(1) Write out (with the help of the literature, e.g., Bredon's book on Sheaf Theory) the proof of the theorem that singular cohomology agrees (under mild assumptions on the topological space, say paracompact and locally contractible) with sheaf cohomology with coefficients in a constant sheaf. (Note: we assume the space is paracompact. In Bredon's book we can then take for Φ the collection of all closed subsets of X, in which case his cohomology groups $H^i_\Phi(X,\mathscr{F})$ are the usual sheaf cohomology groups $H^i(X,\mathscr{F})$. This remark should greatly simplify the reading of Bredon's book. A similar remark applies if you prefer to read Godement's book.)

(2) Singular cohomology: make an overview of all the structures and results that we have available. Some keywords: functoriality, cup-product, excision, Künneth formula, ... Then try to figure out (use literature!) how much of this has an analogue in the context of sheaf cohomology.

(3) If X is a complex manifold, prove that

$$0 \longrightarrow 2\pi i\mathbb{Z} \longrightarrow \mathscr{O}_X \xrightarrow{\exp} \mathscr{O}_X^* \longrightarrow 0$$

is a short exact sequence of sheaves on X. (This is too easy!)

(4) If X is a C^∞-manifold, and $\mathscr{A}^n_{X,\mathbb{R}}$ is the sheaf of $\mathbb{R}$-valued C^∞-forms of degree n, show that this sheaf is acyclic. (E.g., prove that this is a soft sheaf and then apply the general result that soft sheaves are acyclic.) If X is a complex manifold and Ω^n_X is the sheaf of holomorphic differential forms of degree n, give examples that show that this sheaf is in general definitely not acyclic. (Do you understand the difference between these two cases?)

(5) Try to calculate (perhaps with the help of some books) the dimensions of the cohomology groups $H^q(\mathbb{P}^n,\Omega^p)$ for all n, p and q. Note that Ω^p denotes the sheaf of *holomorphic* p-forms. (In many cases you should find zero.) More generally, try to calculate the dimension of $H^q\big(\mathbb{P}^n,\Omega^p\otimes\mathscr{O}(r)\big)$. If you know what is an algebraic variety, ask yourself whether it matters if you calculate these groups analytically or algebraically (Zariski topology, etc.).

(6) Let $f\in\mathbb{C}[x]$ be a monic polynomial of odd degree $2n+1$. Let $F\in\mathbb{C}[x,z]$ be the corresponding homogeneous form of degree $2n+1$. (So if $f = x^{2n+1}+a_{2n}x^{2n}+\cdots+a_1x+a_0$ then $F = x^{2n+1}+a_{2n}x^{2n}z+\cdots+a_1xz^{2n}+a_0z^{2n+1}$.) Show that the equation $y^2z^{2n-1}=F$ defines a 1-dimensional complex submanifold $C\subset\mathbb{P}^2$. Try to calculate the dimension of $H^1(C,\mathscr{O}_C)$.

(7) For those who know some algebraic geometry: Read Appendix B from HAG.