

Bridging between Contexts and Concepts: How Data Video and Computer Modelling Can Help

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Abstract

It is generally assumed that students best learn mathematics and science by doing mathematics and science. Concrete experiences are considered as a good way to start the study of any topic in mathematics and science. The teacher's job is to help students move from the concrete to the abstract, and vice versa. In this paper we will describe how data video and computer modelling can support the process of helping students move between concrete contexts and concepts in mathematics and science. At the same time, this paper will illustrate the advantage of an integrated learning environment in which different activities like data collection, data analysis, and modelling can be combined. Examples are taken from practical investigations of Dutch secondary school students and from experimental learning materials.

Keywords

ICT enhanced learning, data video, computer modelling, practical investigation

INTRODUCTION

Many teachers like to provide their students with opportunities to be involved in the active process of learning mathematics and science. They want them to collect, process, and analyze data, to use digital images and videos, to develop and run mathematical models, and so on. They do this because they know from experience that active learners get a better understanding of concepts, methods and techniques than those who are taught in a traditional style of lectures, textbooks, and exercises. Technology allows students to work with high-quality, real-time data and it enables students to look at realistic applications of science and to do investigative work that resembles professional practice. Hope and expectation is also that this teaching and learning method makes mathematics and science more attractive and challenging for students, especially for those who are less motivated by abstract study.

Practical investigation tasks are part of the Dutch examination programme of senior secondary education. In mathematics and science, students are expected to develop a broad range of research skills, which includes connecting real world phenomena with the scientific world, understanding the problems at hand and asking the right questions, making a project plan, designing and carrying out an experiment, and collecting, representing, analysing, and interpreting information. Students need ICT tools that make such investigation tasks feasible and that enable them to work at an appropriate level.

However, the students' work is partly obscured because active students need many tools to do their work and they need time to learn these tools. Educational researchers and developers at the AMSTEL Institute of the University of Amsterdam put, for many years already, great effort in creating a single, versatile computer learning environment that offers many of the tools that one wants to use in mathematics and science classes in an easy and integrated way: its name is Coach. Coach activities may contain:

- Texts with activity explanations and instructions;
- Pictures with illustrations of experiments, equipment, and context situation, or to make image-based measurements;
- Video clips to illustrate phenomena or to make video-based measurements;
- Measured data presented as table, graphs, meters, or digital values
- Models (in graphical, equation, or textual mode) to describe and simulate phenomena;
- Animations of modelled phenomena;
- Programs to control devices and to do mathematical computations;
- Links to Internet sites as extra resources for students.

In addition, teachers have powerful, easy-to-learn and easy-to-use authoring tools to prepare activities for their students. They can select and prepare text, graphs, video clips, mathematical models, and measurement settings, and they can choose the right level according to age and skills of their students. A short overview of the learning environment Coach has been presented elsewhere by Mioduszezwska and Ellermeijer (2001). In this paper we will present illustrative examples of its use in secondary pre-university education.

MEASURING THE PUPIL LIGHT REFLEX

The pupil light reflex is the change of pupil area in response to a change of light. Constriction of the pupil of the eye happens quickly and automatically in response to a step-increase of light intensity. But a pupil cannot instantaneously reach its new size when the level of illumination is suddenly increased: there is a delay in reaction time ($\approx 200\text{-}500\text{ms}$) and then constriction can be approximated reasonably by an exponential decay function. Hereafter, in case the step-increase was not too large, the pupil redilates slowly almost back to its original size. This response is called 'pupil escape'. If, however, the increase in light intensity is very large, the pupil simply constricts without redilation, a response named 'pupil capture'. Pupil escape and capture have been successfully modelled via nonlinear delay-differential equations; see e.g. Bressloff et al (1996).

The pupil light reflex affects both eyes, even if only one eye is stimulated. This fact is used in the following experiment: the right eye of a person is stimulated by an oscillatory light stimulus via a bicycle lamp, which is periodically switched on and off, and the pupil motion is recorded at the same time with a web cam operating at a frame rate of 60 fps. A Coachlab interface connected with the computer is used to control the lamp and to measure the light intensity in the test tube. At the beginning of the experiment, the test tube with the lamp is held in front of the web cam so that the recorded video and the light intensity measurement can be synchronized by matching the first increase of light intensity with the first time that the bicycle lamp is switched on. Hereafter the lamp is put in front of the right eye and the data collection continues. Figure 1 shows the screen shot of the Coach activity. The lower left window is a visual representation of the Coachlab interface with the connected light sensor and lamp. The upper right window shows the control program to switch the lamp on and off. The upper left window contains the recorded video and the lower left window shows the graph of the measured light intensity during the experiment. All windows are linked: so by comparing the step-increases of the light intensity with the frames in which the pupil constricts, you can find the delay in reaction time. The measured reaction time is 0.25s.

The recorded video is used to measure the diameter of the left pupil during the second phase of the experiment. Point tracking makes this measurement an easy task: two opposite points at the boundary of the pupil are selected in the starting frame and the coordinates of these points are automatically recorded in subsequent frames. These recordings are used to compute the pupil diameter at any time during the second phase of the experiment. For more details about the

algorithm implemented in Coach for point tracking we refer to Heck and Uylings (2006b). Figure 2 is a screen shot of the video measurement activity. The graph of the measured pupil diameter is shown in the right window. This diagram also contains the graph of the sinusoidal regression curve that fits the data. The period of the pupil oscillation is 4.3s, which is in agreement with the period of the measured light intensity.

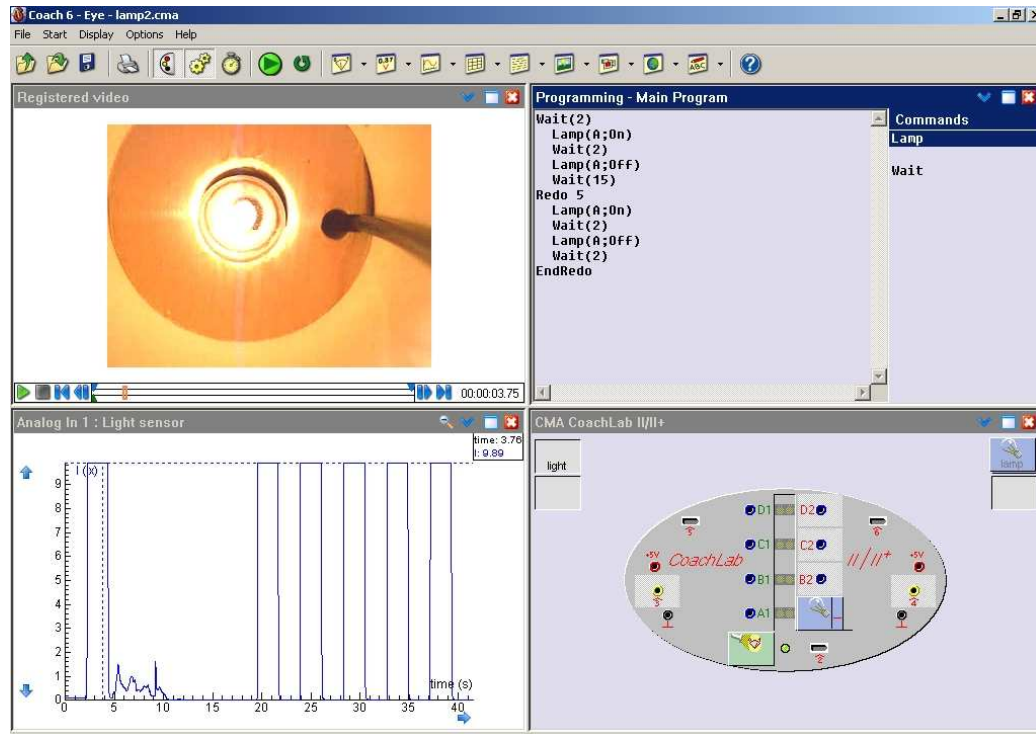


Figure 1. Screen shot of a data collection activity in Coach.

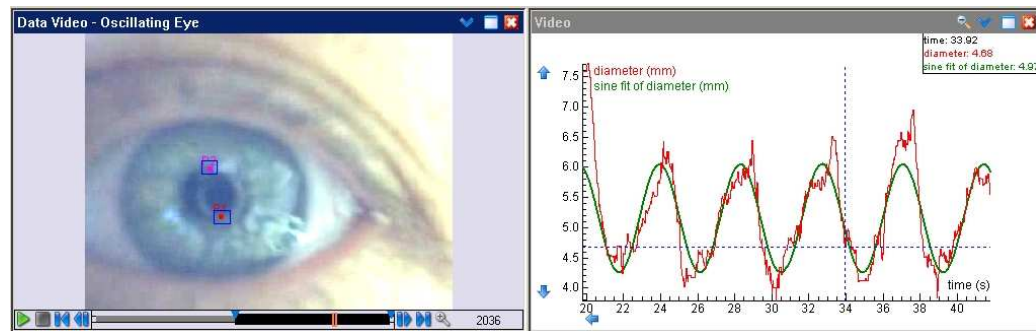


Figure 2. Screen shot of a video analysis activity in Coach.

This example shows that students doing experiments that resemble professional research practice often need a computer environment in which various tools can be used simultaneously or in combination with each other. It is not good enough to focus in ICT enhanced learning on separated experimental control, data collection, and data analysis.

MEASURING AND MODELLING A FALLING AND BOUNCING BALL

The next example shows that a computer learning environment with advanced facilities for data video and mathematical modelling offers students many opportunities to investigate motions of objects. We begin the investigation of a free falling object with a video analysis of a recorded experiment and an experimental

way of modelling the motion via regression. A research study of Larkin-Hein and Zollman (2000) showed that such an approach serves as an effective way to permit students to become more active learners. In addition, it enhances student motivation and attitudes as well as encourages longer time on task.

Figure 3 is a screen shot of the video analysis. In the lower left window is the original video clip of the experiment in which a person standing on a ladder lets a ball fall down from a height of 4.5m. Data collection using this video clip is problematic because of the perspective distortion. Luckily, Coach provides a tool to correct the perspective view of an image plane of motion based on the method of Liebowitz (2001). Details about the software implementation and more inspiring examples of its use can be found in Heck and Uylings (2006b). The upper left window in the screen shot below shows the result of rectifying the video clip to a front-parallel view of the scene. Point tracking makes the data collection in the rectified video clip a piece of cake. The windows to the right show the tabular and graphical results of measuring the height of the ball with respect to time. In the graph window, the (numerical) derivative of the vertical position has been computed as well: it is a straight line of which the slope is close to the gravitational constant. It also motivates a parabolic regression curve to fit the data.

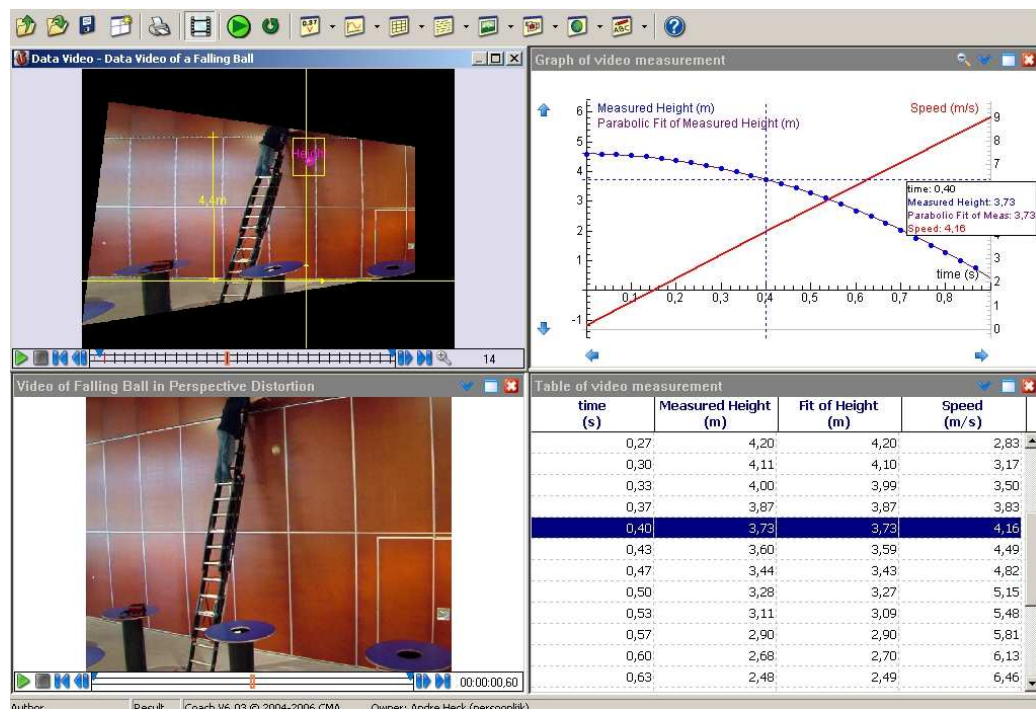


Figure 3. Screen shot of video measurement and function fit.

Experimental modelling via regression is practical and gives excellent results, but it would be nicer if you could underpin it with a mathematical model using elementary concepts of kinematics. Here, computer modelling comes into play. There exist basically three types of computer tools for simulation dynamical systems: a system dynamics approach, event-based modelling, and agent-based modelling. Coach is a hybrid system that combines a traditional system dynamics approach with event-based modelling in order to allow modellers to deal in their description of a phenomenon with sudden effects. We will see examples of this later on in this paper. Figure 4 is a screen shot of the modelling activity using only the system dynamics part of the tool. The graphical model is shown in the upper window. The meanings of the icons are similar to those of system dynamics software like Stella (Steed, 1992): there are icons for state variables, flows, auxiliary

variables, constants, and connectors. The graphical model represents a system of differential equations that originate from Newton's laws of motion. The diagram in the upper right corner of Figure 4 illustrates that the vertical position of the ball computed in the model matches very well with the parabolic fit of the measured motion.

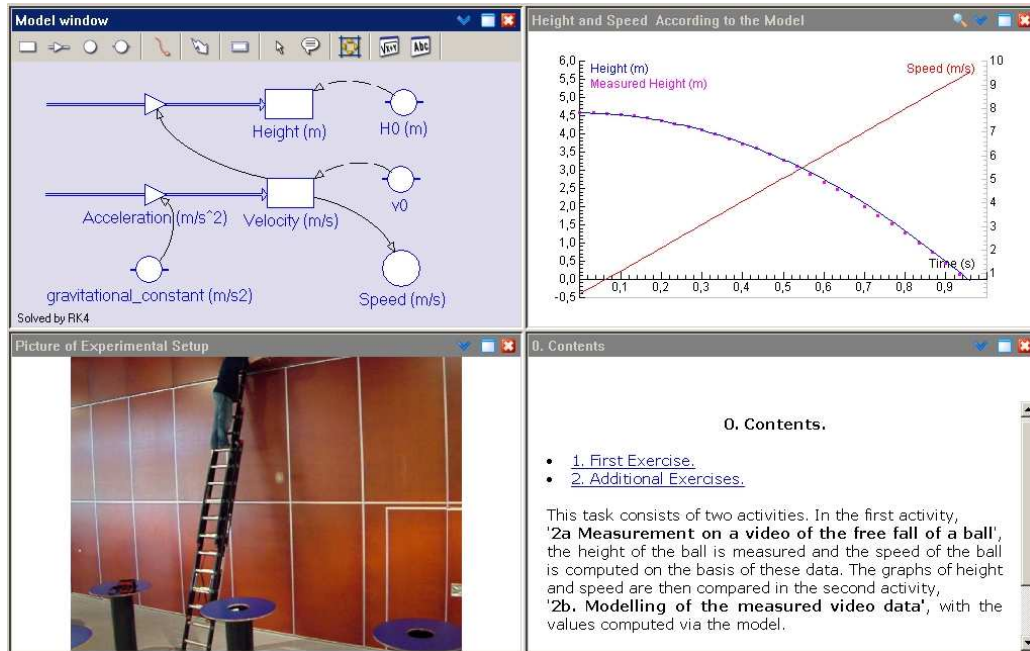


Figure 4. Computer modelling and comparison with measurement.

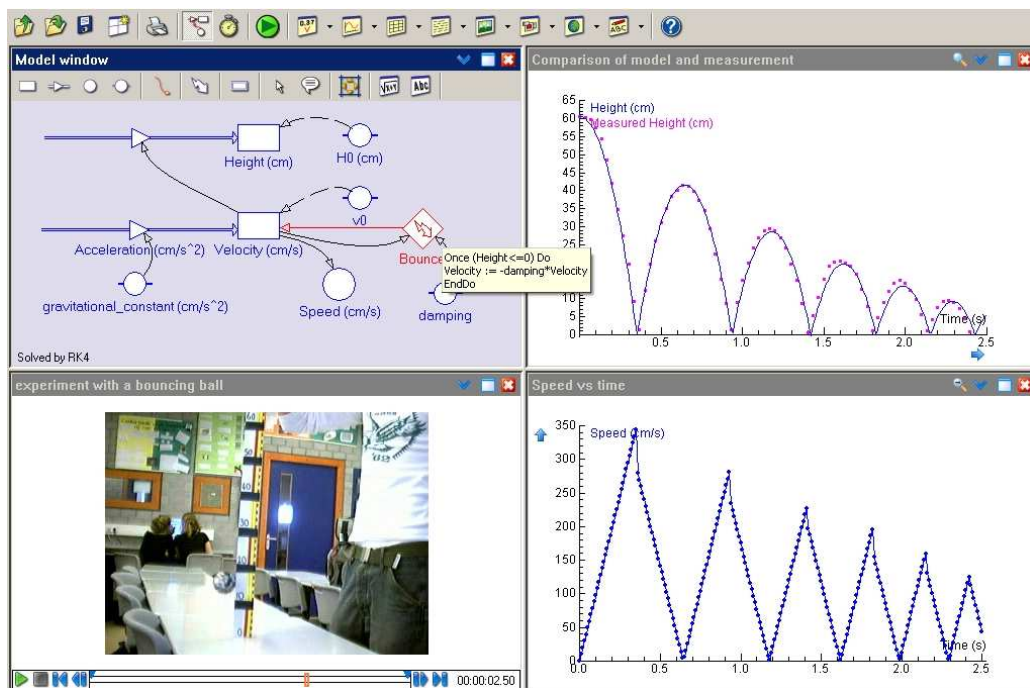


Figure 5. Modelling a bouncing ball and comparing the model with reality.

Let us continue with the more complicated situation of a bouncing ball. As you can see in Figure 5, the graphical modelling window is just an extension of the previous model of a free falling ball with an event. At the discrete time event when the ball hits the table we must update the velocity in order to change the direction of

motion, i.e., to change the velocity. Let us look at the details of this event: it is triggered when the height becomes less than or equal zero. Because of energy loss due to inelastic collision – one notices that the bouncing ball loses height in time – the downward velocity changes into an upward velocity and its magnitude is decreased by multiplication with a damping factor between 0 and 1. The yellow sticker, which pops up when you keep the cursor for a while on top of the events icon (the thunderbolt symbol), contains the following computer code:

```
Once (Height<=0) Do Velocity := -damping*Velocity EndDo
```

The above experimental approach to study the motion of the bouncing ball perhaps satisfies a student who models just this particular system, but he or she must start all over again for every new kind of ball. In other words, what do you learn from it about the motion of a bouncing ball in general? To this end, mathematics and physics must come into play, possibly in combination with a system dynamics approach that can handle discrete events. For example, the damping can be computed as the square root of the ratio of the maximum height after and before the bounce when aerodynamic effects are not taken into account. The reasoning is not beyond the level of secondary school students in physics: the speed v_1 of the ball with mass m falling freely from a maximum height h_1 when it hits the table is given by $v_1 = \sqrt{2gh_1}$, where g is the gravitational constant. This is true because the potential energy mgh_1 is completely transformed into the kinetic energy $\frac{1}{2}mv_1^2$. For the speed v_2 of the ball when it bounces we take $v_2 = k \cdot v_1$, where k is the damping factor, and the height h_2 to which the ball returns is determined by $v_2 = \sqrt{2gh_2}$.

Therefore $k = v_2/v_1 = \sqrt{h_2/h_1}$. The damping found experimentally in this way from the measured heights in the video is 0.84; this is in excellent agreement with the damping 0.83 used in the computer model to match the data. Another question that can be successfully investigated by a student is the following: how much time does it take for the ball to come to rest? The mathematical reasoning could be that the time needed for the first and second fall are $t_1 = \sqrt{2h_1/g}$ and $t_2 = \sqrt{2h_2/g} = k \cdot t_1$, respectively. If one takes into account that the ball in the first phase only falls on the table and that hereafter it goes up and down, then it is clear that the time T it takes for the ball to come to rest is given by

$$T = t_1 + 2t_2 + 2t_3 + \dots = t_1 + 2kt_1(1 + k + k^2 + \dots) = t_1 + 2kt_1/(1 - k)$$

$$= t_1 \cdot (1 - k)/(1 + k) = \sqrt{2h_1/g} \cdot (1 - k)/(1 + k)$$

What is important about this algebraic work for education is that it is not just a hobby of the teacher, but that the formulas really help students to understand the phenomenon better. For parameter values $k = 0.84$, $h_1 = 0.60\text{m}$, $g = 9.81\text{m/s}^2$ we get $T = 4.0\text{s}$, which is actually one second more than the time that is found in the experiment recorded in the video clip. The reason for the difference between mathematical model and experiment is that the bouncing process is more complicated and changes when time goes on and when the last bounces of the ball take place. Another point that may surprise students is that although the mathematical model predicts that the ball bounces infinitely many times, the total time for the ball to come to rest is finite. As a matter of fact, all these issues make the investigation not just more complicated, but more interesting and challenging.

Using discrete events, modelling of complex system dynamics is not beyond most of the students anymore. It really widens the scope of computer modelling in education. In our case, it has led to a realistic, physics-based computer model of a bouncing ball, the simulation of which matches in an excellent way with the

experimental data at the beginning of this process. A similar success of investigating the motion of a yoyo via video analysis and graphical modelling has been reported by Heck and Uylings (2006a).

MODELLING STOCHASTIC WALKS

In this section we illustrate the usefulness of a hybrid modelling tool in Coach, which combines event-based modelling with a system dynamics approach, by the example of a two-dimensional random walk. We also discuss how the text mode of the modelling tool can be effectively used to analyze a stochastic walk.

The stochastic walk that we investigate is a random walk on a 2-dimensional square grid in which each step consists of a motion from a grid point to a neighbouring grid point. We choose as neighbourhood the so-called Moore neighbourhood that consists of eight directions: north, south, east, west, north-east, north-west, south-east, and south-west. They correspond with steps $(0,1)$, $(0,-1)$, $(1,0)$, $(-1,0)$, $(1,1)$, $(1,-1)$, $(-1,1)$, $(-1,-1)$. In a symmetric stochastic walk, all directions have equal chance $1/8$. This random choice of a direction in each step can be implemented in a computer program by drawing a random integer between 0 and 7. In Coach you can use for this purpose the `Rand` function that draws a random number between 0 and 1 according to a uniform distribution: the formula `Entier(8*Rand)` does the job. The generation and visualization of the random walk becomes easy when one uses events to implement the handling of the various cases. Figure 6 shows the Coach model and the graphs of a simulation.

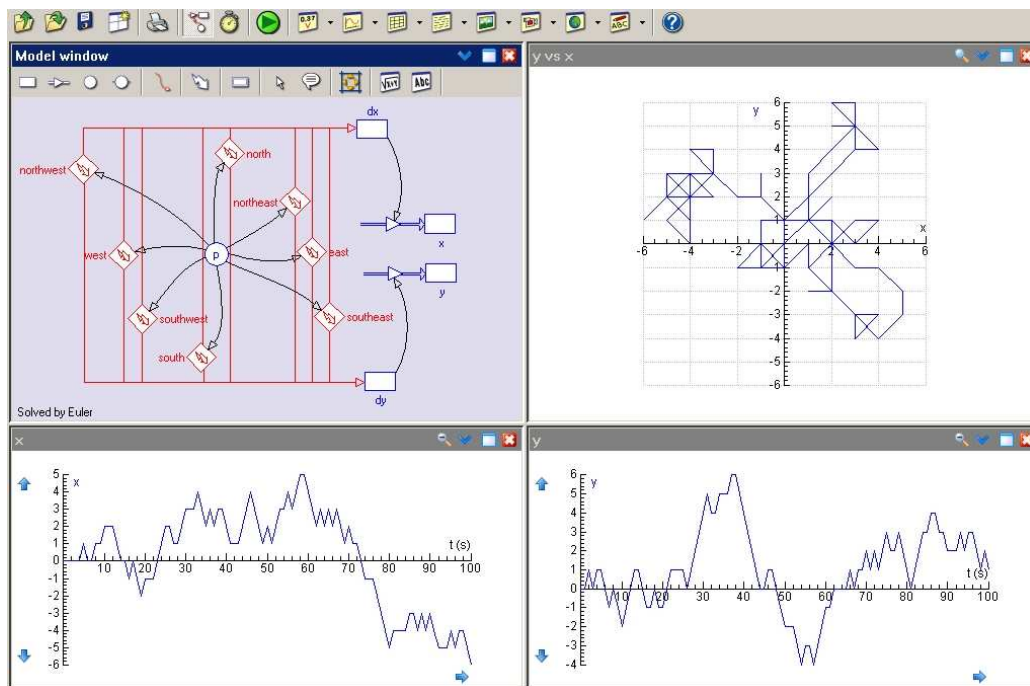


Figure 6. A graphical model and a simulation of a 2D random walk.

The events play in the above model the role of choices of direction. In the model it is explicitly stated what should happen when an event occurs. The Coach program contains fragments of code like:

```
Once (p=0) Do
  dx := 0
  dy := 1
EndDo
```

Boundary conditions to keep the random walk restricted to a particular area of the grid can also be implemented via events. The graphical model of a random

walk in a Moore neighbourhood (Figure 6) can be easily extended to a model in which the position of the random walker is bounded in east-west and north-south direction. Figure 7 shows a graphical model and a simulation under such boundary conditions. To this end we have introduced two events for imminent crossings of borderlines (by the way, in the equation mode of the modelling tool you can easily order the handling of events). Table 1 shows the computer code for bouncing walls at a distance of 10 units from the origin.

<i>wallEastWest</i>	<i>wallNorthSouth</i>
Once ($(x+dx)<-10$) or ($(x+dx)>10$)	Once ($(y+dy)<-10$) or ($(y+dy)>10$)
Do	Do
$dx := -dx$	$dy := -dy$
EndDo	EndDo

Table 1. Coach code for horizontal and vertical events specifying bouncing walls.

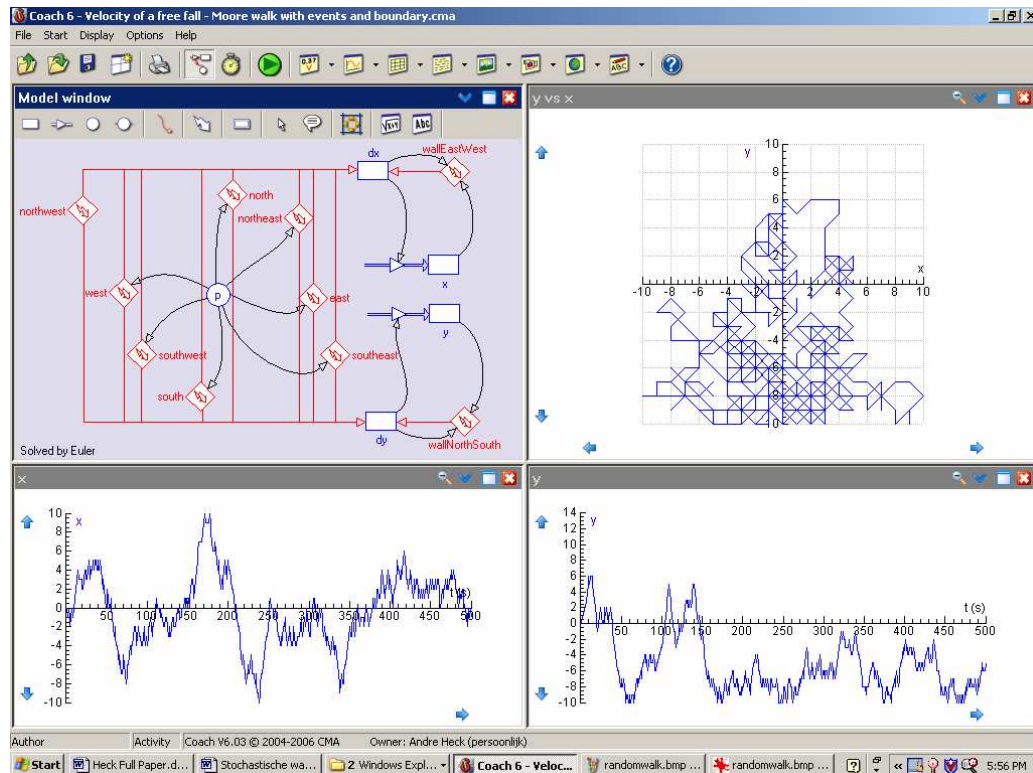


Figure 7. A graphical model and a simulation of a random walk in a restricted area.

It becomes more fun for students when they make an animation of the random motion and when they implement two interacting random walkers in a similar way. We will not work this out here but end this section with the use of the text mode of the Coach modelling tool for analyzing a two-dimensional random walk in statistical terms via computer simulations. We consider the random walk in a square grid using the Moore neighbourhood without borders.

For a statistical analysis of stochastic walks via computer simulations it is important that you can compare results of numerous computer runs of the model. Because the graphical mode of the modeling tool in Coach provides no means for repetition loops, you have to write a computer program in the Coach programming language. You can still use the modeling tool for this purpose by taking the simulation number as the independent variable and write a program like the one shown in the left part of Figure 8. By the way, the code inside the repetition loop has been copied from the earlier graphical model. So, the programming of the simulation is rather easy and straightforward. Figure 8 also shows the table of the distance

from the position reached after 200 random steps to the origin for 25000 simulation runs, a statistical overview with the mean distance and standard deviation, and the histogram of the distances found in the simulation. We have also approximated the

histogram by a bell-shaped regression curve: $\text{percentage} = 3.84 e^{-\frac{1}{2} \left(\frac{\text{distance} - 25.5}{10.4} \right)^2}$. This result is pretty close to the normal distribution with mean value 25.5 and standard deviation 10.4. So, the statistical notion of normal distribution pops up naturally in the analysis and is not pre-arranged by the teacher or inside the learning material.

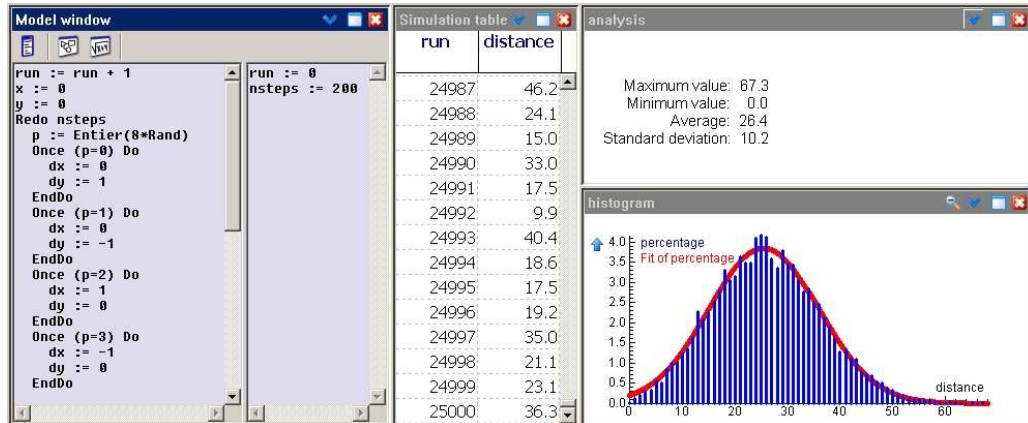


Figure 8. Statistical analysis of a random walk via a computer simulation.

CONCLUSION

In our point of view, the real educational issue in the investigative work that we described and in the examples that were given in this paper is the ICT-supported interaction between experimental work and modelling, in which the interpretation of results is based on methods from mathematics and physics. The role of ICT is here to allow students to collect high-quality, real-time data, to construct and use computer models of dynamic systems in much the same way as professionals do, and to compare results from experiments, models, and theory. Furthermore, students can develop and practise through their activities research skills, and the fact that they must apply their knowledge of mathematics and physics in a meaningful way in a concrete context leads at the same time to deepening and consolidation of this knowledge. Important research competencies that students practise through this kind of practical investigations, which are by the way part of the upper level curriculum in the Dutch secondary school system, are being able to

- formulate good research questions that guide the work;
- observe and discuss with others about the behaviour of an object in motion;
- design and implement an experiment for collection of relevant data;
- apply mathematical knowledge and physical concepts in new situations;
- construct, test, evaluate, and improve computer models, and have insight in their role in an investigation;
- interpret and theoretically underpin results;
- reflect on own work;
- collaborate with others in an investigation task.

ICT plays an important role in enabling students to carry out investigations at a high level of quality. It also brings the real world into mathematics and physics education in an attractive way. Computer models can be constructed by students themselves on the basis of general physical concepts in situations where algebraic formulas are out of reach or even impossible. In short, ICT provides the students with tools to be actively involved in the process of finding solutions to challenging problems that they

come up with themselves. The fact that these tools are bundled and integrated in a single computer learning environment comes in handy: this makes it for students easier to apply these tools in their attempts to solve problems or to get a better understanding of given situations. Actually, we consider the modelling process, the underlying thinking processes, and the discussions with fellow students during the research as more important in the students' work than the obtained results. All the same it is joyful when experiment, model, and theory are in full agreement, as is the case for example in the study of the motion of a free falling and bouncing ball.

REFERENCES

- Bressloff, P. C., Wood C. V. and Howarth, P. A. (1996) Nonlinear Shunting Model of the Pupil Light Reflex. *Proceedings of the Royal Society B: Biological Sciences*, Vol. 263, No. 1773, 953–960.
- Heck, A. and Uylings, P. (2006a) Yoyo Joy. In *Proceedings of the 7th International Conference on Technology in Mathematics Teaching*. Edited by F. Olivero and R. Sutherland. Bristol: University of Bristol, Volume 2, pp. 237–244.
- Heck, A. and Uylings, P. (2006b) Capturing the Real World in the Classroom. *The International Journal for Technology in Mathematics Education*, Vol. 13, No. 3, 107–116.
- Larkin-Hein, T. and Zollman, D.A. (2000) Digital Video, Learning Styles, and Student Understanding of Kinematic Graphs. *Journal of SMET Education*, 1/2 May–August 2000, 17–30.
- Liebowitz, D. (2001) Camera Calibration and Reconstruction of Geometry of Images. PhD thesis, Chapter 3. Available www.robots.ox.ac.uk/~vgg
- Mioduszewska, E. and Ellermeijer, T. (2001) Authoring Environment for Multimedia Lessons. In *Physics Teacher Education Beyond*. Edited by R. Pinto and S. Rusinach. Paris:Elsevier. pp. 689–690.
- Steed, M. (1992) Stella, A Simulation Construction Kit: Cognitive Process and Educational Implications. *Journal of Computers in Mathematics and Science Teaching*, 11, 39–52.

Biographies



André Heck is project manager at the AMSTEL Institute, at the University of Amsterdam. Besides his work in curriculum innovation at university level, he is actively involved in the research and development that aims for a computer learning environment for mathematics and science education at secondary level. This environment contributes to the realization of cross-disciplinary and challenging practical work of students.



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