Homework 8

Due Monday, April 27 2009, 11 am with Dick de Jongh.

1. (a) Let $\mathfrak{M}$ and $\mathfrak{N}$ be two **IPC**-models, and f a **frame**-p-morphism from $\mathfrak{M}$ to $\mathfrak{N}$. Let $\psi_1, \ldots, \psi_n$ be such that for each $w \in W$ and each $i \leq n$, $w \models \psi_i$ iff $f(w) \models p_i$. Prove that for each $\varphi(p_1, \ldots, p_n)$ we have $w \models \varphi(\psi_1, \ldots, \psi_n)$ iff $f(w) \models \varphi(p_1, \ldots, p_n)$. [4 pts]

   (b) Assume that $\mathfrak{M}$ is a model with a root w_0 such that $w_0 \models \neg\neg\psi$ and $w_0 \not\models \psi$. Define a **frame**-p-morphism f from $\mathfrak{M}$ to $\mathcal{RN}_{w_2}$ in such a way that, for each $w \in W$, $w \models \psi$ iff $f(w) \models p$. Here $\mathcal{RN}_{w_2}$ is the submodel of the Rieger-Nishimura ladder generated by w_2. [3 pts]

2. Show that if $\vdash_{\mathbf{IPC}} g_{n+5}(\varphi)$ then $\vdash_{\mathbf{IPC}} g_i(\varphi)$ for some $i < n + 5$.

   This is a step in a more proof-theoretic proof of Theorem 40 of the lecture notes.

   Use the fact that if $\vdash_{\mathbf{IPC}} (\varphi \to \psi) \to \chi \vee \theta$ then $\vdash_{\mathbf{IPC}} (\varphi \to \psi) \to \chi$ or $\vdash_{\mathbf{IPC}} (\varphi \to \psi) \to \theta$ or $\vdash_{\mathbf{IPC}} (\varphi \to \psi) \to \varphi$.

   Use also that $\vdash_{\mathbf{IPC}} \varphi(p) \leftrightarrow \psi(p)$ iff $\varphi(p)$ and $\psi(p)$ have the same value in the Rieger-Nishimura lattice (see page 56 slides) where the value of implications is calculated as on page 64 slides (and disjunctions in the obvious manner). [5 pts]

3. Assume $\mathfrak{M} \models \neg\neg\varphi$ and $\mathfrak{M} \not\models \varphi$. Assume that $\mathfrak{N} \models \neg\varphi$. Now add nodes w'_n for $n \geq 3$ to the union of $\mathfrak{M}$ and $\mathfrak{N}$ to obtain a new model $\mathfrak{M}^*$. Describe the accessibility relation of the new model.

   With the help of the p-morphism of exercise 1(b), construct a p-morphism from $\mathfrak{M}^*$ to the Rieger-Nishimura ladder. Use then exercise 1(a) to show that $\mathfrak{M}^* \not\models g_n(\varphi)$ for each n.

   Conclude Theorem 40. [6 pts]