CATEGORY THEOREMS FOR STABLE SEMIGROUPS

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Abstract. Inspired by the classical category theorems of Halmos and Rohlin for discrete measure preserving transformations, we prove analogous results in the abstract setting of unitary and isometric C_0-semigroups on a separable Hilbert space. More presicely, we show that the set of all weakly stable unitary groups (isometric semigroups) is of first category, while the set of all almost weakly stable unitary groups (isometric semigroups) is residual for an appropriate topology.

1. Introduction

In 1944 Halmos [9] showed that a "typical" dynamical system is weakly mixing, or, more precisely, the set of all weakly mixing transformations on a measure space is residual and hence of second category. Four years later, Rohlin [19] showed that the set of all strongly mixing transformation is of first category (see also Halmos [10, pp. 77–80]). In this way they proved the existence of weakly but not strongly mixing systems (see e.g. Petersen [18, Section 4.5] for later concrete examples and for a method to construct such systems).

In the time continuous case, i.e., for measure preserving flows, analogous results are not stated in the standard literature. However, Bartoszek and Kuna [1] recently proved a category theorem for Markov semigroups on the Schatten class C_1, while an analogous result for stochastic semigroups can be found in Lasota, Myjak [15].

In this paper we consider the time continuous case in the more general setting of unitary and isometric semigroups on a Hilbert space and prove corresponding category results.

Recall that a (continuous) dynamical system $(\Omega, \mu, (\varphi_t)_{t\geq 0})$ induces a unitary/ isometric C_0-(semi)group $(T(t))_{t\geq 0}$ on the Hilbert space $H := L^2(\Omega, \mu)$ via the formula $T(t)f(\omega) = f(\varphi_t(\omega))$. Moreover, the decomposition into invariant subspaces $H = \langle \mathbf{1} \rangle \oplus H_0$ holds for $H_0 = \{f \in H : \int_\Omega f d\mu = 0\}$. Recall further that the flow (φ_t) is strongly mixing if and only if the restricted semigroup $(T_0(t))_{t\geq 0}$ on H_0 is weakly stable, i.e., $\lim_{t\to\infty}\langle T_0(t)x, y\rangle = 0$ for every $x, y \in H_0$. Analogously, weak mixing of the flow corresponds to so-called *almost weak stability* of the semigroup $T_0(\cdot)$, i.e.,

$$\lim \langle T_0(t)x, y\rangle = 0, \quad t \to \infty,\ t \in M, \quad \text{for every } x, y \in H_0$$

for a set $M \subset \mathbb{R}_+$ with (asymptotical) density one, i.e., with $d(M) := \lim_{t\to\infty} \frac{\lambda(M\cap[0,t])}{t} = 1$ for the Lebesgue measure λ. We refer to e.g. Cornfeld, Fomin, Sinai [2, Section 1.7] for details.

We now consider an arbitrary Hilbert space H and a contractive C_0-semigroup $T(\cdot)$ on H with generator A. By the classical decomposition theorem of Jacobs–Glicksberg–de Leeuw (see Glicksberg, De Leeuw [3] or Engel, Nagel [7, Theorem V.2.8]), H decomposes into the so-called

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reversible and stable parts

$$\begin{aligned} H_r &:= \overline{\text{lin}}\{x : Ax = i\alpha x \text{ for some } \alpha \in \mathbb{R}\}, \\ H_s &:= \{x : 0 \text{ is a weak accumulation point of } \{T(t)x, t \geq 0\}\}. \end{aligned}$$

(Note that the Jacobs–Glicksberg–de Leeuw decomposition is valid for every bounded semigroup on a reflexive Banach space or, more generally, for every relatively weakly compact semigroup.) A result of Hiai [12] being a continuous version of discrete results of Nagel [16] and Jones, Lin [13] shows that the semigroup $T(\cdot)$ restricted to X_s is almost weakly stable as defined above. (For a survey on weak and almost weak stability see Eisner, Farkas, Nagel, Serény [4].)

Weak mixing (almost weak stability) can be characterised by a simple spectral condition, while the more natural property of strong mixing (weak stability) *"is, however, one of those notions, that is easy and natural to define but very difficult to study..."*, see the monograph of Katok, Hasselblatt [14, p. 748].

In this paper we extend the results of Halmos and Rohlin and show that weak and almost weak stability differ fundamentally. More precisely, we prove that for an appropriate (and natural) topology the set of all weakly stable unitary groups on a separable infinite-dimensional Hilbert space is of the first category and the set of all almost weakly stable unitary groups is residual and hence of the second category (Section 2). An analogous result holds for isometric semigroups (Section 3). In Section 4 we discuss the case of contractive semigroups.

For analogous results on the time discrete case, see Eisner, Serény [5].

2. Unitary case

Consider the set $\mathcal{U}$ of all unitary C_0-groups on an infinite-dimensional Hilbert space H. The following density result on periodic unitary groups is a first step in our construction.

Proposition 2.1. *For every $N \in \mathbb{N}$ and every unitary group $U(\cdot)$ on H there is a sequence $\{V_n(\cdot)\}_{n=1}^{\infty}$ of periodic unitary groups with period greater than N such that $\lim_{n\to\infty} \|U(t) - V_n(t)\| = 0$ uniformly on compact time intervals.*

Proof. Take $U(\cdot) \in \mathcal{U}$ and $n \in \mathbb{N}$. By the spectral theorem (see, e.g., Halmos [11]), H is isomorphic to $L^2(\Omega, \mu)$ for some locally compact space Ω and measure μ and $U(\cdot)$ is unitary equivalent to a multiplication group $\tilde{U}(\cdot)$ with

$$(\tilde{U}(t)f)(\omega) = e^{itq(\omega)} f(\omega), \quad \forall \omega \in \Omega,\ t \in \mathbb{R},\ f \in L^2(\Omega, \mu)$$

for some measurable $q : \Omega \to \mathbb{R}$.

We approximate the (unitary) group $\tilde{U}(\cdot)$ as follows. Take $n > N$ and define

$$q_n(\omega) := \frac{2\pi j}{n}, \quad \forall \omega \in q^{-1}\left(\left[\frac{2\pi j}{n}, \frac{2\pi(j+1)}{n}\right]\right),\ j \in \mathbb{Z}.$$

Denote now by $\tilde{V}_n(t)$ the multiplication operator with $e^{itq_n(\cdot)}$ for every $t \in \mathbb{R}$. The unitary group $\tilde{V}_n(\cdot)$ is periodic with period greater than or equal to n and therefore N. Moreover,

$$\|\tilde{U}(t) - \tilde{V}_n(t)\| \leq \sup_{\omega} |e^{itq(\omega)} - e^{itq_n(\omega)}| \leq |t| \sup_{\omega} |q(\omega) - q_n(\omega)| \leq \frac{2\pi|t|}{n} \underset{n\to\infty}{\longrightarrow} 0$$

uniformly in t on compact intervals and the proposition is proved. □

Remark 2.2. By a modification of the proof of Proposition 2.1 one can show that for every $N \in \mathbb{N}$ the set of all periodic unitary groups with period greater than N with bounded generators is dense in $\mathcal{U}$ with respect to the strong operator topology uniform on compact time intervals.

For the second step we need the following lemma. From now on we assume the Hilbert space H to be separable.

Lemma 2.3. *Let H be a separable infinite-dimensional Hilbert space. Then there exists a sequence $\{U_n(\cdot)\}_{n=1}^{\infty}$ of almost weakly stable unitary groups with bounded generator satisfying* $\lim_{n\to\infty} \|U_n(t) - I\| = 0$ *uniformly in t on compact intervals.*

Proof. By the isomorphism of all separable infinite-dimensional Hilbert spaces we can assume without loss of generality that $H = L^2(\mathbb{R})$ with respect to the Lebesgue measure.

Take $n \in \mathbb{N}$ and define $U_n(\cdot)$ on $L^2(\mathbb{R})$ by

$$(U_n(t)f)(s) := e^{\frac{itq(s)}{n}} f(s), \quad s \in \mathbb{R}, \quad f \in L^2(\mathbb{R}),$$

where $q : \mathbb{R} \to (0,1)$ is a strictly monotone increasing function.

Then all $U_n(\cdot)$ are almost weakly stable by the theorem of Jacobs–Glicksberg–de Leeuw and we have

$$\|U_n(t) - I\| = \sup_{s\in\mathbb{R}} |e^{\frac{itq(s)}{n}} - 1| \leq [\text{for } t \leq \pi n] \leq |e^{\frac{it}{n}} - 1| \leq \frac{2t}{n} \to 0, \quad n \to \infty,$$

uniformly on t in compact intervals. □

The metric we introduce now on the space $\mathcal{U}$ is given by

$$d(U(\cdot), V(\cdot)) := \sum_{n,j=1}^{\infty} \frac{\sup_{t\in[-n,n]} \|U(t)x_j - V(t)x_j\|}{2^{j+n}\|x_j\|} \quad \text{for } U, V \in \mathcal{U},$$

where $\{x_j\}_{j=1}^{\infty}$ some fixed dense subset of H with all $x_j \neq 0$. Note that the topology coming from this metric is a continuous analogue of the so-called strong* operator topology for operators, see, e.g., Takesaki [21, p. 68]), and the corresponding convergence is the strong convergence uniform on compact time intervals of (semi)groups and their adjoints. This metric makes $\mathcal{U}$ a complete metric space.

We denote by $\mathcal{S}_{\mathcal{U}}$ the set of all weakly stable unitary groups on H and by $\mathcal{W}_{\mathcal{U}}$ the set of all almost weakly stable unitary groups on H. The following proposition shows the density of $\mathcal{W}_{\mathcal{U}}$ which will play an important role later.

Proposition 2.4. *The set $\mathcal{W}_{\mathcal{U}}$ of all almost weakly stable unitary groups with bounded generators is dense in $\mathcal{U}$.*

Proof. By Proposition 2.1 it is enough to approximate periodic unitary groups by almost weakly stable unitary groups. Let $U(\cdot)$ be a periodic unitary group with generator A and period τ. Take $\varepsilon > 0$, $n \in \mathbb{N}$, $x_1, \ldots, x_n \in H$ and $t_0 > 0$. We have to find an almost weakly stable unitary group $T(\cdot)$ with $\|U(t)x_j - T(t)x_j\| \leq \varepsilon$ for all $j = 1, \ldots, n$ and $t \in [-t_0, t_0]$.

By Engel, Nagel [7, Theorem IV.2.26] we have the orthogonal space decomposition

$$H = \overline{\bigoplus_{k\in\mathbb{Z}} \ker\left(A - \frac{2\pi ik}{\tau}\right)}. \tag{1}$$

Assume first that $\{x_j\}_{j=1}^{n}$ is an orthonormal system of eigenvectors of A.

Our aim is to use Lemma 2.3. For this purpose we first construct a unitary group $V(\cdot)$ which coincides with $U(\cdot)$ on every x_j and has infinite-dimensional eigenspaces only.

Define the $U(\cdot)$-invariant subspace $H_0 := \mathrm{lin}\{x_1, \dots, x_n\}$ and the unitary group $V_0(\cdot) := U(\cdot)|_{H_0}$ on H_0. Since H is separable, we can decompose H in an orthogonal sum

$$H = \bigoplus_{k=0}^{\infty} H_k \quad \text{with } \dim H_k = \dim H_0 \text{ for every } k \in \mathbb{N}.$$

Denote by P_k an isomorphism from H_k to H_0 and define $V_k(\cdot) := P_k^{-1} V_0(\cdot) P_k$ on H_k being copies of $V_0(\cdot)$. Consider now the unitary group $V(\cdot) := \bigoplus_{k=0}^{\infty} V_k(\cdot)$ on H which is periodic by Engel, Nagel [7, Theorem IV.2.26]. (Note that its generator is even bounded.)

For the periodic unitary group $V(\cdot)$ the space decomposition analogous to (1) holds. Moreover there are only finitely many eigenspaces (less or equal to n, depending on $x_1, ..., x_n$) and they all are infinite-dimensional by the construction. Applying Lemma 2.3 to each eigenspace we find an almost weakly stable unitary group $T(\cdot)$ with $\|U(t)x_j - T(t)x_j\| \leq \varepsilon$ for all $j = 1, \dots, n$ and $t \in [-t_0, t_0]$.

Take now arbitrary $x_1, \dots, x_n \in H$. Take further an orthonormal basis of eigenvectors $\{y_k\}_{k=1}^{\infty}$. Then there exists $N \in \mathbb{N}$ such that $x_j = \sum_{k=1}^{N} a_{jk} y_k + o_j$ with $\|o_j\| < \frac{\varepsilon}{4}$ for every $j = 1, \dots, n$.

We can apply the arguments above to $y_1, \dots, y_N$ and find an almost weakly stable unitary group $T(\cdot)$ with $\|U(t)y_k - T(t)y_k\| < \frac{\varepsilon}{4NM}$ for $M := \max_{k=1,\dots,N, j=1,\dots,n} |a_{jk}|$ and every $k = 1, \dots, N$ and $t \in [-t_0, t_0]$. Hence

$$\|U(t)x_j - T(t)x_j\| \leq \sum_{k=1}^{N} |a_{jk}| \|U(t)y_k - T(t)y_k\| + 2\|o_j\| < \varepsilon$$

for every $j = 1, \dots, n$ and $t \in [-t_0, t_0]$, and the proposition is proved. □

We are now ready to prove a category theorem for weakly and almost weakly unitary groups. For this purpose we extend the argument used in the proof of the category theorems for operators induced by measure preserving transformation in ergodic theory (see Halmos [10, pp. 77–80]).

Theorem 2.5. *The set $\mathcal{S}_\mathcal{U}$ of weakly stable unitary groups is of first category and the set $\mathcal{W}_\mathcal{U}$ of almost weakly stable unitary groups is residual in $\mathcal{U}$.*

Proof. We first prove the first part of the theorem. Fix $x \in H$ with $\|x\| = 1$ and consider the sets

$$M_t := \left\{ U(\cdot) \in \mathcal{U} : \ |\langle U(t)x, x \rangle| \leq \frac{1}{2} \right\}.$$

Note that all sets M_t are closed.

For every weakly stable $U(\cdot) \in \mathcal{U}$ there exists $t > 0$ such that $U \in M_s$ for all $s \geq t$, i.e., $U(\cdot) \in N_t := \cap_{s \geq t} M_t$. So we obtain

$$\mathcal{S}_\mathcal{U} \subset \bigcup_{t>0} N_t. \tag{2}$$

Since all N_t are closed, it remains to show that $\mathcal{U} \setminus N_t$ is dense for every t.

Fix $t > 0$ and let $U(\cdot)$ be a periodic unitary group. Then $U(\cdot) \notin M_s$ for some $s \geq t$ and therefore $U(\cdot) \notin N_t$. Since by Proposition 2.1 periodic unitary groups are dense in $\mathcal{U}$, $\mathcal{S}$ is of first category.

To show that $\mathcal{W}_{\mathcal{U}}$ is residual we take a dense subspace $D = \{x_j\}_{j=1}^{\infty}$ of H and define

$$W_{jkt} := \left\{ U(\cdot) \in \mathcal{U} : \ | \langle U(t)x_j, x_j \rangle | < \frac{1}{k} \right\}.$$

All these sets are open. Therefore the sets $W_{jk} := \bigcup_{t>0} W_{jkt}$ are open as well.

We show that

$$\mathcal{W}_{\mathcal{U}} = \bigcap_{j,k=1}^{\infty} W_{jk} \tag{3}$$

holds.

The inclusion "$\subset$" follows from the definition of almost weak stability. To prove the converse inclusion we take $U(\cdot) \notin \mathcal{W}_{\mathcal{U}}$. Then there exists $x \in H$ with $\|x\| = 1$ and $\varphi \in \mathbb{R}$ such that $U(t)x = e^{it\varphi}x$ for all $t > 0$. Take $x_j \in D$ with $\|x_j - x\| \leq \frac{1}{4}$. Then

$$\begin{aligned} | \langle U(t)x_j, x_j \rangle | &= | \langle U(t)(x - x_j), x - x_j \rangle + \langle U(t)x, x \rangle - \langle U(t)x, x - x_j \rangle - \langle U(t)(x - x_j), x \rangle | \\ &\geq 1 - \|x - x_j\|^2 - 2\|x - x_j\| > \frac{1}{3} \end{aligned}$$

for every $t > 0$. So $U(\cdot) \notin W_{j3}$ which implies $U(\cdot) \notin \cap_{j,k=1}^{\infty} W_{jk}$. Therefore equality (3) holds. Combining this with Proposition 2.4 we obtain that $\mathcal{W}_{\mathcal{U}}$ is residual as a dense countable intersection of open sets. □

3. Isometric case

In this section we extend the result from the previous section to isometric semigroups on H which we again assume to be infinite-dimensional and separable.

Denote by $\mathcal{I}$ the set of all isometric C_0-semigroups on H. On $\mathcal{I}$ we consider the metric given by the formula

$$d(T(\cdot), S(\cdot)) := \sum_{n,j=1}^{\infty} \frac{\sup_{t\in[0,n]} \|T(t)x_j - S(t)x_j\|}{2^{j+n}\|x_j\|} \quad \text{for } T(\cdot), S(\cdot) \in \mathcal{I},$$

where $\{x_j\}_{j=1}^{\infty}$ is a fixed dense subset of H. This corresponds to the strong convergence uniform on compact time intervals. The space $\mathcal{I}$ endowed with this metric is a complete metric space.

Analogously to Section 2 we denote by $\mathcal{S}_{\mathcal{I}}$ the set of all weakly stable and by $\mathcal{W}_{\mathcal{I}}$ the set all almost weakly stable isometric semigroups on H.

The key for our results in this section is the following classical structure theorem on isometric semigroups on Hilbert spaces.

Theorem 3.1. *(Wold decomposition, see* [20, Theorem III.9.3]*.) Let $V(\cdot)$ be an isometric semigroup on a Hilbert space H. Then H can be decomposed into an orthogonal sum $H = H_0 \oplus H_1$ of $V(\cdot)$-invariant subspaces such that the restriction of $V(\cdot)$ on H_0 is a unitary (semi)group and the restriction of $V(\cdot)$ on H_1 is a continuous unilateral shift, i.e., H_1 is unitarily equivalent to $L^2(\mathbb{R}_+, Y)$ for some Hilbert space Y such that the restriction of $V(\cdot)$ to H_1 is equivalent to the right shift semigroup on $L^2(\mathbb{R}_+, Y)$.*

We further need the following lemma, see also Peller [17] for the discrete version.

Lemma 3.2. *Let Y be a Hilbert space and let $R(\cdot)$ be the right shift semigroup on $H := L^2(\mathbb{R}_+, Y)$. Then there exists a sequence $\{U_n(\cdot)\}_{n=1}^{\infty}$ of periodic unitary (semi)groups on H converging strongly to $R(\cdot)$ uniformly on compact time intervals.*

Proof. For every $n \in \mathbb{N}$ we define $U_n(\cdot)$ by

$$(U_n(t)f)(s) := \begin{cases} f(s), & s \geq n; \\ R_n(t)f(s), & s \in [0,n], \end{cases}$$

where $R_n(\cdot)$ denotes the n-periodic right shift on the space $L^2([0,n], Y)$. Then every $U_n(\cdot)$ is a C_0-semigroup on $L^2(\mathbb{R}_+, Y)$ which is isometric and n-periodic, and therefore unitary.

Fix $f \in L^2(\mathbb{R}_+, Y)$ and take $0 \leq t < n$. Then we have

$$\|U_n(t)f - R(t)f\|^2 = \int_{n-t}^{n} \|f(s)\|^2 ds + \int_{n}^{\infty} \|f(s) - f(s-t)\|^2 ds \leq 2 \int_{n-t}^{\infty} \|f(s)\|^2 ds \underset{n\to\infty}{\longrightarrow} 0$$

uniformly in t on compact intervals and the lemma is proved. □

As a consequence of the Wold decomposition and Lemma 3.2 we obtain the following density result for periodic unitary (semi)groups in $\mathcal{I}$.

Proposition 3.3. *The set of all periodic unitary semigroups is dense in $\mathcal{I}$.*

Proof. Let $V(\cdot)$ be an isometric semigroup on H. Then by Theorem 3.1 the orthogonal decomposition $H = H_0 \oplus H_1$ holds, where the restriction $V_0(\cdot)$ of $V(\cdot)$ to H_0 is unitary, H_1 is unitarily equivalent to $L^2(\mathbb{R}_+, Y)$ for some Y and the restriction $V_1(\cdot)$ of $V(\cdot)$ on H_1 corresponds by this equivalence to the right shift semigroup on $L^2(\mathbb{R}_+, Y)$. By Proposition 2.1 and Lemma 3.2 we can approximate both semigroups $V_0(\cdot)$ and $V_1(\cdot)$ by unitary periodic ones and the assertion follows. □

We further have a density result for almost weakly stable isometric semigroups.

Proposition 3.4. *The set $\mathcal{W}_{\mathcal{I}}$ of almost weakly stable isometric semigroups is dense in $\mathcal{I}$.*

Proof. Let V be an isometry on H, H_0, H_1 the orthogonal subspaces from Theorem 3.1 and V_0 and V_1 the corresponding restrictions of V. By Lemma 3.2 the operator V_1 can be approximated by unitary operators on H_1. The assertion now follows from Proposition 2.4. □

By using the same arguments as in the proof of Theorem 2.5 we obtain with the help of Propositions 3.3 and 3.4 the following category result for weakly and almost weakly stable isometries.

Theorem 3.5. *The set $\mathcal{S}_{\mathcal{I}}$ of all weakly stable isometric semigroups is of first category and the set $\mathcal{W}_{\mathcal{I}}$ of all almost weakly stable isometric semigroups is residual in $\mathcal{I}$.*

4. A remark on the contractive case

It is not clear how to prove an analogue to Theorems 2.5 and 3.5 for contractive semigroups, while this is done in the discrete case in Eisner, Serény [5]. We point out one of the difficulties.

Let $\mathcal{C}$ denote the set of all contraction semigroups on H endowed with the metric

$$d(T(\cdot), S(\cdot)) := \sum_{n,i,j=1}^{\infty} \frac{\sup_{t\in[0,n]} |\langle T(t)x_i, x_j\rangle - \langle S(t)x_i, x_j\rangle|}{2^{i+j+n}\|x_i\|\|x_j\|} \quad \text{for } T, S \in \mathcal{C},$$

where $\{x_j\}_{j=1}^{\infty}$ is a fixed dense subset of H. The corresponding convergence is the weak convergence of semigroups uniform on compact time intervals. Note that this metric is a continuous analogue of the metric used in Eisner, Serény [5].

Since for $H := l^2$ there exists a Cauchy sequence $\{T_n(\cdot)\}_{n=1}^{\infty}$ in $\mathcal{C}$ of semigroups (with bounded generators) such that the pointwise limit $S(\cdot)$ does not satisfy the semigroup law (see Eisner, Serény [6]), the metric space $\mathcal{C}$ is *not* complete (or compact) in general, therefore the standard Baire category theorem cannot be applied. It is an open question whether $\mathcal{C}$ is at least Čech complete.

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