True or false?

Decide if the following assertions are true or not. If you think a statement is true, give an argument. If you think a statement is false, give an example demonstrating this.

(1) For any natural number n the group $(\mathbb{Z}/n\mathbb{Z})^*$ is cyclic.

(2) If R is an integral domain and $x \in R$ is an irreducible element then $(x) \subset R$ is a prime ideal.

(3) Let $f,\ g \in \mathbb{Q}[X]$ be irreducible polynomials of the same degree. Then the fields $\mathbb{Q}[X]/(f)$ and $\mathbb{Q}[X]/(g)$ are isomorphic.

(4) Let K be a field. If $R \subset K$ is a subring with $1 \in R$ then R is an integral domain.

(5) Let $f,\ g \in \mathbb{Z}[X]$ with $g \neq 0$. Then there exist $q,\ r \in \mathbb{Z}[X]$ with $f = q \cdot g + r$ and $r = 0$ or $\deg(r) < \deg(f)$.

(6) Let R be a commutative ring. If $f,\ g \in R[X]$ are nonzero polynomials, $\deg(fg) = \deg(f) + \deg(g)$.

(7) Let R be a commutative ring. If $f = a_0 + a_1X + \cdots + a_nX^n$ is a unit in $R[X]$ then $f = a_0$ is a constant polynomial and $a_0 \in R^*$.

(8) Let $f \in \mathbb{Z}[X]$ be a primitive polynomial. Then f is irreducible in $\mathbb{Z}[X]$ if and only if f is irreducible in $\mathbb{Q}[X]$.

(9) Consider fields $K \subset L \subset M$. Suppose $\alpha \in M$ is algebraic over L. Further suppose that L is algebraic over K. Then α is algebraic over K.

(10) The fields $\mathbb{R}$ and $\mathbb{C}$ have the same prime field.

(11) A subring of a non-commutative ring is itself also non-commutative.

(12) Let $\alpha \in \mathbb{C}$. Then there exists a polynomial $f \in \mathbb{R}[X]$ with $f(\alpha) = 0$.

(13) The fields $\mathbb{Q}(\pi)$ and $\mathbb{Q}(e)$ are isomorphic. (Here $\pi = 3,1415\ldots$ and $e = 2,71828\ldots$ are the usual constants.)

(14) Let K be a field. If $f \in K[X]$ is an irreducible polynomial of degree $d > 0$ and $K \subset L$ is decomposition field of f over K then $[L : K] = d$.

(15) The ring $\mathbb{C}[X]$ has infinitely many prime ideals.

(16) If $R_1 \subset R_2$ is a subring and $I \subset R_2$ is a principal ideal then $R_1 \cap I$ is a principal ideal of R_1.

(17) Let K be a field, and let $M_n(K)$ be the ring of $n \times n$ matrices with coefficients in K. Then $M_n(K)$ has no zero divisors.

(18) An infinite field has characteristic 0.

(19) Let $\mathbb{Q} \subset K$ be a field extension of degree 6. If $\alpha \in K$ and $\alpha \notin \mathbb{Q}$ then the minimum polynomial of α over $\mathbb{Q}$ has degree 6.

(20) Same question, but now with $[K : \mathbb{Q}] = 7$.