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On the stellar luminosity of the universe

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ABSTRACT
It has been noted at times that the rate of energy release in the most violent explosive events in the Universe, such as supernovae and gamma-ray bursts, rivals the stellar luminosity of the observable universe, $\mathcal{L}_*$. The underlying reason turns out to be that both can be scaled to $c^5/G \equiv \mathcal{L}_{\rm G}$, albeit that for the explosions $L/\mathcal{L}_{\rm G}$ follows from first principles, whereas for $\mathcal{L}_*$ the scaling involves quantities too complex to derive from elementary considerations at the present time. Under fairly general circumstances, $\mathcal{L}_*$ is dominated by stars whose age is similar to the Hubble time.

Key words: cosmology – stellar evolution – gamma-rays: bursts – supernovae

1 INTRODUCTION

When a massive star dies, its core collapses to, or close to, the Schwarzschild radius, $R_{\rm S} \equiv 2GM/c^2$. Since of the order of the rest energy of the mass can thus be released, in a time close to the dynamical time ($R_{\rm S}/c$), this can lead to the release of energy at a rate $Mc^2/(R_{\rm S}/c) \sim c^5/G \equiv \mathcal{L}_{\rm G}(= 3.7\times10^{59}{\rm erg\,s^{-1}})$. This number is independent of the collapsing mass, and might be achieved in practice, for example, as the gravity-wave luminosity from two merging black holes[1]. Since gamma-ray bursts (at least of the long-soft variety) are thought to result from the core collapse of massive stars, it is not surprising that their luminosity should be a fair fraction of $\mathcal{L}_{\rm G}$ (for a review on gamma-ray bursts, see Van Paradijs et al. (2000)). Even the deviation from $\mathcal{L}_{\rm G}$ can be understood quantitatively. For example, the neutrino luminosity of a supernova is down by a factor 0.1 because only that fraction of the rest mass is released, and by another factor 10^5 because the neutrinos are released on the much slower diffusion time rather than the dynamical time, leading to a luminosity of order $10^{53}{\rm erg\,s^{-1}}$.

Is it, however, understandable that this luminosity should roughly rival the stellar luminosity, $\mathcal{L}_*$, of the observable universe, or is that a mere numerical coincidence? A rough estimate puts $\mathcal{L}_*$ at a few times $10^{55}{\rm erg\,s^{-1}}$, since the observable universe contains of order 10^{11} galaxies, each with on average 10^{11} stars, that emit a few times $10^{33}{\rm erg\,s^{-1}}$ per star. In sect. 2 I show that one can actually write $\mathcal{L}_*$ in terms of fundamental quantities, albeit that some microscopic processes enter via parameters; in sect. 3 I then eliminate the dependence on the stellar nuclear burning time. The main findings are summarized in sect. 4.

2 THE LUMINOSITY OF ALL STARS

In this work I am after finding characteristic numbers and global scalings, not precise values. I therefore do not calculate corrections for and integrations over redshift, but simply count the observable universe as the Euclidean volume out to the Hubble radius, $r_{\rm H} = ct_{\rm H}$. I also approximate $t_{\rm H} = H_0^{-1}$, which is strictly valid only for an empty Universe, but close enough for plausible others. (For these, and other elementary cosmology relations, see, e.g., Peebles (1993).)

First we express the luminosity of one star in terms of its mass and lifetime:

$$L_* = \eta \frac{M_* c^2}{t_*} \quad (1)$$

Since we are going to average over stellar populations eventually, we are only interested in the average luminosity and count t_* as its total lifetime, rather than the instantaneous nuclear burning time of a specific evolutionary phase. This in turn is well approximated by the main-sequence lifetime. η is the nuclear burning efficiency, which is 0.007 for the fractional mass loss due to fusion, times another factor 0.5 to account for the fact that only about half the initial mass of a star will go through nuclear burning. Assuming that a fraction f_* of all the mass within the Hubble radius, $M_{\rm H}$, is in stars, we can write the total stellar luminosity within the Hubble radius as

$$\mathcal{L}_* = \eta f_* M_{\rm H} c^2 / t_*. \quad (2)$$

In order to proceed from here, it helps to note that the

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[1] As is well known, $\mathcal{L}_{\rm G}$ is also the only quantity of dimension luminosity that can be constructed out of the fundamental constants of gravity and relativity, and thus the only natural luminosity in the large-scale world (as that is uncharged and not quantum-mechanical).

Schwarzschild radius of all the matter within the Hubble radius is of order $\Omega r_{\rm H}$. This can be seen as follows:

$$R_{\rm S,H} = \frac{2GM_{\rm H}}{c^2} = \frac{8\pi G}{3c^2}\rho_{\rm m}(ct_{\rm H})^3, \tag{3}$$

where we have made the Euclidean-volume approximation, and $\rho_{\rm m}$ is the total matter density (baryonic plus dark). Now we can write $\rho_{\rm m} = \Omega_{\rm m}\rho_{\rm c}$, where the critical density, $\rho_{\rm c} = 3H_0^2/8\pi G$, and use $H_0 t_{\rm H} = 1$ to get

$$R_{\rm S,H} = \Omega_{\rm m} c t_{\rm H} = \Omega_{\rm m} r_{\rm H}, \tag{4}$$

as stated at the start. (Somewhat crudely, one could say that a flat universe comes close to living inside its own black hole.) We can now use this result to rewrite $M_{\rm H}$ in the expression for $\mathcal{L}_*$:

$$M_{\rm H} \equiv \frac{c^2 R_{\rm S,H}}{2G} = \frac{c^3\Omega_{\rm m} t_{\rm H}}{2G}. \tag{5}$$

Putting everything together, we then get

$$\mathcal{L}_* = \frac{f_*\eta\Omega_{\rm m}}{2}\frac{t_{\rm H}}{t_*}\frac{c^5}{G} = \frac{f_*\eta\Omega_{\rm m}}{2}\frac{t_{\rm H}}{t_*}\mathcal{L}_{\rm G}. \tag{6}$$

Recalling that $\eta \simeq 1/300$, $\Omega_{\rm m} \simeq 0.3$, and about 10% of the baronic mass is in stars[2] (e.g., Fukugita & Peebles (2004)), so $f_* \sim 0.01$, this implies $\mathcal{L}_* \sim 10^{55} t_{\rm H}/t_*$. This is in reasonable agreement with the crude observational estimate, provided that the relevant stellar age is close to the Hubble time.

3 THE AGE OF THE DOMINANT STARS

The next step is to show that indeed usually the relevant mean stellar age equals the Hubble time. Note again that we average over scales that are a significant fraction of the horizon scale, so that we parametrize the average of all types of star formation (from slow and gradual to massive starbursts) simply by what is the total mass in stars, and what is the representative mean initial mass function (IMF) of stars. Let the formation rate of stars of mass M at time t since the Big Bang be given by

$$S(M,t) = Kt^{-q}M^{-1-x}, \tag{7}$$

where $x \simeq 1.5$ (Salpeter 1955; Miller & Scalo 1979), and q is positive, but not large, currently (i.e., we take S simply to be proportional to the (time-independent) IMF and to change gradually with time). The present day mass function $P(M)$ can then be found by integrating $S(M,t)$ over time. For small masses, all stars formed since the beginning are still around, and so we simply integrate the time dependence. For large masses, only the stars formed during a time $t_*(M) \propto M/L$ in the recent past count. The transition between the two regimes happens at the mass $\tilde{M}$, for which $t_*(\tilde{M}) = t_{\rm H}$, so we have

$$\begin{aligned} P(M) &\propto t^{1-q}M^{-1-x} && M < \tilde{M} \\ &\propto t^{-q}M^{-x}/L(M) && M > \tilde{M}. \end{aligned} \tag{8}$$

To get the contibution to the total luminosity from mass M, we finally multiply by $M.L(M)$:

$$\begin{aligned} \mathcal{L}(\mathcal{M}) &\propto t^{1-q}M^{-x}L(M) && M < \tilde{M} \\ &\propto t^{-q}M^{1-x} && M > \tilde{M}. \end{aligned} \tag{9}$$

Since L scales very steeply with M, typically $L \propto M^{3.5}$ on the lower main sequence, this implies that $\mathcal{L} \propto M^2$ up to $\tilde{M}$, and $\mathcal{L} \propto M^{-0.5}$ thereafter, so indeed the total luminosity from a gradually-forming population of stars is dominated by those for which $M = \tilde{M}$, hence $t_* = t_{\rm H}$. Note that this result stems from the steep negative slope of the initial mass function combined with the steep positive slope of the mass-luminosity relation for stars. Specially, for a significantly flatter IMF than the present-day $x \simeq 1.5$, the most massive stars in the population might dominate the total luminosity, and then the result is not valid. With that caveat, however, we may conclude that under conditions of star formation as we now know it, it is valid to set $t_* = t_{\rm H}$ and thus

$$\mathcal{L}_* \simeq f_*\eta\Omega_{\rm m}\mathcal{L}_{\rm G}. \tag{10}$$

4 CONCLUSIONS

Starting from the nuclear burning rate of a star, and using Euclidean approximations to get rough numbers for the mass in the observable universe, I find that the total stellar luminosity in the observable universe can be expressed as a fraction of the characteristic gravitational luminosity, $\mathcal{L}_{\rm G} = c^5/G$. The constants of proportionality in the final equation, eq. 10, hide our ignorance about the details of star formation in the form of f_*, the fraction of mass in the universe that is in stars. Part of the reason why a simple expression of this type could be found is that both the birth rate of stars and the luminosity of a star are steep functions of stellar mass, which conspire to make the stars that dominate the total luminosity to be always those that are about the same age as the universe (or of any other closed system in which star formation is a slow function of time).

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[2] Recall that f_* is defined as the mass in stars divided by the total (baryonic plus dark) matter mass.