Grand Tour of *Derive*

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Chapter 1

General Information

1.1 Short Description

Derive is a general purpose symbolic calculator which is continually being developed by David Stoutemyer and Albert Rich of Soft Warehouse, Inc. It is the successor of muMATH. It can handle many of the mathematical computations learned in secondary school, polytechnic, and university mathematics, and it can handle two- and three-dimensional graphing. *Derive* can be thought of as having a relationship to algebra and calculus similar to that of any ordinary calculator to basic arithmetic. *Derive* also incorporates some functional programming facilities that allow the user to define mathematical procedures. All of *Derive*'s own utility functions are written in the *Derive* language.

1.2 Current Version

The current version is *Derive* 3. At present, the exact version number is 3.02, indicating that minor improvements have been made since the major release upgrade.

1.3 Hardware Requirements

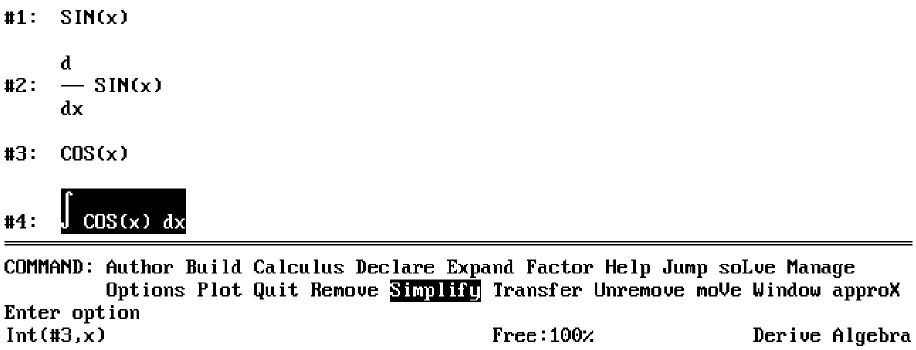
There are two versions of the *Derive* program: *Classic Derive*, and *Profesional Derive.* They operate identically; the difference is in memory usage.

Classic Derive requires a PC compatible computer with at least 512K of main memory, including RM Nimbus 186/286/386 and Archimedes with PC emulator, running MS-DOS (version 2.1 or higher), and with a 3.5" diskette drive.

Profesional Derive requires a 386 or 486 based PC compatible computer with at least 2MB of extended memory and can make use of up to 4GB of extended memory to solve large problems.

At present, *Derive* is the only computer algebra system which is available on palmtop PCs such as POQET PC, HP95LX, HP100LX, and HP200LX. *Derive* supports both the 95LX graphics mode and the CGA graphics modes of the 100LX and 200LX computers. Therefore, version 3 can be downloaded and run on all three computers (the 95LX, 100LX, and 200LX) provided it has at least one megabyte (1Mb) of RAM. Also a PCMCIA RAM card is recommended for storing DERIVE program files. Also, Version 3 automatically takes measures to conserve power when running HP palmtops off a battery.

Derive supports common monochrome monitors (MDA and Hercules) and color graphics monitors (CGA, EGA, MCGA, and VGA). Supported printers include HP LaserJet Series II and III, Epson FX80, IBM Proprinter, and compatible printers. Screen images can be saved as TIF (tagged image format) files for further processing in word processing or desktop publishing programs.



```
#1:  SIN(x)

     d
#2:  — SIN(x)
     dx

#3:  COS(x)

     ⌠
#4:  ⌡ COS(x) dx
COMMAND: Author Build Calculus Declare Expand Factor Help Jump soLve Manage
         Options Plot Quit Remove Simplify Transfer Unremove moVe Window approX
Enter option
Int(#3,x)                                 Free:100%            Derive Algebra
```

Figure 1.1: A *Derive* session in progress

1.4 User Interface

Derive is a menu-driven computer algebra system. Therefore, most operations can be carried out using single key-strokes. The interface can be best illustrated by a simple example: see Figure 1.1 for a screen dump of a *Derive* session in progress.

The *Derive* screen consists of three parts: the *working arrea*, the *command menu*, and the *status line*. You can select a menu item by tapping the space bar (or < TAB > key) enough times until the requested is high-lighted followed by pressing the < ENTER > key. A more experienced user will probably select a menu item by immediately entering of the upper case character in the name of the requested menu item. In Figure 1.1 the sine function has been entered by the **Author** command, the derivative has been computed, and the indefinite integral of the cosine function is going to be computed (as the annotation in the status line indicates).

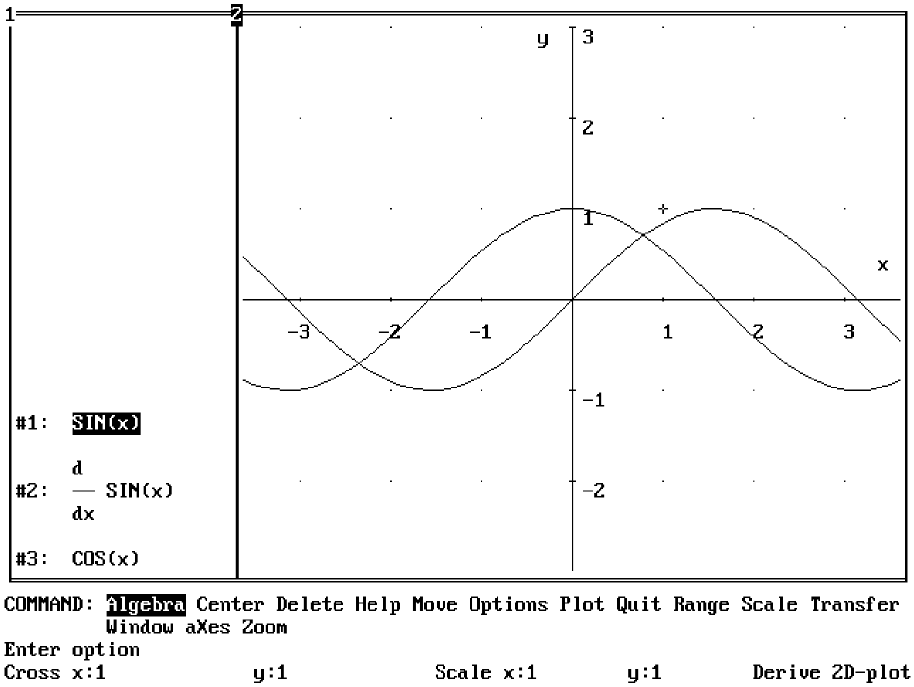
The *Derive* screen may consist of several windows. There are two types of windows: an algebra window and a plot window. An algebra window contains the numbered list of expressions you have created. A plot window contains graphs of functions of one or two variables that have been selected from an algebra window. A typical *Derive* session is illustrated in Figure 1.2.

There is on-line information available via help screens. A typical help screen looks like Figure 1.3.

Derive can also be used in demonstration mode by reading the commands that correspond with menu choices from a file with name-extension `.DMO`. *Derive* will then pause after executing a command A short explanation of the current command can be found in the status line. *Derive* proceeds with the next command in a demo file after a key stroke.

Although *Derive* is a menu-driven system, all commands and declarations of state variables that control precision, notation, domain, and so on, can be made on the **Author** line.

The *Derive* menu and the annotations can be customized. E.g., frequently used menu commands can be moved to the top-level menu, commands can be hidden, and menu command names or annotations can be translated into other languages. In Figure 1.4 you see a Dutch menu that only allows plotting of functions.



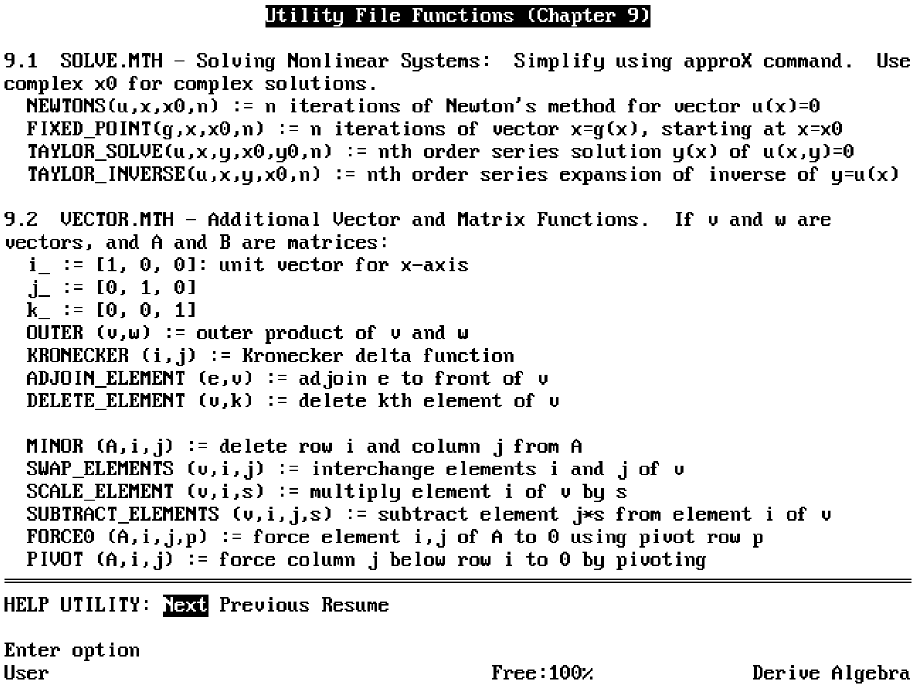
```
#1:  SIN(x)

     d
#2:  — SIN(x)
     dx

#3:  COS(x)

COMMAND: Algebra Center Delete Help Move Options Plot Quit Range Scale Transfer
         Window aXes Zoom
Enter option
Cross x:1           y:1           Scale x:1       y:1         Derive 2D-plot
```

Figure 1.2: An algebra and plot window on the screen



```
                  Utility File Functions (Chapter 9)

9.1  SOLVE.MTH - Solving Nonlinear Systems:  Simplify using approX command.  Use
complex x0 for complex solutions.
  NEWTONS(u,x,x0,n) := n iterations of Newton's method for vector u(x)=0
  FIXED_POINT(g,x,x0,n) := n iterations of vector x=g(x), starting at x=x0
  TAYLOR_SOLVE(u,x,y,x0,y0,n) := nth order series solution y(x) of u(x,y)=0
  TAYLOR_INVERSE(u,x,y,x0,n) := nth order series expansion of inverse of y=u(x)

9.2  VECTOR.MTH - Additional Vector and Matrix Functions.  If v and w are
vectors, and A and B are matrices:
  i_ := [1, 0, 0]: unit vector for x-axis
  j_ := [0, 1, 0]
  k_ := [0, 0, 1]
  OUTER (v,w) := outer product of v and w
  KRONECKER (i,j) := Kronecker delta function
  ADJOIN_ELEMENT (e,v) := adjoin e to front of v
  DELETE_ELEMENT (v,k) := delete kth element of v

  MINOR (A,i,j) := delete row i and column j from A
  SWAP_ELEMENTS (v,i,j) := interchange elements i and j of v
  SCALE_ELEMENT (v,i,s) := multiply element i of v by s
  SUBTRACT_ELEMENTS (v,i,j,s) := subtract element j*s from element i of v
  FORCE0 (A,i,j,p) := force element i,j of A to 0 using pivot row p
  PIVOT (A,i,j) := force column j below row i to 0 by pivoting

HELP UTILITY: Next Previous Resume

Enter option
User                                     Free:100%              Derive Algebra
```

Figure 1.3: A help screen

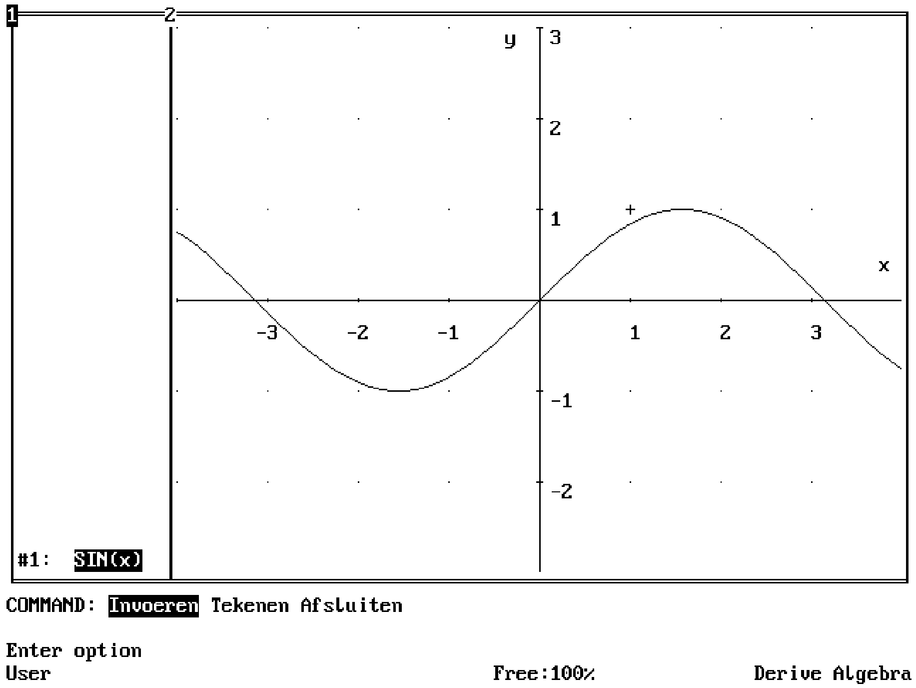


Figure 1.4: A customized Dutch Derive menu

1.5 Software Documentation

The *Derive* User Manual is the main source for information about *Derive*. In this book of 373 pages is explained how to use the software and what is available. Many examples explain the functionality of the software. The *Derive* User Manual has been translated into other languages.

1.6 Licensing Possibilities

A single license copy of *Derive* allows usage of the software on at most one computer at a time or on one computer terminal in a network at a time. Educational Lab licenses, site licenses, and home study licenses are also available.

1.7 Distributors/Dealers

In the following European countries official Distributors/Dealers of Derive are present (December 22, 1994):

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Bromley, Kent BR1 2NE
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Fax: +44 81 4671754

United Kingdom

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Weston on the Green
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Fax: +44 869 89654

United Kingdom

System Science
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London N1 9JF
Phone: +44 71 8376411
Fax: +44 71 8331022

1.8 Derive Addresses

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Fax: +1 808 735 1105
Email: swh@aloha.com

Soft Warehouse Europe GmbH
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Austria
Phone: +43 7236 3297
Fax: +43 7236 3769
Email: kutzler@swp.co.at

Derive User Group
Attn. Josef Böhm
D'Lust 1
A-3042 Würmla
Austria
Phone/Fax: +43 2275 8207

International Derive Journal
Paramount Publishing
Campus 400, Mayland Avenue
Hemel Hempstead
Herts. HP2 7EZ, United Kingdom
Phone: +44 442 882 164
Fax: +44 442 882 265

The **Derive Internet Mailing List** is a new electronic discussion list set up after the First International Derive Conference at Plymouth in July 1994. It covers matters relating to the computer algebra package *Derive* and its use in teaching. You can join by sending a message to `mailbase@mailbase.ac.uk` with the single line `join derive-news firstname lastname` as the text of the message. To contribute to the derive mailing list, send email to the address `derive-news@mailbase.ac.uk`. The bulletin is maintained by the CTI Centre for Mathematics and Statistics in Birmingham, UK.

1.9 Long Description

1.9.1 General Description

Derive is a general purpose symbolic calculator which is continually being developed by David Stoutemyer and Albert Rich of Soft Warehouse, Inc. It is the successor of muMATH. A design goal that distinguishes *Derive* from all other computer algebra systems is the impressive effort to make computer algebra possible on simple, cheap, and readily available hardware platforms. This design goal is already met in the sense that the software fits on a diskette and works on almost any personal computer. *Derive* is also the only computer algebra system that is available on the palmtop calculator.

Another important design issue is *Derive*'s ease of use. Most mathematical operations and manipulations can be carried out in the software via its menu-driven user interface. Furthermore, via arrow keys you can easily and quickly select part of an expression to work further with. The *Derive* screen can be split into special purpose parts such as algebra windows and graphics windows. Screen dumps and saving of plots in TIF files enable the user to combine *Derive* with his/her favorite word processor and prepare good-quality mathematical documents. In classroom situations, the demonstration mode of *Derive* is convenient: Step by step one can go through a calculation. The *Derive* menu and the annotations can be customized. E.g., frequently used menu commands can be moved to the top-level menu, menu commands can be hidden, and menu command names or annotations can be translated into other languages.

Derive can handle many of the mathematical computations learned in secondary school, polytechnic, and university mathematics, and it can handle two- and three-dimensional graphing. To mention a few features, *Derive* offers mathematical knowledge of common mathematical functions and their simplification rules, it provides you with calculus tools for differentiation, integration, computing of limits and Taylor series, and analytical solving of ordinary differential equations, and it allows you to do all basic arithmetic of vectors and matrices (addition and multiplication, inner and outer product, determinant, transpose, inverse, eigenvalues and eigenvectors, and so on). With *Derive* you can graph functions in one or two unknowns in various coordinate systems. Many options like scales, center, viewpoint, grid, and color can be changed via the menu. The 3rd party AcroSpin software allows you to rotate, translate, and scale 3-dimensional wire frames and point clouds on the fly.

Derive also incorporates some functional programming facilities that allow the user to define mathematical procedures. In the *Derive* language you have iteration, the if-then-else control structure, logical and relational operators, and recursion at your disposal. All of *Derive*'s own utility functions are written in the *Derive* language. Their use is well-documented in the Derive User Manual, which is translated into other languages.

1.9.2 Features

Features of *Derive* include:

Arithmetic

Exact arithmetic to many digits;
Approximate arithmetic to user-defined precision;
Mixed exact/approximate arithmetic;
Integer factoring, greatest common divisors, and prime numbers;
Complex and infinite arithmetic.

Algebra

Simplification formulae, expansion or factorization of polynomials;
Declaration of variable domains and values;
Numeric solving of equations at desired precision;

Algebraic solving of equations and inequalities;
Solving of systems of linear equations.

Graphics

2D Plotting:
  Plotting in rectangular or polar coordinates;
  Parametric plotting and space curves;
  Implicit Plots;
  Locating and tracing points by movable cross;
  Optional automatic vertical scaling;
  Choice of center of plot;
  Easy-to-handle plot range specification;
  Zooming in or out.

3D Plotting:
  Wire frame plots with hidden line removal;
  Choice of viewpoint and plot scale;
  Optional automatic vertical centering and scaling.

Calculus

Symbolic limits, derivatives, and integrals;
Arc lengths, area, volume, and related functions;
Taylor and Fourier series;
Laplace transforms;
Exact solving of ordinary differential equations;
Runge-Kutta method for systems of ODE's.

Matrix and Vector Calculus

Transpose, determinant, trace, and inverse of matrices;
Eigenvalues and eigenvectors;
Dot, cross and outer products;
Differential and integral vector calculus.

Functions

Exponential, logarithmic, trigonometric, and hyperbolic functions;
Complex-valued functions;
Probability, statistics, financial, and annuity functions;
Bessel, hypergeometric, Chi-square, Zeta, and other special functions;
Pseudo-random number generator.

Derive Language

If-then-else conditional programming;
Logical and relational operators;
Recursive programming;
Iteration and recurrence relations;
Selection in vectors.

Menu-Driven Interface

Split and/or overlaid algebra and plot windows;
2D mathematical display of formulae;
Highlighting and extraction of subexpressions;
Demo facilities;
CGA, EGA, VGA, MCGA, and Hercules graphics;
Custamizable menu;
Simple annotation of expressions.

Input/Output

Saving and loading of expressions in files;
Multi-file loading;
(Color) printing of plots, expressions, and screens;
Saving of graphical images as TIF files;
Loading of numeric data;
Code generation in C, Fortran, Pascal, and Basic.

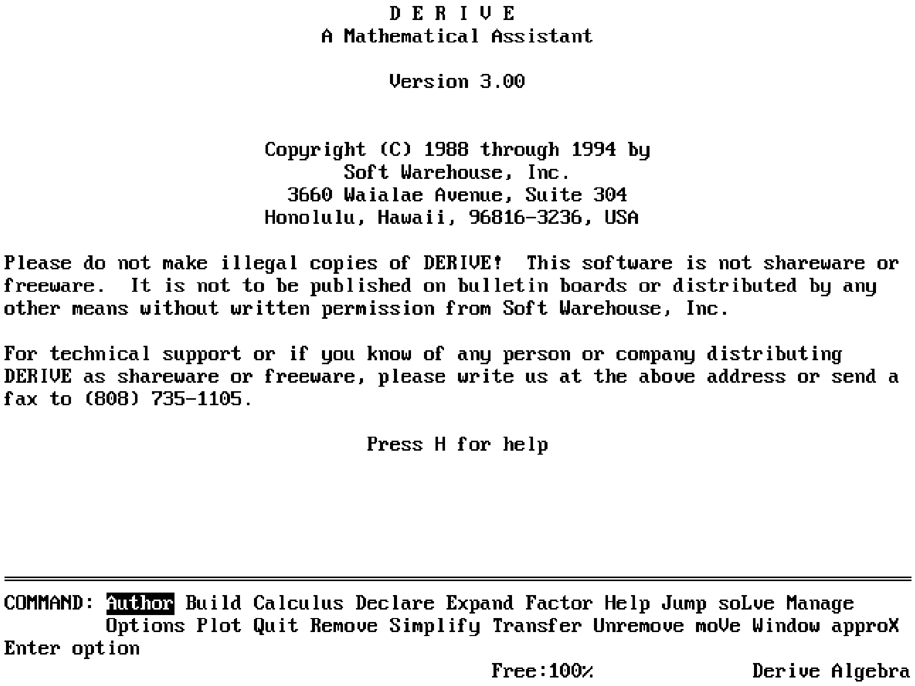
Chapter 2

A Tour of Derive

In this chapter we show most features of *Derive* in the form of a pseudo *Derive* session interleaved with explanatory remarks.

2.1 Launching Derive

To start *Derive* enter DERIVE from the DOS shell. The system comes up with the initial screen (see Figure 2.1).



```
                              D E R I V E
                      A Mathematical Assistant

                            Version 3.00


                  Copyright (C) 1988 through 1994 by
                         Soft Warehouse, Inc.
                    3660 Waialae Avenue, Suite 304
                  Honolulu, Hawaii, 96816-3236, USA

Please do not make illegal copies of DERIVE!  This software is not shareware or
freeware.  It is not to be published on bulletin boards or distributed by any
other means without written permission from Soft Warehouse, Inc.

For technical support or if you know of any person or company distributing
DERIVE as shareware or freeware, please write us at the above address or send a
fax to (808) 735-1105.

                            Press H for help




COMMAND: Author Build Calculus Declare Expand Factor Help Jump soLve Manage
         Options Plot Quit Remove Simplify Transfer Unremove moVe Window approX
Enter option
                                          Free:100%           Derive Algebra
```

Figure 2.1: The initial *Derive* screen

Like any other *Derive* screen it consists of three parts: the working area, the command menu, and the status line. When you select the menu item **Author** by tapping the space bar (or < TAB > key) enough times until **Author** is high-lighted followed by pressing the < ENTER > key, or when you immediately select the menu item **Author** by pressing the **A** key, you will be prompted to enter an expression. You enter for example

```
sin(x)
```

and *Derive* responds in the working area with

```
1:    SIN(x)
```

when *Derive* is in case-insensitive word input mode (use the **Otion Input** menu to change the settings if necessary).

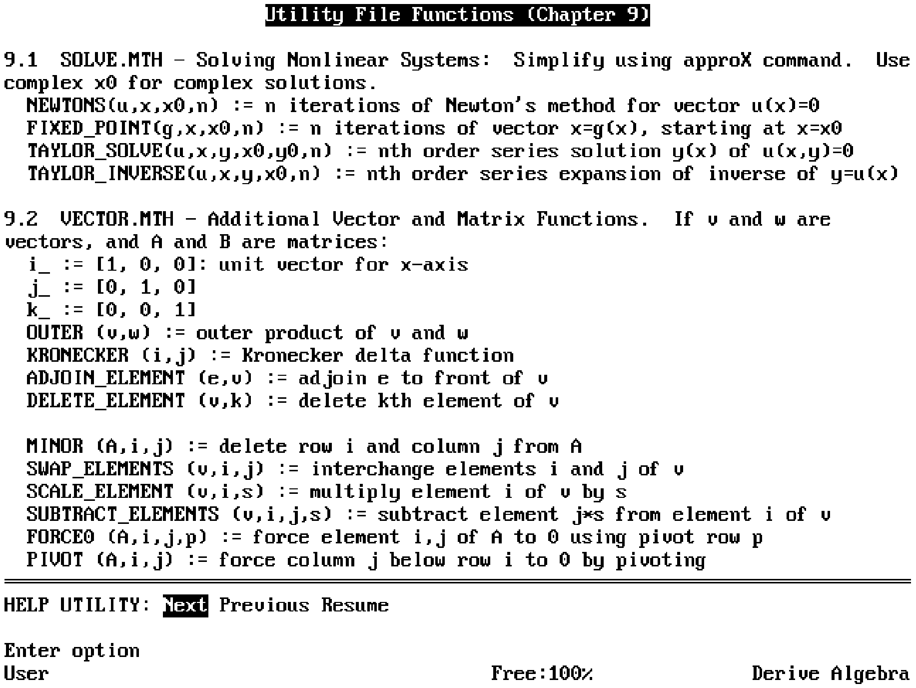
The computer algebra system *Derive* is a symbolic calculator. So, it can for example compute the derivative of the expression. For this, you issue the **Calculus Differentiate** command, and choose to compute the first derivative with respect to x. The system responds with

```
      d
2:    -- SIN(x)
      dx
```

This is what you wanted, so you can select **Simplify** from the menu to activate the "real computation" and get from *Derive*

```
3:    COS(x)
```

There is on-line information available via help screens. A typical help screen looks like Figure 2.2. You can interrupt a lengthy computation in *Derive* or cancel your menu choice by pressing the <Esc> key.



```
                    Utility File Functions (Chapter 9)

9.1  SOLVE.MTH - Solving Nonlinear Systems:  Simplify using approX command.  Use
complex x0 for complex solutions.
  NEWTONS(u,x,x0,n) := n iterations of Newton's method for vector u(x)=0
  FIXED_POINT(g,x,x0,n) := n iterations of vector x=g(x), starting at x=x0
  TAYLOR_SOLVE(u,x,y,x0,y0,n) := nth order series solution y(x) of u(x,y)=0
  TAYLOR_INVERSE(u,x,y,x0,n) := nth order series expansion of inverse of y=u(x)

9.2  VECTOR.MTH - Additional Vector and Matrix Functions.  If v and w are
vectors, and A and B are matrices:
  i_ := [1, 0, 0]: unit vector for x-axis
  j_ := [0, 1, 0]
  k_ := [0, 0, 1]
  OUTER (v,w) := outer product of v and w
  KRONECKER (i,j) := Kronecker delta function
  ADJOIN_ELEMENT (e,v) := adjoin e to front of v
  DELETE_ELEMENT (v,k) := delete kth element of v

  MINOR (A,i,j) := delete row i and column j from A
  SWAP_ELEMENTS (v,i,j) := interchange elements i and j of v
  SCALE_ELEMENT (v,i,s) := multiply element i of v by s
  SUBTRACT_ELEMENTS (v,i,j,s) := subtract element j*s from element i of v
  FORCE0 (A,i,j,p) := force element i,j of A to 0 using pivot row p
  PIVOT (A,i,j) := force column j below row i to 0 by pivoting
================================================================================
HELP UTILITY: Next Previous Resume

Enter option
User                                     Free:100%              Derive Algebra
```

Figure 2.2: A help screen

Now everything is ready for our tour of *Derive*. Throughout the tour we shall use some conventions: typewriter style words like **soLve** refer to a menu command in *Derive*. The boldface character is the capital character that can be typed to trigger the menu command. We shall refer to a sequence of commands entered from successive menus by concatenating the menu option names, like e.g. in the above phrase "issue the **Declare Variable** command". We shall often use *Derive* screen dumps to show results. Note that this also means that the commands are not displayed.

2.2 Arithmetic

Derive provides three modes of computing with numbers: exact, arbitrary precision, and mixed arithmetic. Let us first look at exact arithmetic. First, issue the **Option Precision** command and make sure that the `Exact` mode is chosen. Next, **Author** `3^5` to get the number

```
       5
4:    3
```

Derive only displays the number in exponential format, but to get it in ordinary number format you must **Simplify** it further

```
5:    243
```

Unlike a numerical calculator, *Derive* can compute exactly with large numbers. For example, you can enter 333^{555} and simplify it.

```
         555
6:    333
```

```
7:    904793758405842302339751965847805693334010290411675875418030355725136884.........
```

On the screen, the number is displayed in one line; to see more of the output, repeatedly press the < Ctrl > and right arrow key. Although the number is too large to fit on the screen, *Derive* has internally stored the number in exact mode. So, you can process it further and, for example, **Factor** it.

```
       1110   555
8:    3    37
```

You can compute approximate results. For example, the previous result can be approximated by the **approX** command. After a few seconds, you get

```
                1399
9:    9.04793 10
```

In *Derive*, you can compute at any desired precision. Let us compute the numeric value of $\sqrt{5+2\sqrt{6}}$ in forty digits. On dumb terminals this number is displayed as

```
10:  SQRT(5 + 2 SQRT(6))
```

On PC compatible computers you can also define the square root by pressing < Alt > Q and it is displayed as $\surd$. In exact mode the square roots can be denested (issue the **Simplify** command).

```
10:   SQRT(2) + SQRT(3)
```

Now, set the precision to forty digits by selecting **Option Precision** `Digits 40`.

```
11:   PrecisionDigits := 40
```

By the way, you get to the **Digits** field by pressing the < Tab > key. Henceforth, numerical computations are done in a precision of forty digits; then number $\sqrt{2}+\sqrt{3}$ is approximated to

```
12:   3.146264369941972342329135065715570445512
```

In approximate mode, *Derive* simplifies a number to the simplest rational number that approximates the original number accurate to the current precision and converts it in scientific notation. *Derive* also provides a mixed mode of arithmetic in which irrational number are approximated but rational numbers are not. Note the difference in the next two approximations of $\sqrt{1-8/9}$; the first result is obtained in approximate mode, the second in mixed mode.

```
          /     8 \
13:   SQRT|1 - ---|
          \     9 /

14:   0.3333333333333333333333333333333333333333

       1
15:   ---
       3
```

Derive can also handle complex numbers (the complex number is denoted by $\sharp i$ and can be entered on PC compatible computers by pressing <ALT>i).

```
           1
17:   ----------
       2 + 3 #i

        2     3 #i
18:   ---- - ------
       13      13

      /  /     1    \    /     1    \\
19:   |RE|----------|, IM|----------||
      \  \ 2 + 3 #i /    \ 2 + 3 #i //

      /  2       3 \
20:   |----, - ----|
      \ 13      13 /
```

Derive allows easy access to different branches of complex multi-valued functions. For example, the principle branch choice of $(-8)^{\frac{1}{3}}$ is $1+\sqrt{3}i$, but selecting the **Real** branch option under the **Manage Branch** menu command gives the result -2.

```
21: Branch := Real

        1/3
22: (-8)

23: -2

24: Branch := Principal

        1/3
25: (-8)

26: 1 + SQRT(3) #i
```

Derive “knows” about standard mathematical functions.

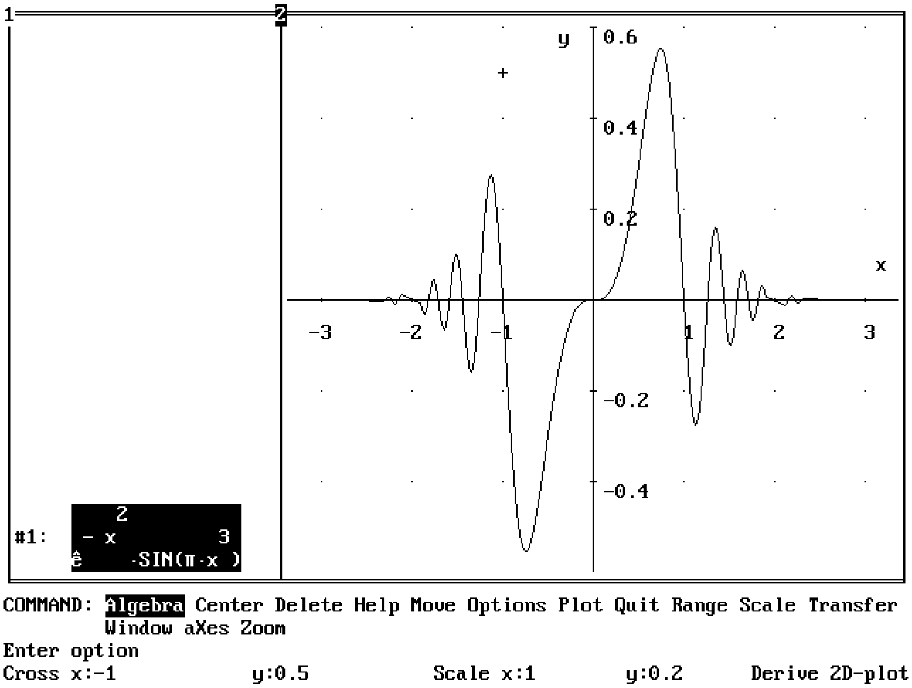
```
27:   SIN(pi/6)

       1
28:   ---
       2
```

```
29:   ACOS(1/2)

       pi
30:   ----
        3
```

The number π can be entered in *Derive* as `pi` or by pressing <ALT>p.

2.3 Graphics

You can plot the graph of a function, say of $e^{-x^2}\sin(\pi x^3)$, over some interval. You select the formula that you want to plot and choose the menu item **`Plot`**. You are asked whether you want the graph beside, below, or on top of the algebra window. In Figure 2.3 we have chosen to split vertically the working area into an algebra window and a plot window. You can switch between the split windows via the `F1` function key. In the plot window we have rescaled the vertical axis and zoomed in to make the picture as large as possible on the screen. The cross is at position $(-1, 0.5)$ as can be seen from the status line.



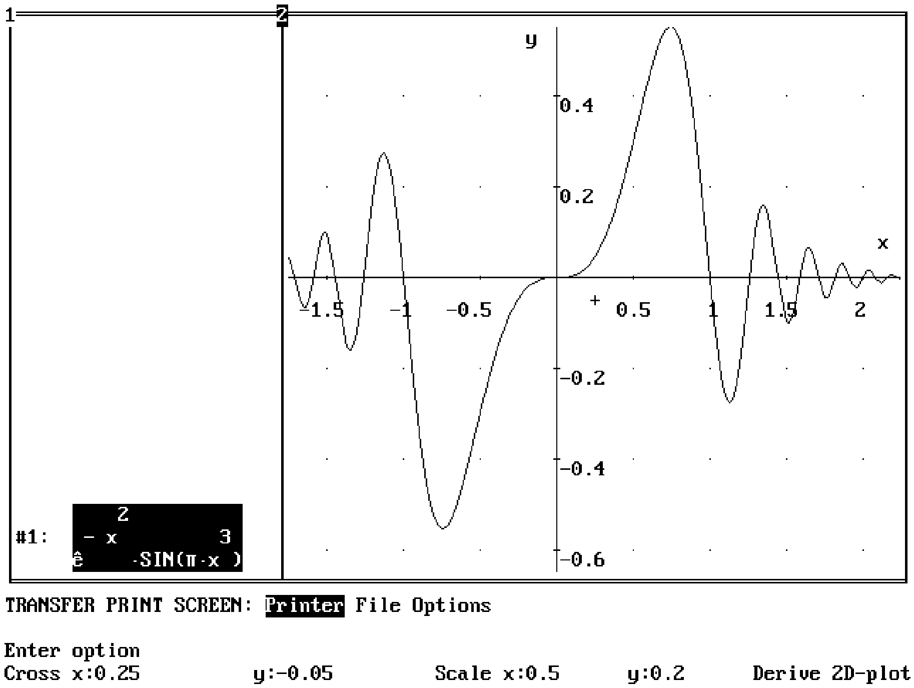
```
           2
#1:     - x          3
       ê      ·SIN(π·x )
```

```
Enter option
Cross x:-1          y:0.5          Scale x:1       y:0.2      Derive 2D-plot
```

Figure 2.3: 2D Plot

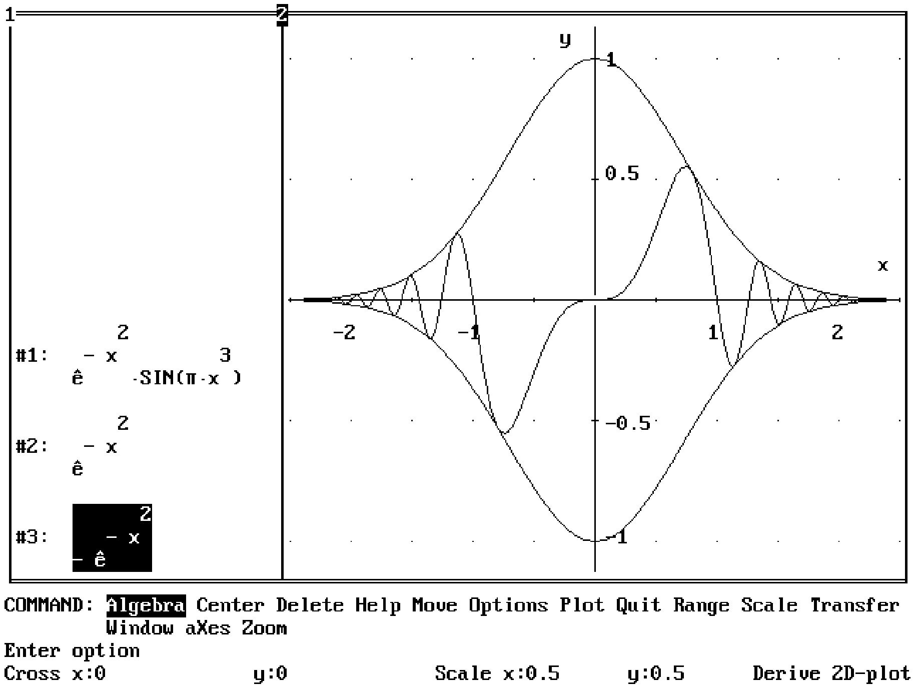
You can change many of the options that determine how the graph actually looks like. In Figure 2.4 we have centered the plot around the point $(0.25, -0.05)$ via the menu item **`Center`**, we have zoomed in along the vertical axis, and have changed the horizontal scaling via the **Range** command.

More than one graph in a plot window is no difficulty: you simply select a formula that you want to plot, too, and plot it in the usual way. It will appear in a new color on the screen. In Figure 2.5 the graphs of e^{-x^2} and e^{-x^2} are also present.



```
#1:       2
       - x        3
      ê     ·SIN(π·x )

TRANSFER PRINT SCREEN: Printer File Options

Enter option
Cross x:0.25          y:-0.05         Scale x:0.5      y:0.2       Derive 2D-plot
```

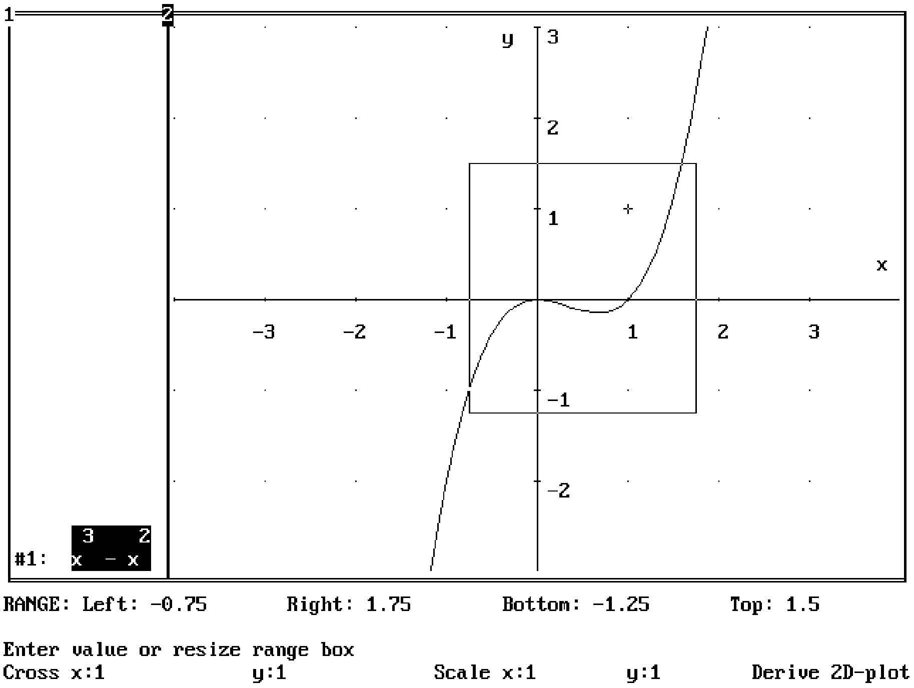
Figure 2.4: 2D Plot with center (0.25,-0.05)



```
          2
#1:    - x         3
      ê     ·SIN(π·x )

          2
#2:    - x
      ê

             2
#3:       - x
      - ê

COMMAND: Algebra Center Delete Help Move Options Plot Quit Range Scale Transfer
         Window aXes Zoom
Enter option
Cross x:0             y:0             Scale x:0.5      y:0.5       Derive 2D-plot
```

Figure 2.5: Three graphs in one 2D plot

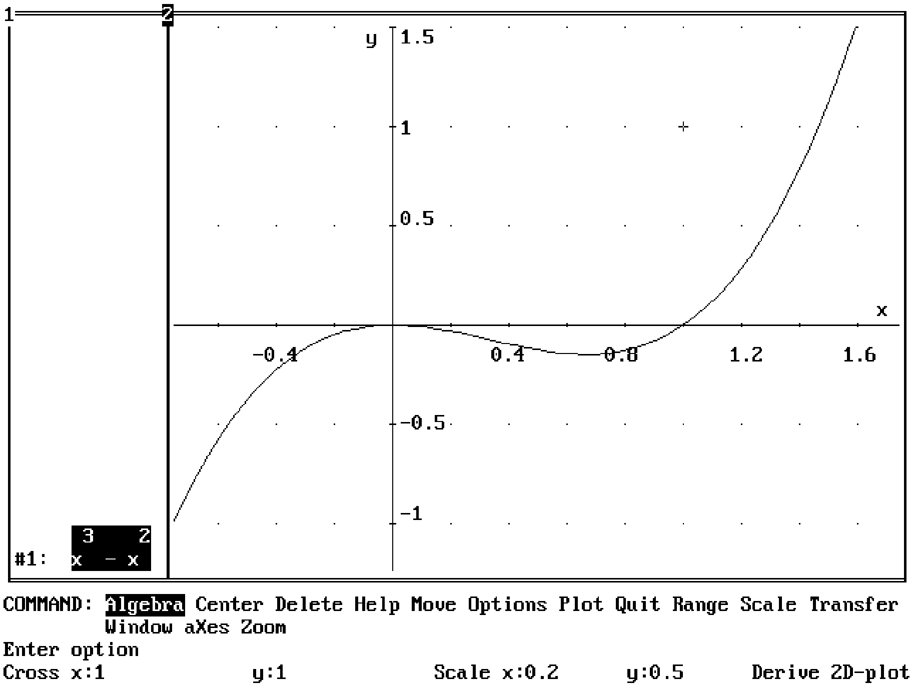
The **Range** command allows you to interactively specify the plot range of 2D plot windows; by the arrow keas you can size and position a rectangular zoom box as shown in Figure 2.6 below.



```
RANGE: Left: -0.75         Right: 1.75         Bottom: -1.25         Top: 1.5

Enter value or resize range box
Cross x:1              y:1              Scale x:1        y:1        Derive 2D-plot
```

Figure 2.6: 2D Plot with zoom box

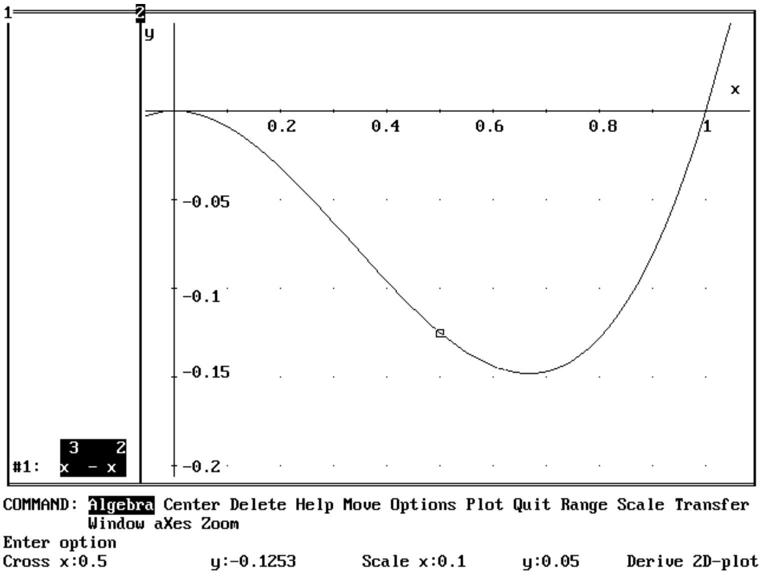
The selection is replotted in Figure 2.7.



```
Enter option
Cross x:1              y:1              Scale x:0.2      y:0.5      Derive 2D-plot
```

Figure 2.7: 2D Plot after zooming in

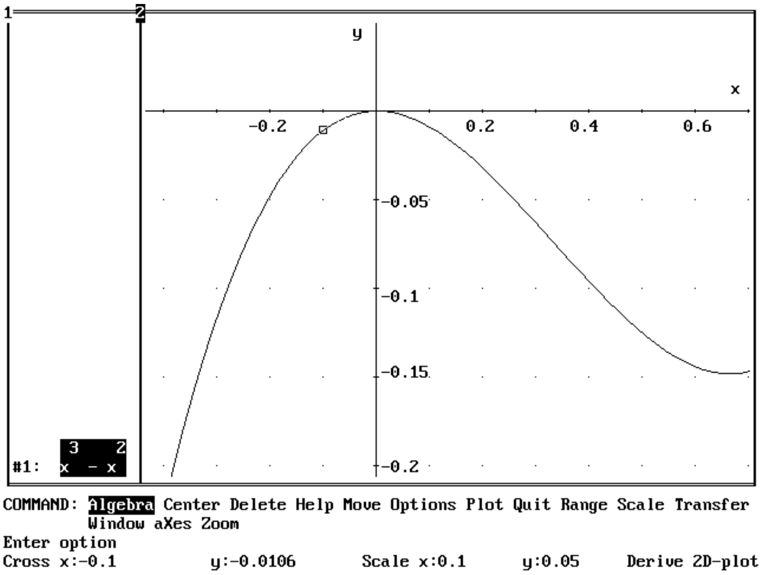
In *Derive* 3 you can trigger `trace mode` by the `F3` function key so that the cross is confined to moving along a plot line. The cross displays in trace mode as a small square box whose center point lines on the plot line (see Figure 2.8).



```
COMMAND: Algebra Center Delete Help Move Options Plot Quit Range Scale Transfer
         Window aXes Zoom
Enter option
Cross x:0.5           y:-0.1253       Scale x:0.1       y:0.05      Derive 2D-plot
```

Figure 2.8: 2D Plot in trace mode

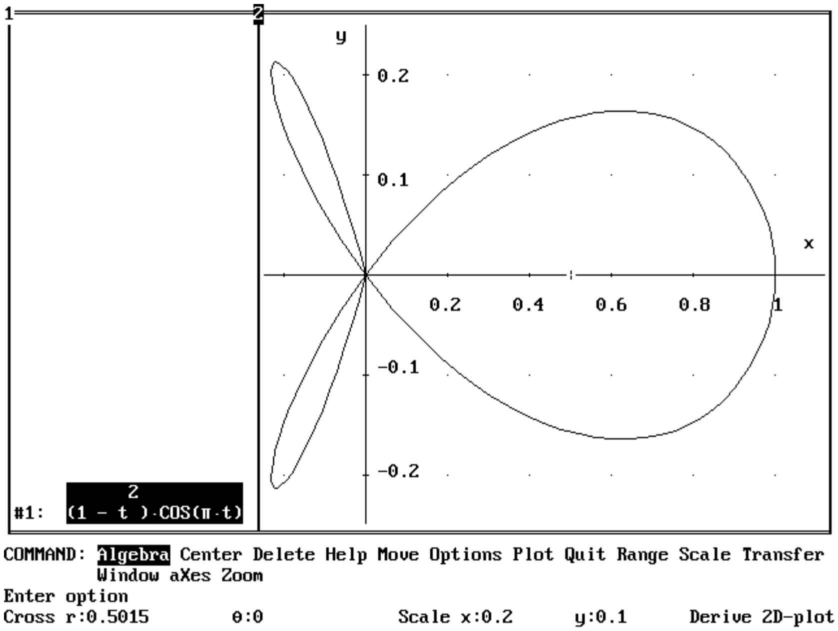
When you issue in the plot 2D window **Options State** command and select the `follow cross mode`, then the plot range is automatically adjusted whenever the cross is moved out of the current plot range. In Figure 2.9 you see the effect of moving the cross in Figure 2.8 to the left out of the plot range: the plot range is adjusted so the cross remains visible.



```
COMMAND: Algebra Center Delete Help Move Options Plot Quit Range Scale Transfer
         Window aXes Zoom
Enter option
Cross x:-0.1          y:-0.0106       Scale x:0.1       y:0.05      Derive 2D-plot
```

Figure 2.9: 2D Plot in trace mode

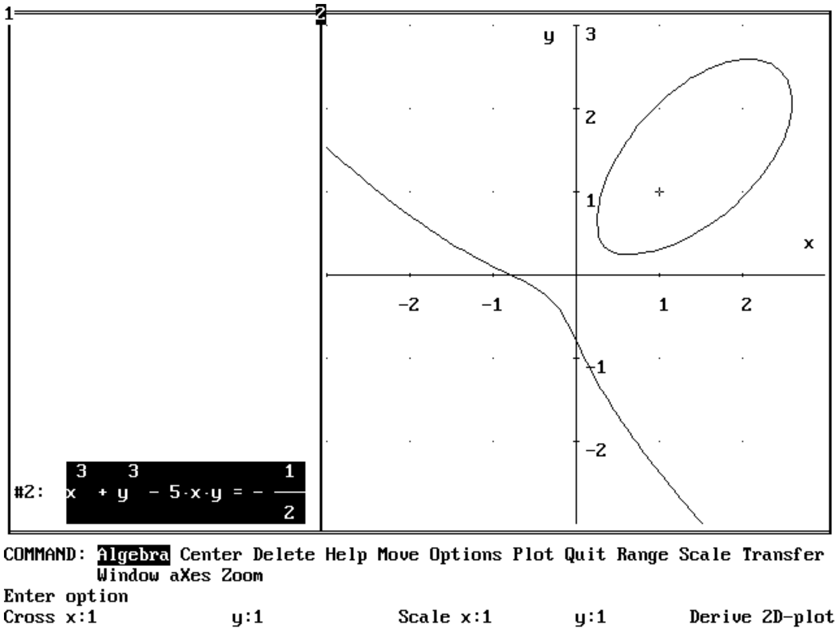
Special two-dimensional graphics is provided, too. For example, you can plot a parametric curve in polar coordinates (see Figure 2.10). All you have to do is to set **Options State** `Coordinates: Polar`.



```
#1:  (1 - t^2)·COS(π·t)
COMMAND: Algebra Center Delete Help Move Options Plot Quit Range Scale Transfer
         Window aXes Zoom
Enter option
Cross r:0.5015        θ:0              Scale x:0.2      y:0.1       Derive 2D-plot
```

Figure 2.10: 2D parametric curve plot

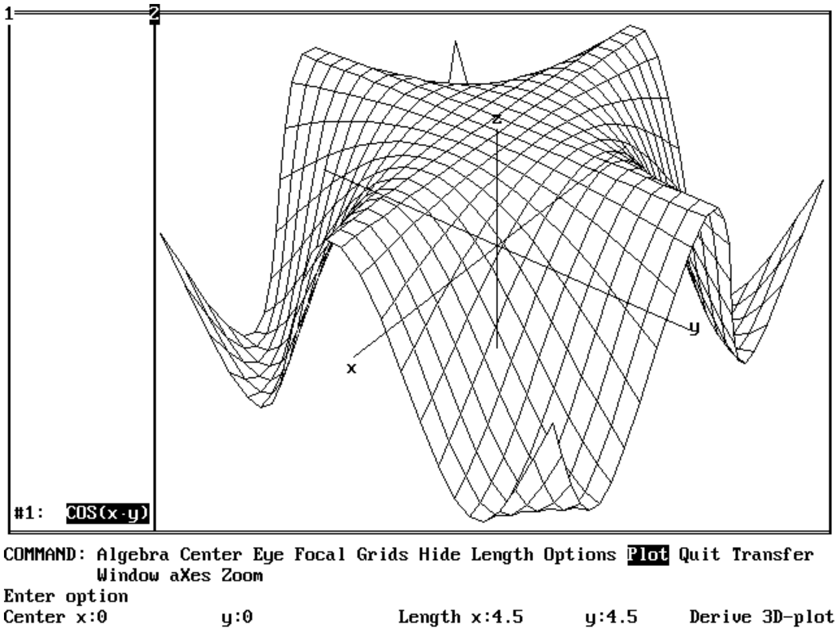
Or you make an implicit plot as shown in Figure 2.11 below.



```
#2:  x^3 + y^3 - 5·x·y = - 1/2
COMMAND: Algebra Center Delete Help Move Options Plot Quit Range Scale Transfer
         Window aXes Zoom
Enter option
Cross x:1             y:1              Scale x:1        y:1         Derive 2D-plot
```

Figure 2.11: implicit plot

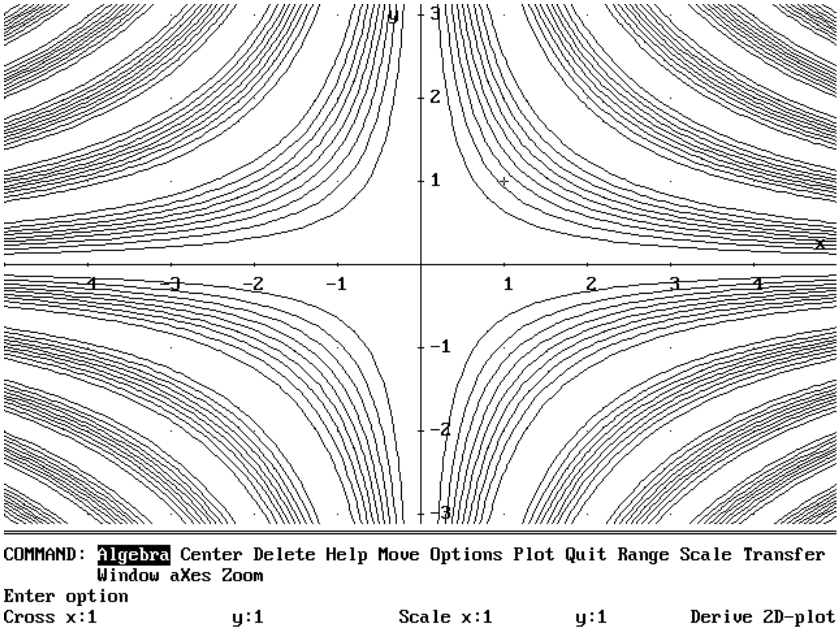
A three-dimensional surface plot is shown in Figure 2.12. Again, many options can and have been changed interactively. The Figure 2.12 was drawn with a 25×25 grid, with a higher eye point than usual.



```
Enter option
Center x:0           y:0                Length x:4.5       y:4.5      Derive 3D-plot
```

Figure 2.12: 3D surface plot

Special 3D graphics is provided, too. For example, Figure 2.13 shows a contour plot of the function $z = \cos(xy)$ as z varies from -1 to 1 in steps of 0.2



```
Enter option
Cross x:1            y:1                Scale x:1        y:1          Derive 2D-plot
```

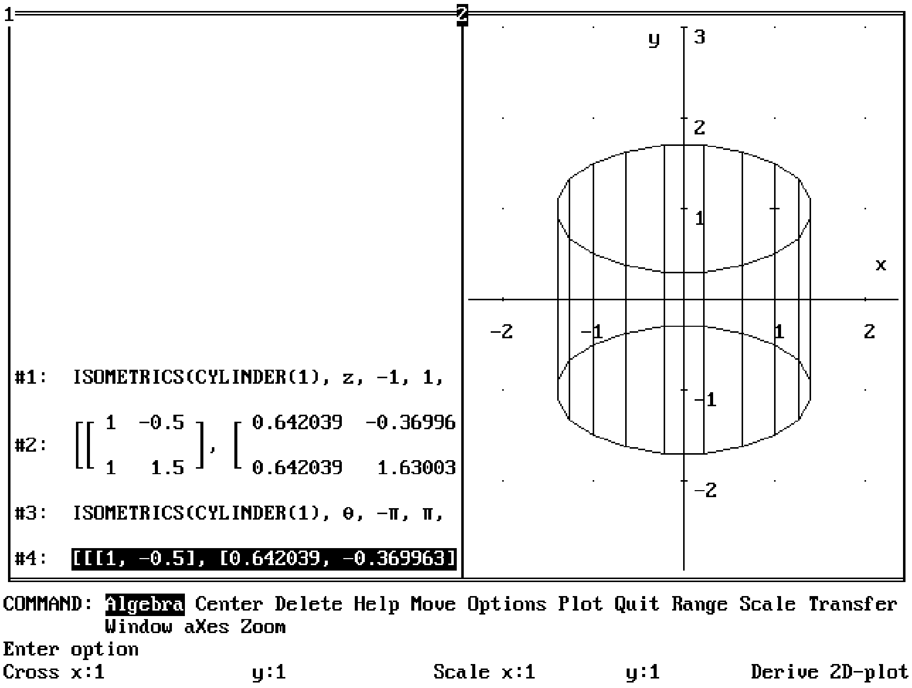
Figure 2.13: contour plot

In the algebra window you enter

```
1: VECTOR( z = COS(x y), z, -1, 1, 0.2 )
```

Simplify and **P**lot the expression.

Figure 2.14 shows a cylinder obtained by the following sequence of commands.



```
#1:   ISOMETRICS(CYLINDER(1), z, -1, 1,

        ┌┌ 1  -0.5 ┐  ┌ 0.642039  -0.36996
#2:     ││           │, │
        └└ 1   1.5 ┘  └ 0.642039   1.63003

#3:   ISOMETRICS(CYLINDER(1), θ, -π, π,

#4:   [[[1, -0.5], [0.642039, -0.369963]

COMMAND: Algebra Center Delete Help Move Options Plot Quit Range Scale Transfer
         Window aXes Zoom
Enter option
Cross x:1            y:1            Scale x:1       y:1       Derive 2D-plot
```

Figure 2.14: 3D Cylinder plot

First, you have to issue the **T**ransfer **L**oad **U**tility command and load the utility file GRAPHICS.MTH. In the algebra window you enter

```
1:   ISOMETRICS( CYLINDER(1), z, -1, 1, 1, theta, -pi, pi, 20 )
```

Issue the appro**X** command and **P**lot the data structure. You can plot it in one color by selecting **O**ptions **C**olor **A**uto `Change: no`; you must also choose **O**ptions **S**tate `Coordinates: Rectangular`, `Mode: Connected`, and `Size: Small` to connect the sample points.

```
     // 1  -0.5 \  / 0.642039  -0.369963 \  / 0.221231  -0.301598 \
2:   ||         |, |                     |, |                     |, ...
     \\ 1   1.5 /  \ 0.642039   1.63003  /  \ 0.221231   1.69840  /
```

At this stage, only vertical lines are shown. By interchange of arguments, you can also connect the points horizontally and get a “real” cylinder.

```
3:   ISOMETRICS( CYLINDER(1), theta, -pi, pi, 20, z, -1, 1, 1 )

4:   [[[1, -0.5], [0.642039, -0.369963], .....
```

In the output we have omitted part of the expressions.

2.4 Algebraic Facilities

2.4.1 Calculus

Differentiation

Though calculus is the study of real functions with infinitesimal tools like differentiation, integration and series expansion, many of its computations are in fact of algebraic nature. For instance, differentiation can be implemented as a set of transformation rules applied to a formula consisting of elementary functions like powers, trigonometric functions, exponential mapping, logarithm, etc. One of the transformation rule that can be realized in *Derive* is

$$\frac{d}{dx}x^n = nx^{n-1}.$$

First you **`Author`** `x^n` to get the formula

```
       n
1:    x
```

Next you issue the **`Calculus Differentiate`** command and you choose to compute the first derivative with respect to x. The system responds with

```
      d   n
2:    -- x
      dx
```

This is what you wanted, so you can **`Simplify`** the expression.

```
         n - 1
3:    n x
```

You can issue successive **`Calculus Differentiate`** commands to obtain mixed partial derivatives. For example, take the expression

```
4:    x TAN (y) + TAN (x)
```

Note that you may also use in input mode a space instead of the * symbol for multiplication. The mixed second-order partial derivative of $x\tan y + \tan x$ with respect to x and y displays as

```
      d  d
5:    -- -- (x TAN (y) + TAN (x))
      dy dx
```

It simplifies to

```
             2
6:    TAN (y)  + 1
```

By the **`Factor`** command it is transformed into

```
          1
7:    --------
             2
      COS (y)
```

Function Definition

Defining a function. e.g.,

$$f : x \mapsto \sqrt{x^2 + \frac{1}{y}}$$

can be done in *Derive* in two ways. You can issue the **Declare Function** command and enter its name and definition (by `SQRT(x^2+1/y)`). *Derive* displays it on dumb screens as

```
                      / 2    1 \
8:   F (x, y) := SQRT |x  + ---|
                      \      y /
```

Alternatively, you **Author** this line from scratch. On PC compatible computers you can also define the square root by pressing <ALT>`Q` and it is displayed as √ . Now, `F (u, v)` simplifies to

```
          /  2       \
          | u  v + 1 |
9:   SQRT |----------|
          \     v    /
```

`F (3, 4)` simplifies to the exact number

```
      SQRT (37)
10:  -----------
          2
```

The appro**X**imate command does credit to its name and yields the corresponding numerical value. Before this you can issue the **Options Precision** command to set the number of digits. E.g., $\frac{\sqrt{37}}{2}$ approximates in 20 digits to

```
11:  3.0413812651491098444
```

Integration

Use the **Calculus Integrate** command to compute antiderivatives and definite integrals. After you select the expression, *Derive* prompts you to enter the name of the variable, the lower limit and the upper limit. When you do not specify values for the limits *Derive* will compute the antiderivative. *Derive* knows all classical integration tricks. For example, it computes integral

$$\int \frac{1}{x^4\sqrt{1+x^2}}\,dx$$

and comes back with the answer

```
             2          2
12:   SQRT (x  + 1) (2 x  - 1)
     ---------------------------
                    3
                 3 x
```

Note that *Derive* follows the style of integral tables and leaves out the integration constant. In contrast with integral tables, *Derive* offers you the possibility to check the answer by differentiation (check the example!).

Derive is quite powerful in computing definite integrals. The following definite integral can only be computed without any effort with *Derive* and not in other computer algebra systems such as Mathematica 2.3, Maple V Release 3, Macsyma 419, or Axiom 2. It is entered as

```
INT( ASIN(1/SQRT(4-x^2)), x, 0, 1 ) =
```

and *Derive* displays the result as

```
      1
     /      /       1      \       pi          / 1 \
     |  ASIN|--------------| dx = ---- - 3*ASIN|---|
13:  |      |           2  |        2          \ 3 /
    /      \ SQRT(4 - x ) /
      0
```

This is a general scheme: if an expression is entered on the author line followed by an = sign, then *Derive* displays an equation whose left side is the unsimplified form of the expression and whose right side is the simplified form of the expression.

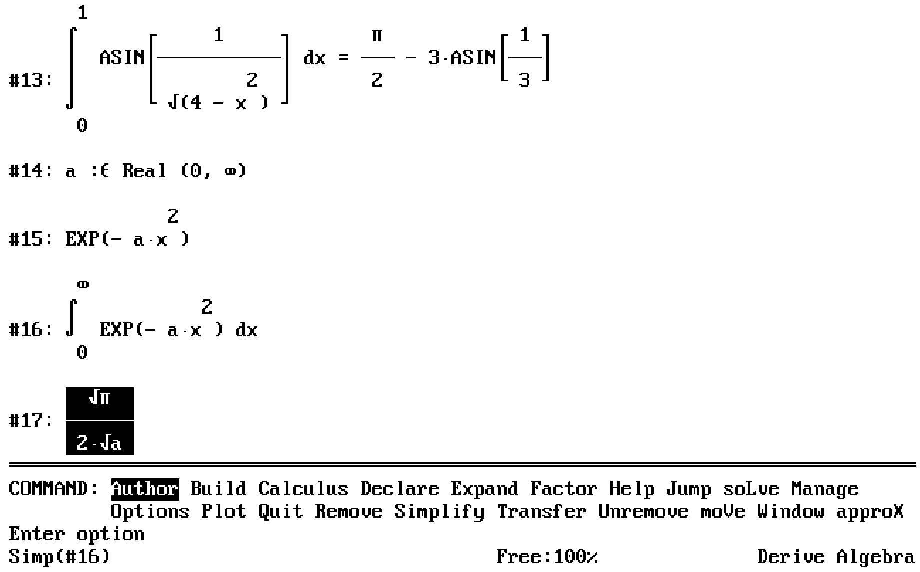
Sometimes you have to specify the range of a variable which occurs in the integrand. For example,

$$\int_0^\infty e^{-ax^2}\,dx$$

can only be computed by *Derive* if you first issue the **`Declare Variable`** command to specify that a is a positive real number. The corresponding *Derive* session looks on a dumb screen like

```
14:   a :epsilon Real (0, inf)


                2
15:   EXP (- a x )


       inf
      /              2
16:   |    EXP (- a x ) dx
      /
       0

       SQRT (pi)
17:   ------------
       2 SQRT (a)
```

On most screens it looks however much better (see Figure 2.15).



```
        1
      ⌠      ┌     1      ┐        π
#13:  │ ASIN │ ────────── │ dx = ─── - 3·ASIN┌1┐
      ⌡      │         2  │        2         └3┘
             └ √(4 - x )  ┘
        0

#14: a :∈ Real (0, ∞)

                  2
#15: EXP(- a·x )

        ∞
      ⌠              2
#16:  ⌡  EXP(- a·x ) dx
        0

      √π
#17: ────
     2·√a
COMMAND: Author Build Calculus Declare Expand Factor Help Jump soLve Manage
         Options Plot Quit Remove Simplify Transfer Unremove moVe Window approX
Enter option
Simp(#16)                              Free:100%            Derive Algebra
```

Figure 2.15: assumptions

When a closed-form of an integral does not exist you can always try to find a numerical approximation by the `approXimate` command.

```
        1
       /
18:    |  SIN (SIN (x)) dx
       /
        0

19:    0.430606
```

Taylor Series

Use the **`Calculus Taylor`** command to compute the Taylor series approximation of an expression around some expansion point and up to some order. For example, the series expansion of $\frac{1}{2-x}$ around 0 up to order 8 is found by *Derive* as

```
              /   1           \
19:    TAYLOR |-------, x, 0, 8|
              \ 2 - x         /

         8       7       6      5      4      3     2
        x       x       x      x      x      x     x     x     1
20:    ----- + ----- + ----- + ---- + ---- + ---- + --- + --- + ---
        512     256     128     64     32     16     8     4     2
```

Limits

Use the **Calculus Limits** command to compute limits or use the `LIM` command in the author line followed by the = sign to show the limit and the result in equational format.

```
                         2
             1 - COS (x)     1
21:   lim  --------------  = ---
      x->0   x SIN (2 x)     2
```

Derive can also compute directional limits.

```
                   SQRT (p)
22:   lim  (1 + p)
      p->0-

23:   1
```

2.4.2 Solving Systems of Equations

Derive can solve systems of *linear* equations by the `soLve` command.

```
24:   [x + a y = 1, a x - y = -1]

      /      1 - a         a + 1 \
25:   |x = --------, y = --------|
      |      2             2     |
      \     a  + 1        a  + 1 /
```

Systems of *nonlinear* equations cannot be solved automatically by Derive. For example, consider the following system of polynomial equations

```
       2    2
26:   x  + y  = 1

27:   x y = 1
```

With the `soLve` command you can rewrite the second equation as

```
           1
28:   x = ---
           y
```

Issue the **Manage** `Substitute` command to substitute this expression for `x` into equation `26`.

```
      / 1 \2    2
29:   |---|  + y  = 1
      \ y /
```

and solve it for `y`. This is in this example basically the same as solving a univariate polynomial equation of fourth degree, which can be done analytically by the `soLve` command.

```
           SQRT(3)     #i
30:   y = --------- + ----
             2          2
```

```
           SQRT(3)     #i
31:   y = --------- - ----
             2         2

             SQRT(3)     #i
32:   y = - --------- + ----
               2         2

             SQRT(3)     #i
33:   y = - --------- - ----
               2         2
```

Four complex solutions. The reciprocal values are the corresponding `x` values. For example, the first `y` value has

```
               1
      ------------------
34:     SQRT(3)     #i
       --------- + ----
          2         2

       SQRT(3)     #i
35:   --------- - ----
         2         2
```

as corresponding `x` values.

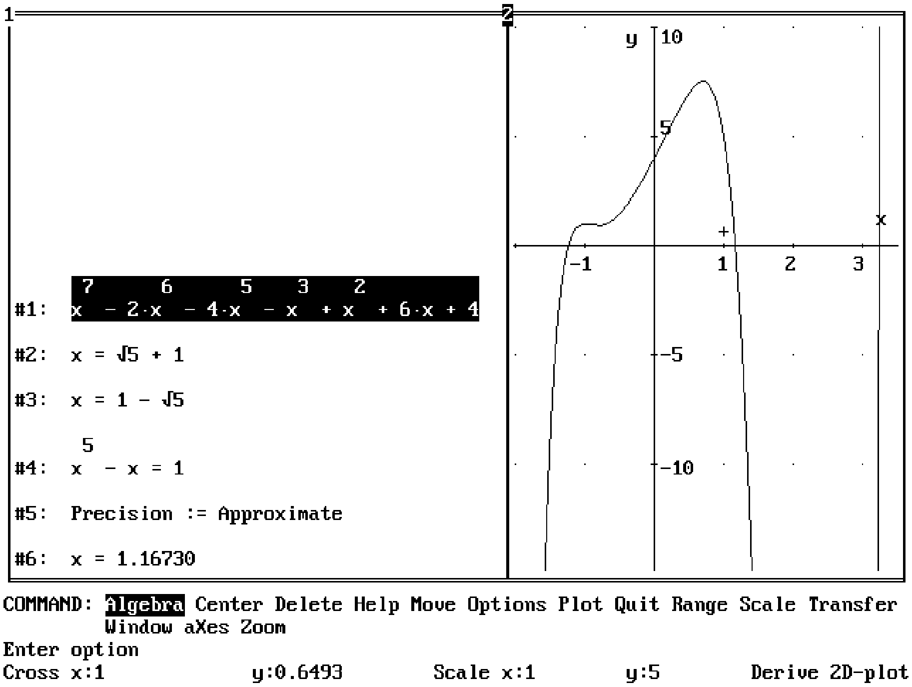
The `soLve` command can also be used to find roots of a polynomial of degree less than four. For higher degree polynomials or polynomial equations, the `soLve` command will try to find a numerical approximation of a real root after you have turned *Derive* in approximate mode. The system will ask you for the domain to search for a solution. A graph of the polynomial may indicate where to look for real roots. Figure 2.16 on the next page illustrates the root finding of a seventh degree polynomial. In this screen dump you see that the original polynomial has two real roots that can be found analytically. One is left with the fifth degree polynomial equation $x^5 - x - 1 = 0$ for which one real root can be approximated, viz., $x = 1.16730$.

2.4.3 Ordinary Differential Equations

For first and second order ordinary differential equations provides facilities to solve the ODEs analytically and numerically. We show the classical example of the step response of a linear damped mathematical pendulum

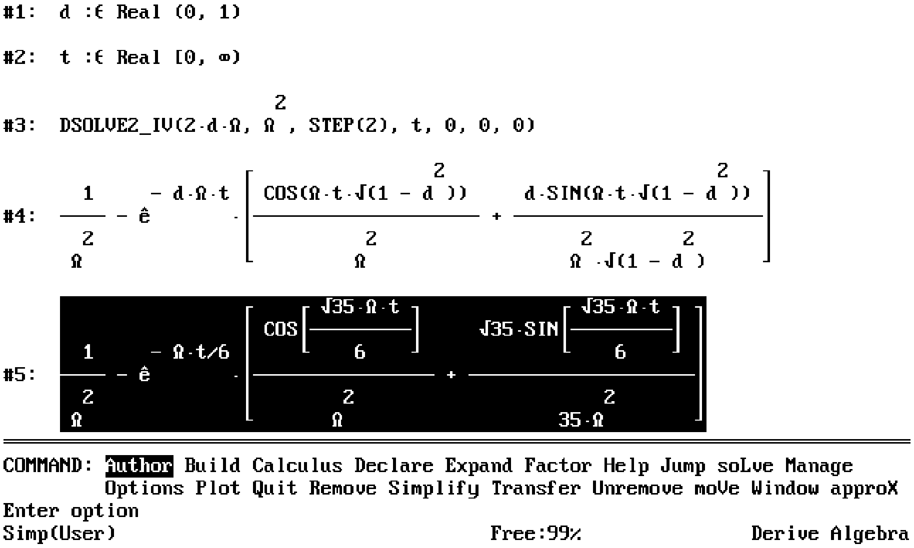
$$y'' + 2\,d\,\omega\,y' + \omega^2 y = \delta(t),$$

with initial values $y(0) = 0, y'(0) = 0$, and underdamping $0 < d < 1$. First, you issue the **`Transfer Load Utility`** command and load the file ODE2.MTH. Next, you issue the **`Declare Variable`** command and specify for `d` the interval $(0, 1)$ and declare `t` as a nonnegative real number. Now, you are ready to enter the initial value problem into *Derive* and solve the ODE analytically. In Figure 2.17 you see the analytical solution and the specialization $d = 1/6$.



```
       7      6      5     3     2
#1:  x  - 2·x  - 4·x  - x  + x  + 6·x + 4

#2:  x = √5 + 1

#3:  x = 1 - √5

      5
#4:  x  - x = 1

#5:  Precision := Approximate

#6:  x = 1.16730

COMMAND: Algebra Center Delete Help Move Options Plot Quit Range Scale Transfer
         Window aXes Zoom
Enter option
Cross x:1            y:0.6493        Scale x:1        y:5        Derive 2D-plot
```

Figure 2.16: Root finding



```
#1:  d :∈ Real (0, 1)

#2:  t :∈ Real [0, ∞)

                          2
#3:  DSOLVE2_IV(2·d·Ω, Ω , STEP(2), t, 0, 0, 0)

                                        2                          2
      1       - d·Ω·t   ┌ COS(Ω·t·√(1 - d ))     d·SIN(Ω·t·√(1 - d )) ┐
#4:  ──── - ê         · │ ─────────────────── + ───────────────────── │
       2                │          2                2         2       │
      Ω                 └         Ω                Ω ·√(1 - d )       ┘

                        ┌      ┌ √35·Ω·t ┐               ┌ √35·Ω·t ┐ ┐
                        │  COS │ ─────── │       √35·SIN │ ─────── │ │
      1       - Ω·t/6   │      └    6    ┘               └    6    ┘ │
#5:  ──── - ê         · │ ──────────────────── + ─────────────────── │
       2                │          2                      2          │
      Ω                 └         Ω                   35·Ω           ┘

COMMAND: Author Build Calculus Declare Expand Factor Help Jump soLve Manage
         Options Plot Quit Remove Simplify Transfer Unremove moVe Window approX
Enter option
Simp(User)                              Free:99%             Derive Algebra
```

Figure 2.17: Solving a 2nd order ODE

2.4.4 Matrices

To generate a 3×3 matrix whose (i,j)-th element is $\gcd(i,j)$ you can issue the **Declare Matrix** command and fill in on request all matrix elements.

```
         / 1  1  1 \
         |         |
1:       | 1  2  1 |
         |         |
         \ 1  1  3 /
```

Alternatively, you can enter the following command to assign `M` the requested matrix.

```
2:   m := VECTOR(VECTOR(GCD(i, j), i, 1, 3), j, 1, 3)
```

```
         / 1  1  1 \
         |         |
3:       | 1  2  1 |
         |         |
         \ 1  1  3 /
```

Standard matrix operations are available.

```
          / 3  4   5 \
      2   |          |
4:   m  = | 4  6   6 |
          |          |
          \ 5  6  11 /
```

```
5:   DET(m) = 2
```

```
           /   5           1  \
           |  ---   -1  - --- |
           |   2           2  |
      -1   |                  |
6:   m   = |   -1    1    0   |
           |                  |
           |    1         1   |
           | - ---   0   ---  |
           \    2         2   /
```

```
     /   5           1  \
     |  ---   -1  - --- |
     |   2           2  |   / 1  1  1 \   / 1  0  0 \
     |                  |   |         |   |         |
7:   |   -1    1    0   | . | 1  2  1 | = | 0  1  0 |
     |                  |   |         |   |         |
     |    1         1   |   \ 1  1  3 /   \ 0  0  1 /
     | - ---   0   ---  |
     \    2         2   /
```

```
8:   EIGENVALUES(m, alpha)

     /                          /     / 3 SQRT(111) \ \
     |                          | ATAN|-------------| |
     |                          |     \      37     / |
9:   |             4 SQRT(3) SIN|---------------------|
     |                          \          3          /
     |alpha = 2 - --------------------------------------,
     \                               3


                           /     / 3 SQRT(111) \        \
                           | ATAN|-------------|        |
                           |     \      37     /     pi |
              4 SQRT(3) SIN|--------------------- + ----|
                           \          3               3 /
     alpha = --------------------------------------------- + 2,
                                      3

                               /     / 3 SQRT(111) \        \ \
                               | ATAN|-------------|        | |
                               |     \      37     /     pi | |
                  4 SQRT(3) COS|--------------------- + ----| |
                               \          3               6 / |
     alpha = 2 - ---------------------------------------------|
                                     3                        /

10:  [alpha = 1.46081, alpha = 4.21432, alpha = 0.324869]
```

The procedure `eigenvalues` tries to compute the eigenvalues as explicitly as possible (therefore, it introduces radicals). The `approX` command gives the numerical approximations. Similarly, eigenvectors can be computed both analytically and numerically.

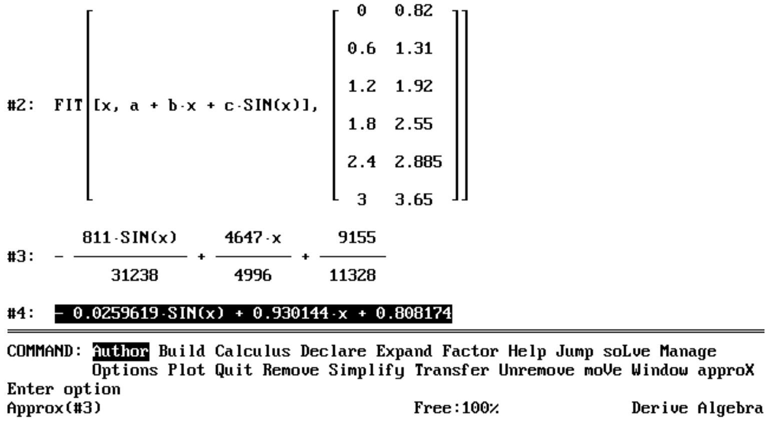
2.5 Numerical Calculus

Besides numerical integration and numerical solving of systems of equations *Derive* offers numerical tools like curve fitting and numerical solving of ODEs. As an example we shall consider linear regression and curve fitting, and Newton-Raphson's method for finding roots of functions.

In an algebra window you issue the **Declare Matrix** and select the number of rows and 2 columns. Enter the data in the row order 0, 0.82, 0.6, 1.31, ...

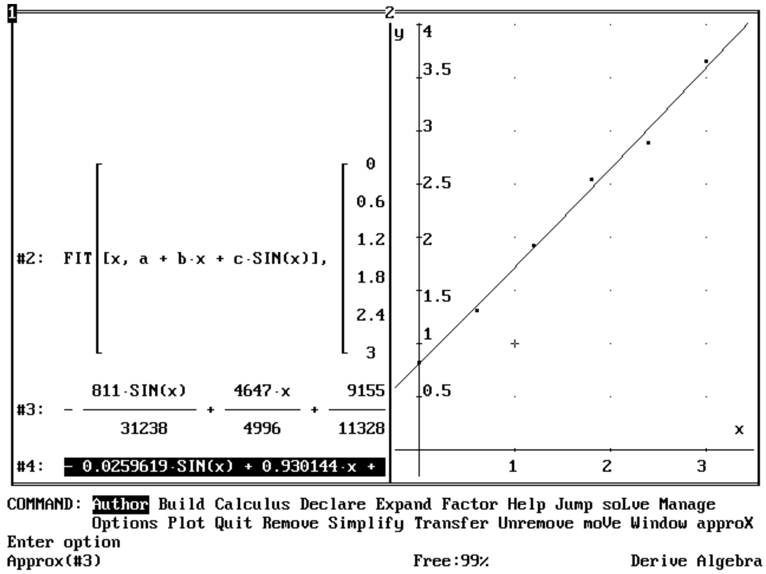
```
     /  0   0.82 \
     |           |
     | 0.6  1.31 |
     |           |
     | 1.2  1.92 |
1:   |           |
     | 1.8  2.55 |
     |           |
     | 2.4  2.85 |
     |           |
     \  3   3.65 /
```

Use the `Fit` command to find the the equation of the best fit of the form $a + bx + c\sin x$. You **`Author`** `FIT( [x, a + b x + c SIN(x)],` and then press the `F3` function key to insert the highlighted expression (in this case, the data matrix). You finish the command with a bracket. **`S`**`implify` to obtain the equation of the best fit. In Figure 2.18 you see the corresponding *Derive* screen. You can plot the matrix data values and the fitted curve (see Figure 2.19).



```
                                           ┌  0    0.82  ┐┐
                                           │ 0.6   1.31  ││
                                           │ 1.2   1.92  ││
#2:  FIT [x, a + b·x + c·SIN(x)],          │ 1.8   2.55  ││
                                           │ 2.4   2.885 ││
                                           └  3    3.65  ┘┘

        811·SIN(x)     4647·x      9155
#3:  − ─────────── + ──────── + ───────
          31238         4996       11328

#4:  − 0.0259619·SIN(x) + 0.930144·x + 0.808174

COMMAND: Author Build Calculus Declare Expand Factor Help Jump soLve Manage
         Options Plot Quit Remove Simplify Transfer Unremove moVe Window approX
Enter option
Approx(#3)                                    Free:100%          Derive Algebra
```

Figure 2.18: Linear regression



```
                                         ┌  0
                                         │ 0.6
                                         │ 1.2
#2:  FIT [x, a + b·x + c·SIN(x)],        │ 1.8
                                         │ 2.4
                                         └  3

       811·SIN(x)      4647·x       9155
#3:  − ─────────── + ──────── + ───────
          31238         4996       11328

#4:  − 0.0259619·SIN(x) + 0.930144·x +

COMMAND: Author Build Calculus Declare Expand Factor Help Jump soLve Manage
         Options Plot Quit Remove Simplify Transfer Unremove moVe Window approX
Enter option
Approx(#3)                                    Free:99%           Derive Algebra
```

Figure 2.19: Curve fitting

In order to be able apply the Newton-Raphson method for root finding of a function you must first issue the **`T`**`ransfer` **`L`**`oad` **`U`**`tility` command and load the utility file SOLVE.MTH. Then you have the

`Newtons` command at your disposal to do the work (issue the `approX` command instead of the `Simplify` command).

```
5:   NEWTONS([COS(x) - x], [x], [1], 4)

     /     1    \
     |          |
     | 0.750363 |
     |          |
6:   | 0.739112 |
     |          |
     | 0.739085 |
     |          |
     \ 0.739085 /
```

2.6 Programming

Derive's programming facilities are limited to procedural programming constructs such as iteration and choice, some functional programming paradigms, and recursion.

2.6.1 Functional Programming

To start with functional constructs, you can enter a vector of squares in a single command by

```
             2
1:   VECTOR(i , i, 1, 10)

2:   [1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
```

or by entering the **Declare** **vectoR** command and filling out all individual vector elements. To multiply each element by two, you can specify this as an operation to be applied on the whole expression (issue the **Build** command)

```
3:   [1, 4, 9, 16, 25, 36, 49, 64, 81, 100] 2

4:   [2, 8, 18, 32, 72, 98, 162, 200]
```

We give three other examples of mapping a function over a data structure.

```
             i
5:   VECTOR(x , i, -3, 3)

     /  1     1    1          2   3\
     |----, ----, ---, 1, x, x , x |
6:   |  3     2    x               |
     \ x     x                     /
```

```
     d  /  1     1    1          2   3\
     -- |----, ----, ---, 1, x, x , x |
7:   dx |  3     2    x               |
        \ x     x                     /

     /    3       2       1                 2\
     |- ----, - ----, - ----, 0, 1, 2 x, 3 x |
8:   |    4       3       2                  |
     \   x       x       x                   /

                    i + j
9:   VECTOR(VECTOR(x      - 1, i, 1, 2), j, 1, 2)

     /  2       3     \
     | x  - 1  x  - 1 |
10:  |                |
     |  3       4     |
     \ x  - 1  x  - 1 /
```

Factorizing all matrix elements gives

```
     /                                   2            \
     |    (x - 1) (x + 1)      (x - 1) (x  + x + 1)   |
11:  |                                                |
     |           2                             2      |
     \ (x - 1) (x  + x + 1)  (x - 1) (x + 1) (x  + 1) /
```

2.6.2 Iteration

With a number x_{old} we associate a new value x_{new} by applying the function f, i.e.

$$x_{new} = f(x_{old}), \quad \text{or} \quad x_{new} \xleftarrow{f} x_{old}.$$

Next, we interpret the number x_{new} as an new x_{old} in the above functional application. We speak of the n-fold iteration when we repeat the update formula $x \xleftarrow{f} x$ n times starting with x_{old}. E.g, for $n = 4$ we get

$$x_{new} = f(f(f(f(x_{old})))), \quad \text{or} \quad x_{new} \xleftarrow{f} x \xleftarrow{f} x \xleftarrow{f} x_{old}.$$

In a procedural programming language such as *Derive*, an n-fold iteration starting at x_0 can be implemented by the function call `ITERATE( f(x) ,x , x_0, n )` followed by the `Simplify` command. As a concrete example, we consider the following arithmetic sequence

$$3, 8, 13, 18, \ldots,$$

i.e.

$$v_0 = 3, \quad \text{and} \quad v_{i+1} = v_i + 5, \text{ for } i > 0.$$

The term v_{10} can be computed in the following way.

```
12:  ITERATE( x+5, x, 3, 10 )

13:  53
```

With `ITERATES` you get the vector of computed result.

```
14:  ITERATES( x+5, x, 3, 10 )

15:  [3, 8, 13, 18, 23, 28, 33, 38, 43, 48, 53]
```

When we want to approximate a zero a of the (2-times differentiable) function f by the Newton-Raphson method from the starting point s, we consider the iteration function

$$it : x \longrightarrow x - \frac{f(x)}{f'(x)}$$

and start applying it to s. We can implement it in *Derive* in a one-line procedure.

```
16:  FINDROOT(f,x,x0,n) := ITERATE(x-f/Dif(f,x), x, x0, n)


                2
17:  FINDROOT(x  - 2, x, 1, 5)
```

S`implify` yields

```
      886731088897
18:  --------------
      627013566048
```

whereas `appro`**X** gives the following result.

```
19:  1.41421
```

Our procedure `FINDROOT` is almost the same as the `Newton` routine in the utility file SOLVE.MTH.

```
NEWTONS(u,x,x0,n):=NEWTON_AUX(APPEND(VECTOR(DIF(u,ELEMENT(x,k_)),k_,1,DIMENSIO~
N(x)),[u])`,x,x0,n)
```

The tilde at the end of the first line in the definition of `NEWTONS` is the continuation character in *Derive*.

2.6.3 Recursion

Recursive procedures are allowed in *Derive*. For example, the factorial function can be defined as follows.

```
1:   FACTORIAL(n) := IF(n < 1, 1, n FACTORIAL(n - 1))

2:   FACTORIAL(5)

3:   120
```

This example also illustrates the `IF` control structure of choice in *Derive*. Although recursive programming is allowed in *Derive*, it may not give fast implementations. Below, the function `F` computes Fibonacci polynomials in a slow way.

```
4:   F(n, x) := IF(n < 2, n, x F(n - 1, x) + F(n - 2, x))

5:   F(15, y)

      14        12        10          8         6         4        2
6:   y   + 13 y   + 66 y   + 165 y  + 210 y  + 126 y  + 28 y  + 1
```

On a 486 WindDX2/50MHz PC this computations takes about twenty seconds, whereas the following iterative procedure computes the polynomial in one second.

```
7:   F(n, x) := element( ITERATE([ELEMENT(v, 2), x ELEMENT(v, 2) +~
                                  ELEMENT(v,1)], v, [0, 1], n), 1 )

8:   F(15, y)

      14        12        10         8         6         4        2
9:   y   + 13 y   + 66 y   + 165 y  + 210 y  + 126 y  + 28 y  + 1
```

The idea in the second procedure is to use a two-element vector `v` to retain the two most recently computed Fibonacci polynomials in the computation.

2.7 Simplification Rules

Derive provides the menu items **`Simplify`**, **`Expand`**, and **`Factor`** to simplify expressions in which special functions like trigonometric functions, logarithm, and exponential occur. *Derive* is very precise in checking whether transformations of formulae are valid or not on the domain of computation. If necessary, you have to issue the **`Declare Variable`** command and restrict the domain of a variable. For functions with simplification rules in two directions such as the rule $\ln(x\,y) = \ln x + \ln y$ you also have to specify the direction via the **`Manage`** menu item. Examples show best what can be achieved.

Let us consider the transformation $\ln(x\,y) = \ln x + \ln y$ in both directions. It is only valid when y is real or complex and x is nonnegative (or with the roles of x and y interchanged). *Derive* refuses to do the transformation unless the necessary condition is satisfied. So, let us first issue the **`Declare Variable`** command and specify `x` as a nonnegative number. Next, we issue the **`Manage Logarithm`** command to control the direction of simplification and choose the `Expand` option.

```
1:   x :epsilon Real [0,inf)

2:   Logarithm := Expand
```

The expression

```
3:   LN(x y)
```

is now **`Simplified`** to

```
4:   LN(x) + LN(y)
```

The `Collect` option in the **`Manage Logarithm`** menu must be chosen to **`Simplify`** in the opposite direction and combine sum of logarithms into the logarithm of a product.

```
5:   Logarithm := Collect

6:   LN(x y)
```

In order to illustrate that *Derive* only does those simplifications about which it is sure that the transformations are valid, we enter under the above circumstances the following sum of logarithms.

```
7:   LN(x) + LN(y) + LN(z)
```

Simplification only partly combines logarithms.

```
8:   LN(x y) + LN(z)
```

You can get the simplification

```
9:   LN(x z) + LN(y)
```

by choosing in the **`Manage Ordering`** command the x, z, y order of variables.

Trigonometric simplification is another highlight of *Derive*. Use the **`Manage Trigonometry`** menu and choose the direction of simplification and the preference of simplification of powers of sines or cosines. The `Auto` option can be chosen when you leave it up to *Derive* to make a heuristic choice. For example, in the `Auto` mode, *Derive* simplifies

```
           2           2
10:  COS(x)  + 2 SIN(x)
```

into

```
           2
11:  SIN(x)  + 1
```

But if you prefer cosines over sines, you select **`Manage Trigonometry`** `Toward: Cosines` and get

```
12:  Trigpower := Cosines
```

```
               2
13:  2 - COS(x)
```

When you enter

```
14:  SIN(x + y)
```

and select **`Manage Trigonometry`** `Direction: Expand`,

```
15:  Trigonometry := Expand
```

then simplification yields

```
16:  COS(x) SIN(y) + SIN(x) COS(y)
```

On its turn, choosing the `Collect` direction, gives back the original expression upon simplification.

2.8 Derive Packages

Many sophisticated mathematical procedures are written in the *Derive* programming language and stored in the so-called *utility files*. Below, we tabulate the utility files and their purposes; we leave out the extension .MTH that indicate that the file is a *Derive* file.

File name	*Purpose*
APPROX	Pade Rational Approximation
BESSEL	Bessel and Airy Functions
DIF_APPS	Applications of Differentiation
ELLIPTIC	Elliptic Integrals
ENGLISH	English Measurement Units
EXP_INT	Exponential, Log, Sine, and Cosine Integrals
FRESNEL	Fresnel Integrals
GRAPHICS	Plotting Space Curves, Complex Expressions
HYPERGEO	Hypergeometric Functions
INT_APPS	Applications of Integration
METRIC	Metric Measurement Units
MISC	Miscellaneous Utilities
NUMBER	Number Theory
NUMERIC	Numerical Differentiation and Integration
ODE1	First-order Ordinary Differential Equations
ODE2	Second-order Ordinary Differential Equations
ODE_APPR	Approximately Solving ODEs
ORTH_POL	Orthogonal Polynomials
PHYSICAL	Fundamental Physical Constants
PLOT2D, PLOT3D, PLOTPARA	Plot examples
PROBABIL	Additional Probability Functions
RECUREQN	Solving Recurrence Relations
SOLVE	Solving Nonlinear Systems & Complex Solutions
VECTOR	Additional Vector and Matrix Functions
ZETA	Zeta, Polylogarithm, and Dilogarithm Functions

We give one example: Laplace transforms. You make the following steps: Load the utility file INT_APPS.MTH, declare the variable `s` to be positive, enter the requested Laplace transform, and simplify. Two concrete functions:

```
               3/2
1:   LAPLACE(t    , t, s)

       inf
     /       - s*t  3/2
2:   /    #e     *t    dt
       0
```

Deliberately we forgot to declare the variable `s` as positive real number. If we do so, *Derive* can compute the Laplace transform.

```
       3 SQRT(pi)
4:   -----------
          5/2
        4 s

             /          2      \
5:           | 2     - t       |
     LAPLACE\t  + #e     , t, s/
```

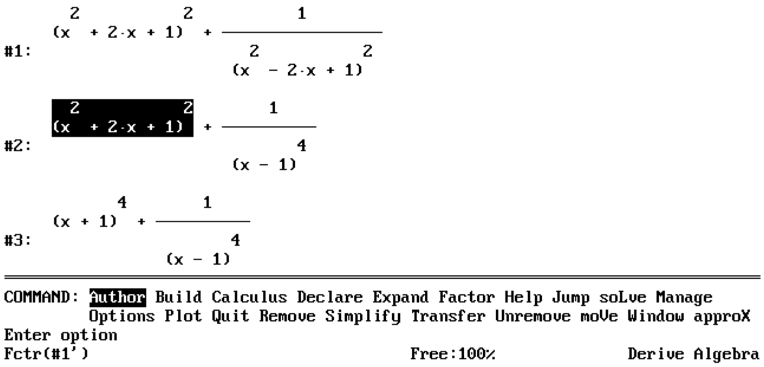
```
                  2
                 s / 4 /       / s \\
      SQRT(pi) #e      |1 - ERF|---||
6:                     \       \ 2 //      2
     --------------------------------- + ----
                     2                     3
                                          s
```

2.9 Formatting

Derive allows saving of expressions into different formats such as C, Fortran, Pascal, or Basic. For example, issue the **Transfer Save Fortran** command if you want to save an expression in Fortran style. But don't expect to much: it is a literal transcription from one language into another and no code optimization is done.

2.10 Interface

Much as already been said about *Derive*'s user interface. Obvious things like command line editing and screen saving are available in *Derive*. In this section we shall only consider two important aspects: manipulation of subexpressions and the demonstration mode.



```
       2          2             1
    (x  + 2·x + 1)  + ----------------
#1:                        2         2
                         (x  - 2·x + 1)

       2          2          1
    (x  + 2·x + 1)  + -----------
#2:                              4
                          (x - 1)

          4         1
    (x + 1)  + -----------
#3:                       4
                   (x - 1)

COMMAND: Author Build Calculus Declare Expand Factor Help Jump soLve Manage
         Options Plot Quit Remove Simplify Transfer Unremove moVe Window approX
Enter option
Fctr(#1')                                   Free:100%            Derive Algebra
```

Figure 2.20: Selecting a subexpression

In *Derive* it is easy to apply operations on parts of formulae. In Figure 2.20 you see how you can simplify the expression

$$(x^2+2x+1)^2+\frac{1}{(x^2-2x+1)^2}$$

into

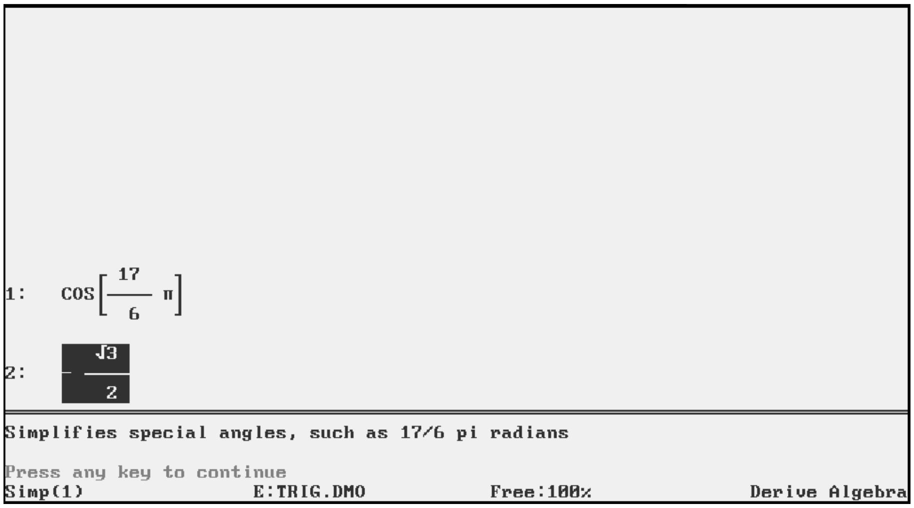
$$(x+1)^4+\frac{1}{(x-1)^4}.$$

Factorization of the whole expression just gives second expression in Figure 2.20. By the arrow keys you can select the second part of the first expression. When you issue the **Factor** command, it will apply factorization only on the highlighted part. This is how the third expression in Figure 2.20 was obtained. You also see on the screen how $(x^2 + 2x + 1)^2$ is selected in this expression to be processed further.

You can issue the **`Transfer` `D`**`emo` command to read expressions from a demonstration file (with default extension DMO). After each expression is read and displayed in the working area, the expression is simplified and the result is displayed. *Derive* then waits for you to press any key to continue or the < Esc > key to suspend the demo mode for free performance of *Derive* commands (issue again the **`Transfer` `D`**`emo` command to resume the demonstration). *Derive* comes with the following demonstration files: ALGEBRA, ARITH, CALCULUS, FUNCTION, MATRIX, and TRIG (with extension DMO). Let us look at one of these files, say `TRIG.DMO`. The beginning of its contents is

```
; Simplifies special angles, such as 17/6 pi radians
COS (17/6 pi)

; Type "DEG" after entering an angle in degrees
SIN (-30 DEG)

; A hidden opportunity for sin^2 + cos^2 --> 1
(SIN x)^3 + SIN x + (COS x)^2 SIN x

; Using different rules in different places
(1 - (COS x)^2)^4 (1 - (SIN x)^2)^3 ((SIN x)^2 + (COS x)^2)^5

; Judicious extraction of phase angles
SIN (pi / 4 + a) - SIN (0.25 (pi - 4 a))

SIN (1.5 pi/2+x) / COS (pi+x) + COS ((3pi-2x)/2) / SIN (pi/2+x)
```

The start of the demonstration is shown in Figure 2.21. Note that the comment "Simplifies special angles, such as 17/6 pi radians" which is the first line in the file starting with a semicolon is shown in the status line.



1: $\cos\left[\frac{17}{6}\pi\right]$

2: $-\frac{\sqrt{3}}{2}$

```
Simplifies special angles, such as 17/6 pi radians

Press any key to continue
Simp(1)              E:TRIG.DMO          Free:100%            Derive Algebra
```

Figure 2.21: *Derive* in demonstration mode

2.11 Saving and Quitting

You can save a *Derive* session in ascii format by the **`Transfer Save Derive`** command. Finally, to quit *Derive*, you select `Quit` from the menu. The system asks you to confirm this action.

Chapter 3

Literature

3.1 Books, Manuals, and Reports

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