. Not every hanging chain has a parabolic shape. Pupils discover this when measuring the hanging necklace of our secretary. More will be said about this in the next section.

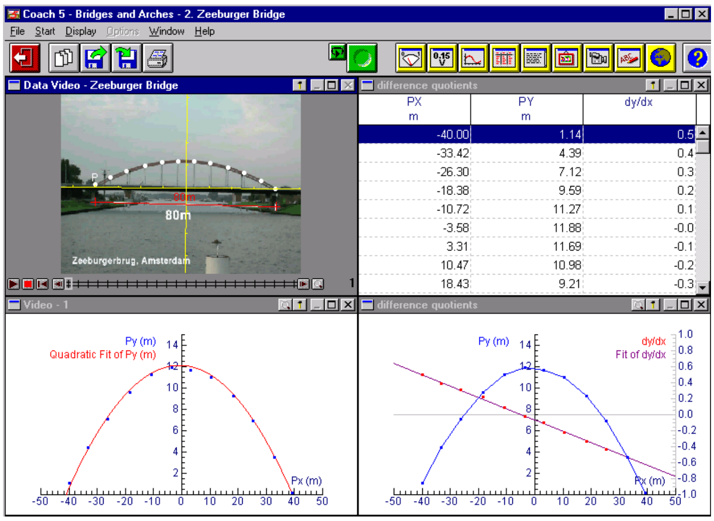

Figure 1. Analysis of the Zeeburger Bridge with Coach.

At the time of the classroom experiment, Coach only allowed measurements on video clips and not on a single digital image. Nevertheless, we can use it to measure on digital images by just converting them into video clips in which all frames are the same. We also go this way in order that the classroom experiment gives us insight into the question whether we really need a separate data image tool, and if yes, so that it provides input for such a tool.[1]

Let us describe briefly how the image measurement activity shown in the above screen dump goes. First of all, the activity allows collection of position data from the digital image. It is possible to place the origin of the coordinate system at any desired position and to rotate the axes, if necessary. You choose the correct scale by matching a ruler with a known distance in the image. Data are gathered by clicking on the location of points of interest. Data can be plotted and used for further analysis. In the lower-left window of Figure 1, the regression tool has been used to find the quadratic function that fits the data best. In the lower-right window, you see the collected points once more, together with the data plot of the difference quotients (dy/dx) of consecutive points (plotted with respect to a second vertical

[1] Newer versions of Coach will indeed contain a data image tool because it offers more possibilities to users.

axis). The difference quotients lie approximately on a straight line. The best line fit can be found with the regression tool. The third column of the table in the upper-right window also shows clearly the pattern for the difference quotients.

Mathematics of the Hanging Chain

Figure 2 shows a screen dump of the activity in which the shape of a perfectly flexible chain hanging under gravity is investigated.

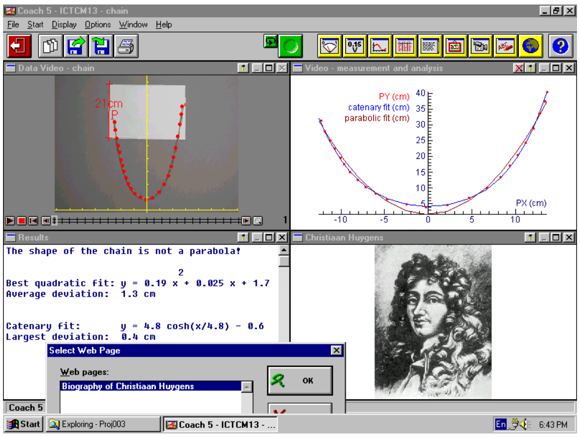

Figure 2 Measuring and computing the shape of a hanging chain.

By collecting positions on the chain and trying a quadratic curve fit on the measured data, a pupil quickly finds out that the form of the chain is not a parabola (as Galilei erroneously claimed). At once, the simple question "How does a chain hang?" becomes a challenging problem. Pupils can follow in the footsteps of Huygens, Bernoulli, and Leibniz. These mathematicians solved the problem of the catenary at the end of the 17th century, when differential calculus was discovered [7].

The function $y(x)$ that describes the vertical position of a point on the chain as a function of the horizontal displacement x satisfies the following differential equation:

$$\frac{d^2y}{dx^2} = \frac{1}{c}\sqrt{1+\left(\frac{dy}{dx}\right)^2},$$

for some positive constant c. If the coordinate system is chosen such that the origin equals the lowest point of the chain, the solution is as follows:

$$y(x) = c\left(\cosh(\frac{x}{c}) - 1\right) = c\left(\frac{e^{x/c} + e^{-x/c}}{2} - 1\right).$$

Of course, a pupil could try to match the data with this formula. But we prefer for our secondary school pupils a different approach to investigate the catenary.

We let the pupils study a similar, but simpler problem: "How does a chain with five objects of equal weight symmetrically attached hang under gravity?" The case of weights at equal horizontal distances is investigated first: the solution is a parabola. A screen dump of an important part of the activity is shown in Figure 3. Measurements in the digital images reveal that the slopes of the right segments of the chain have a fixed ratio, viz., 1:3:5. Measuring in other images would convince pupils that this does not depend on the length of the segments or how far the suspension points are apart from each other. It turns out that one always has the following fixed ratio of positive slopes: 1:3:5:7:9:… Basic physics can explain this: equilibrium of forces holds at each point of application where a weight is attached. This simple observation allows computation of the shape of the system and it explains that the points of application are necessarily on a parabola.

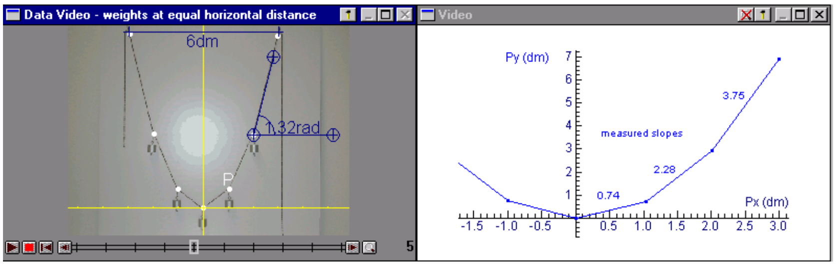

Figure 3 Measuring positions, angles, and slopes in a digital image.

The next step is to realise that it also follows that the curve is not a parabola in case the weights are attached at equal distances *along* the chain. Finally, the fixed ratio can be utilised to approximate the free-hanging ‘ideal’ chain by modelling it as a string of $2n$ beads, for large n, where the beads are close at equal distance of each other. When you choose a starting slope s_0 between the lowest bead and its right neighbour, and when you keep the distance l between beads fixed, you can easily compute the position of subsequent beads, by realising that the slope increases with $2s_0$ at each step in the algorithm. Figure 4 illustrates the use of the modelling environment of Coach for this purpose. Simulation runs with various values of s_0 illustrate how the algorithm can produce a curve that fits the hanging chain.

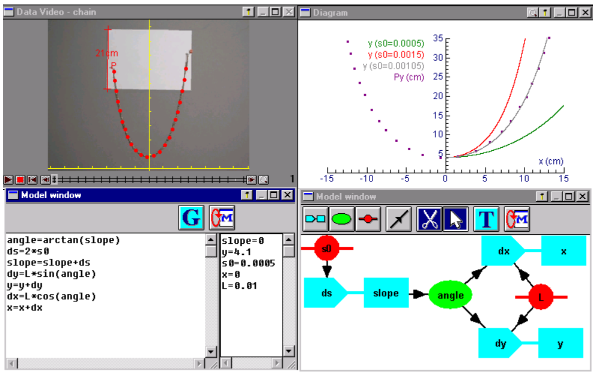

Figure 4. Modelling the hanging chain.

The Classroom Experiment

The profile of the 24 pupils of this research study has already been described at the end of the introduction. The practical assignment was not part of their examination portfolio, but it was graded as a regular test in the semester. The pupils work in pairs, mainly in a computer lab, and complete the assignment within two weeks. The estimated study load is 5 hours. The computer lab was reserved for them in the first week during their regular mathematics lessons (of 45 minutes) and on one afternoon. In the second week the pupils could make use of the computer facilities when available. After these two weeks, they had to hand in the report of their work (written with a text editor), a questionnaire, and a diskette with their Coach activities and results.

In our research experiment we wanted to get in particular answers to the following questions:

- Do the learning material and the chosen educational set-up enable pupils, who have no prior practical experience with Coach, to acquire the right skills and experience to use the data video tool effectively in their study of the shapes of bridges and hanging chains?
- Does Coach, and in particular the data video tool, work without technical problems?
- What do pupils think of the software?
- What do pupils think of the subject, the learning material and of the use of real data?
- Since it was the first time that pupils had to do practical work in the context of mathematics, how do pupils like it and what difficulties did they encounter?

The tools used to get answers to these questions are classroom observations, video recordings (including video capturing of computer work), a questionnaire, and the reports of the pupils.

From the questionnaire we learn about the opinions of 8 teams. The pupils' comments on Coach, in particular the video tool, and on the learning material are positive for the most part. Instructions about the use of Coach are clear and sufficient, so that it is easy to learn to use the video tool. Measuring angles and slopes in the computer environment causes no problems. Most pupils like the quick and easy way to measure on images and to display the data in graphs and/or tables. The 15 minutes demonstration of the learning environment at the beginning of the first activity starts is fine. The assignments link up well with each other and they do not take too long, as far as the collection of data for the report is concerned. All pupils find that theoretical part about the chain with 5 weights links up well with physics, but they differ in opinion about the mathematical background. Half of the pupils write that this part of the assignment does not link up well with their mathematical knowledge. In fact, many of them need assistance from the teacher.

Some pupils are a bit disappointed about the mathematical contents. They find that the assignments focus too much on learning Coach or on physics aspects, and that they less to do with mathematics. In retrospect, we must admit that they are right. We could have gone further into mathematical modelling[2] and we could have paid more attention to the role of symmetry, perspective, and the coordinate system in the problems. Especially the effect that a change of the coordinate system has on the data and on the formulas would have been an interesting topic and a natural introduction to the study of invariance of properties of curves under transformations. We also overlooked some opportunities while preparing the learning material. For example, to convince pupils that really more is needed than curve fitting for deep understanding of a shape, we could have asked them to apply the sinusoidal regression model $y = a\sin(bx + c) + d$ to the data collected for the Zeeburger bridge. They would have seen, maybe to their surprise, that this model works as good as the quadratic model. But which one is correct? And what do we mean by 'correct'? Can any piece of a parabola be

[2] As a matter of fact, we have done a practical assignment about mathematical modelling of parabola and the catenary in a class of pupils in their second year of upper education. It was a success and one pupil commented that this was one of the few times that she was forced without mercy to think deeply. The English translation of this project can be downloaded from the web page `www.science.uva.nl/~heck/research/quadmod`

approximated by a sinusoidal regression model or are there limitations? Similarly, the pupils could have validated that the 'ideal' chain hanging under gravity with suspension points $(-1,1)$ and $(1,1)$, and with its minimum at $(0,0)$ can be approximated very well by the rational formula $y = 9x^2/(11-2x^2)$. What are actually the advantages of the exact formula for the catenary? Such tasks and question would have given the pupils more food for thought and they would have revealed the dangers of experimental mathematical modelling.

Most pupils like the practical work. For example, Jordi and Wester write: "It is more fun than the regular lessons, because the method is different from usual and more varied. Here you have to discover more by yourself." Bianca and Alexandra comment as follows: "It is fun and it makes a change. It was not really difficult, not even the theoretical part. You must 'see it', and then it is easy. Please, do this more often."

75 % of the pupils write their report at home. Therefore, some of them indicate that they would like to have Coach available on their home computer. It turns out that we have been too cautious in our decision not to give the pupils the home version of Coach.

Do our classroom observations and the submitted reports confirm the positive reactions of the pupils? In general, the answer is yes. The enthusiasm of the pupils is great. Most pupils work without stops and we do not notice any negative attitude to work. They do their best to get accurate data in measurements. Pupils collaborate well: there are good discussions between teamworkers about what to do and how to interpret computer results, and fruitful discussions between teams that compare their results. Most teams work rather efficiently: one pupil reads the instructions aloud and the other pupil operates the personal computer. They switch roles many times so that both learn to use the learning environment. Occasionally this division of pupils' roles causes confusion and problems, mainly because the pupil who operates the PC does not wait for complete instructions from his teammate, but goes ahead. Another problem that we notice is that pupils do not always continue reading where the stopped before and miss essential instructions.

The instructions to the pupils about reporting their work are deliberately very short: we want the pupils to think themselves about what and how to report about their activities. So, it is no surprise that reports vary much: some pupils just write down their results like tables and graph without much explanation, others give full report of activities, including their personal experiences. Although the pupils seem to be quite able to express themselves mathematically, it should be pointed out that they use of common mathematical notation. Otherwise they cannot resist the temptation of copying computer results without much questioning. We see in the pupils' reports formulas like `-7.2E-3x^2 + 0.69x + 28.16` and angles of 75.91°.

In the reports, the presence of units of length for slopes indicates that some pupils confuse slope and increase of a quantity. Maybe this is caused by the difference between mathematics, in which tangents are dimensionless, and science, where slope is treated as a quantity. Another interesting difference between the use of diagrams in mathematics and science pops up in the classroom experiment when pupils are making graphs invisible in a plot. To their surprise, pupils get weird diagrams with no coordinate system or no labels near the axes. They are thinking of a graph as a representation of a function, i.e., as a representation of a single object, so that it suffices to work with one variable. This is common in mathematics. In science however, a graph represents a relation between quantities. Then you must work with at least two variables.

Some pupils misunderstand the question in the third activity about the pattern in the angles and/or slopes of the chain segments. They write down that larger angle implies larger slope, and that the tangent of the angle is equal to the slope. They are right of course, but this was not the authors' intention. Most pupils do not find the pattern from their collected data. They find the pattern first in the task to look at angles and slopes for the points $(0,0)$, $(1,1)$, $(2,4)$ and $(3,9)$ in the standard parabola $y=x^2$, and then check if this also occurs in the measured data. This is not a wrong way of doing.

Let us end with possible extensions of the practical assignments. Firstly, pupils suggest the following subjects of practical work in which video and image measurement could play a role: the shape of the atrium at school, the streamline of cars, the supporting power of a structure, movements in sports, collisions, and acceleration, deceleration, and movements of objects. Many of them expect that it is more interesting and fun to work with video clips instead of with still images. We are of opinion that the discrete modelling approach offers an opening to the investigation of other systems of masses acting under gravity on a rope and that the scope of investigation can be broadened to anchor catenaries [8], shapes of suspension bridges, and to architectural structures. This kind of activities would illustrate the use of common mathematical shapes and functions such as straight lines, parabola, exponential and logarithmic curves, and it would reinforce some of the ideas of calculus. But more important, it would bring the real world into mathematics lessons in an attractive way.

Acknowledgements

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