

## Algebraic Number Theory — Final assignment.

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Your grade will depend on how well you have explained your arguments. Please try to be efficient; it should be possible to do the whole exercise in 2 or 3 pages (when typed in  $\text{\TeX}$ ). But also make sure that all steps are clearly motivated and don't make it too terse.

**Algebraic Number Theory — Final assignment, version A.**

Let

$$F := \mathbb{Q}[T]/(f) \quad \text{with} \quad f = t^4 + 8t^2 + 3t + 1.$$

Determine the structure of the class group  $\text{Cl}(O_F)$  and give a unit  $u \in O_F^*$  of infinite order. Your answer for the class group should be stated in the form “The class group is isomorphic with  $\mathbb{Z}/r_1\mathbb{Z} \times \cdots \times \mathbb{Z}/r_t\mathbb{Z}$ ”, where you give explicit integers  $r_i$  such that  $r_i | r_{i+1}$  for all  $i < t$ .

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**Algebraic Number Theory — Final assignment, version B.**

Let

$$F := \mathbb{Q}[T]/(f) \quad \text{with} \quad f = t^4 + 3t^2 + 7t + 4.$$

Determine the structure of the class group  $\text{Cl}(O_F)$  and give a unit  $u \in O_F^*$  of infinite order. Your answer for the class group should be stated in the form “The class group is isomorphic with  $\mathbb{Z}/r_1\mathbb{Z} \times \cdots \times \mathbb{Z}/r_t\mathbb{Z}$ ”, where you give explicit integers  $r_i$  such that  $r_i | r_{i+1}$  for all  $i < t$ .

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**Algebraic Number Theory — Final assignment, version C.**

Let

$$F := \mathbb{Q}[T]/(f) \quad \text{with} \quad f = t^3 + 3t^2 + 27t - 111.$$

Determine the structure of the class group  $\text{Cl}(O_F)$  and give a unit  $u \in O_F^*$  of infinite order. Your answer for the class group should be stated in the form “The class group is isomorphic with  $\mathbb{Z}/r_1\mathbb{Z} \times \cdots \times \mathbb{Z}/r_t\mathbb{Z}$ ”, where you give explicit integers  $r_i$  such that  $r_i | r_{i+1}$  for all  $i < t$ .

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**Algebraic Number Theory — Final assignment, version D.**

Let

$$F := \mathbb{Q}[T]/(f) \quad \text{with} \quad f = t^3 - 4t^2 + 23.$$

Determine the structure of the class group  $\text{Cl}(O_F)$  and give a unit  $u \in O_F^*$  of infinite order. Your answer for the class group should be stated in the form “The class group is isomorphic with  $\mathbb{Z}/r_1\mathbb{Z} \times \cdots \times \mathbb{Z}/r_t\mathbb{Z}$ ”, where you give explicit integers  $r_i$  such that  $r_i | r_{i+1}$  for all  $i < t$ .

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**Algebraic Number Theory — Final assignment, version E .**

Let

$$F := \mathbb{Q}[T]/(f) \quad \text{with} \quad f = t^3 - t^2 - 11t - 20.$$

Determine the structure of the class group  $\text{Cl}(O_F)$  and give a unit  $u \in O_F^*$  of infinite order. Your answer for the class group should be stated in the form “The class group is isomorphic with  $\mathbb{Z}/r_1\mathbb{Z} \times \cdots \times \mathbb{Z}/r_t\mathbb{Z}$ ”, where you give explicit integers  $r_i$  such that  $r_i | r_{i+1}$  for all  $i < t$ .

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**Algebraic Number Theory — Final assignment, version F.**

Let

$$F := \mathbb{Q}[T]/(f) \quad \text{with} \quad f = t^3 - 2t - 15.$$

Determine the structure of the class group  $\text{Cl}(O_F)$  and give a unit  $u \in O_F^*$  of infinite order. Your answer for the class group should be stated in the form “The class group is isomorphic with  $\mathbb{Z}/r_1\mathbb{Z} \times \cdots \times \mathbb{Z}/r_t\mathbb{Z}$ ”, where you give explicit integers  $r_i$  such that  $r_i | r_{i+1}$  for all  $i < t$ .

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**Algebraic Number Theory — Final assignment, version G.**

Let

$$F := \mathbb{Q}[T]/(f) \quad \text{with} \quad f = t^3 - 33t^2 + 361t - 1324.$$

Determine the structure of the class group  $\text{Cl}(O_F)$  and give a unit  $u \in O_F^*$  of infinite order. Your answer for the class group should be stated in the form “The class group is isomorphic with  $\mathbb{Z}/r_1\mathbb{Z} \times \cdots \times \mathbb{Z}/r_t\mathbb{Z}$ ”, where you give explicit integers  $r_i$  such that  $r_i | r_{i+1}$  for all  $i < t$ .

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Determine the structure of the class group  $\text{Cl}(O_F)$  and give a unit  $u \in O_F^*$  of infinite order. Your answer for the class group should be stated in the form “The class group is isomorphic with  $\mathbb{Z}/r_1\mathbb{Z} \times \cdots \times \mathbb{Z}/r_t\mathbb{Z}$ ”, where you give explicit integers  $r_i$  such that  $r_i | r_{i+1}$  for all  $i < t$ .

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In your text, all steps leading to the final answer should be carefully explained. You are *not* allowed to base these steps on computer output. You should give a complete proof that your answers are correct.

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For all results of a trivial computational nature you are allowed to use a pocket calculator or a computer. E.g., for the factorization of an integer, the calculation of the determinant of a matrix, the calculation of the discriminant of a polynomial, evaluation of a polynomial at some integers, etc.: use a computer. In such cases, state the result without giving the computational details. You may use maple/pari/... to do basic calculations (multiplication, division, etc.) in  $O_F$ , in quotient rings  $O_F/\mathfrak{p}$  or in rings like  $\mathbb{Z}/n\mathbb{Z}$ . Also, given some element of your number field  $F$ , you may determine its characteristic polynomial using the computer. On the other hand, to find units, to find prime ideals, to find the decomposition of some ideal into prime factors, to find the index of  $\mathbb{Z}[\alpha]$  in  $O_F$ , etc.: you may first use a computer to find the correct answers (this I even encourage), but after that you should explain how to obtain these results based on abstract arguments. Cf. the examples in the lecture notes, especially Chapter 12.

Your grade will depend on how well you have explained your arguments. Please try to be efficient; it should be possible to do the whole exercise in 2 or 3 pages (when typed in  $\text{\TeX}$ ). But also make sure that all steps are clearly motivated and don't make it too terse.

**Algebraic Number Theory — Final assignment, version Z.**

Let

$$F := \mathbb{Q}[T]/(f) \quad \text{with} \quad f = t^4 + t^3 + 3t^2 + 8t + 8.$$

Determine the structure of the class group  $\text{Cl}(O_F)$  and give a unit  $u \in O_F^*$  of infinite order. Your answer for the class group should be stated in the form “The class group is isomorphic with  $\mathbb{Z}/r_1\mathbb{Z} \times \cdots \times \mathbb{Z}/r_t\mathbb{Z}$ ”, where you give explicit integers  $r_i$  such that  $r_i | r_{i+1}$  for all  $i < t$ .

**Important**

Please turn over for some important information concerning this assignment. Read this information carefully before you begin writing your solutions. The deadline for handing in is January 31st, 2010.