Representation of 5 important ray tracing primitives in 5 models of 3D Euclidean Geometry

	3D LA	3D GA	4D LA	4D GA	5D GA
point	$\mathbf{q}$: vector from the origin to the point	$\mathbf{q}$: vector from the origin to the point	$\vec{q}$: vector from origin to point $\mathbf{q}$ $\mathbf{q} = (\,\vec{q} : 1\,)$	$\vec{q}$: vector from origin $\mathbf{e}_0$ to point $\mathbf{q}$ $\mathbf{q} = \vec{q} + \mathbf{e}_0$	$\vec{q}$: vector from origin $\mathbf{e}_0$ to point $\mathbf{q}$ $\mathbf{q} = \vec{q} + \mathbf{e}_0 - \frac{1}{2}(\vec{q}\cdot\vec{q})\,\mathbf{e}_\infty$
line	$\mathbf{q}$: vector from origin to a point on the line $\mathbf{u}$: direction of the line	$\mathbf{q}$: vector from origin to a point on the line $\mathbf{u}$: direction of the line	$\mathbf{q}_1 = (\,\vec{q}_1 : 1\,)$, $\mathbf{q}_2 = (\,\vec{q}_2 : 1\,)$: two points $\mathbf{l} = (\,\vec{q}_1 - \vec{q}_2 : \vec{q}_1 \times \vec{q}_2)$	$\mathbf{q}_1$, $\mathbf{q}_2$: two points $\mathbf{l} = \mathbf{q}_1 \wedge \mathbf{q}_2$	$\mathbf{q}_1$, $\mathbf{q}_2$: two points $\mathbf{l} = \mathbf{q}_1 \wedge \mathbf{q}_2 \wedge \mathbf{e}_\infty$
plane	$\mathbf{n}$: normal vector of the plane δ: distance of plane to the origin	$\mathbf{p}$: bivector of the plane δ: distance of plane to the origin	$\mathbf{n}$: normal vector of the plane δ: distance of plane to the origin $\mathbf{p} = [\,\mathbf{n} : \delta]$	$\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$: three points $\mathbf{p} = \mathbf{q}_1 \wedge \mathbf{q}_2 \wedge \mathbf{q}_3$	$\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$: three points $\mathbf{p} = \mathbf{q}_1 \wedge \mathbf{q}_2 \wedge \mathbf{q}_3 \wedge \mathbf{e}_\infty$
sphere	$\mathbf{q}$: vector from the origin to the center of the sphere ρ: radius of the sphere	$\mathbf{q}$: vector from the origin to the center of the sphere ρ: radius of the sphere	ρ: sphere radius $\mathbf{q}$: sphere center point	ρ: sphere radius $\mathbf{q}$: sphere center point	$\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$, $\mathbf{q}_4$: four points $\mathbf{s} = \mathbf{q}_1 \wedge \mathbf{q}_2 \wedge \mathbf{q}_3 \wedge \mathbf{q}_4$ or: ρ: sphere radius $\mathbf{q}$: sphere center point $\mathbf{s} = \left(\mathbf{q} + \frac{1}{2}\rho^2\mathbf{e}_\infty\right)^*$
rotation / translation	$\mathbf{R}$: rotation matrix $\mathbf{t}$: translation vector $\mathbf{R}_1, \mathbf{t}_1$: transformation 1 $\mathbf{R}_2, \mathbf{t}_2$: transformation 2 $\mathbf{R}_c = \mathbf{R}_2\mathbf{R}_1$ $\mathbf{t}_c = \mathbf{t}_2 + \mathbf{R}_2\mathbf{t}_1$	$\mathbf{b}$: rotation plane ϕ: rotation angle $\mathbf{R} = \exp^{-\frac{1}{2}\phi\mathbf{b}}$ $\mathbf{t}$: translation vector $\mathbf{R}_1, \mathbf{t}_1$: transformation 1 $\mathbf{R}_2, \mathbf{t}_2$: transformation 2 $\mathbf{R}_c = \mathbf{R}_2\mathbf{R}_1$ $\mathbf{t}_c = \mathbf{t}_2 + \mathbf{R}_2\mathbf{t}_1\mathbf{R}_2^{-1}$	$\mathbf{R}$: 3×3 rotation matrix $\vec{t}$: translation vector $\mathbf{M} = \begin{bmatrix}\mathbf{R} & \vec{t}\\ 0 & 1\end{bmatrix}$ $\mathbf{M}_1, \mathbf{M}_2$: two transformations $\mathbf{M}_c = \mathbf{M}_2\mathbf{M}_1$	f: transformation function satisfying $f(\mathbf{a}\wedge\mathbf{b}) = f(\mathbf{a}) \wedge f(\mathbf{b})$ $\mathbf{M}_f$: outermorphism operator constructed from f $\mathbf{M}_{f1}, \mathbf{M}_{f2}$: two outermorphism operators $\mathbf{M}_c = \mathbf{M}_{f2}\mathbf{M}_{f1}$	$\mathbf{b}$: rotation plane ϕ: rotation angle $\mathbf{R} = \mathrm{e}^{-\frac{1}{2}\phi\mathbf{b}}$ $\vec{t}$: translation vector $\mathbf{T} = \mathbf{1} + \frac{1}{2}\vec{t}\wedge\mathbf{e}_\infty = \mathrm{e}^{\frac{1}{2}\mathbf{t}\wedge\mathbf{e}_\infty}$ $\mathbf{V}_1, \mathbf{V}_2$: two transformations $\mathbf{V}_c = \mathbf{V}_1\mathbf{V}_2$

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