Coursework #2

Deadline: Monday, 27 February 2006, 11:15am

Question 1 (10 marks)

Suppose a newspaper announces the following competition:

> *Every reader may send in a (rational) number between* 0 *and* 100*. The winner is the player whose number is closest to* $\frac{2}{3}$ *times the arithmetic mean of all submissions (in case of a tie the prize money is split equally amongst those with the best guesses).*

(a) Think of some strategies that people might use to decide what number to send in (I really only want you to *think* about it, you do not need to submit an answer for this subquestion).

(b) Does this game have a Nash equilibrium? If yes, what is it?

(c) What changes about Nash equilibria if players can only choose integers?

(d) What changes if players can only choose integers and the mean is being multiplied by $\frac{9}{10}$ rather than $\frac{2}{3}$?

Question 2 (10 marks)

Two candidates, A and B, compete in an election. Of the n citizens, k support candidate A and $m = n - k$ support candidate B. Each citizen decides either to vote, at a cost c (with $0 < c < 1$), for the candidate they support, or to abstain. A citizen who abstains receives the payoffs of 2 if their candidate wins, 1 if there is a draw, and 0 if their candidate loses. A citizen who does vote receives the payoffs $2 - c$, $1 - c$, and $-c$, respectively.

- For $k = m = 1$, is this the same as any of the games introduced in class?
- For $k = m$, identify the set of all Nash equilibria.

  You may find considering some of the following questions helpful: Is the situation where everyone votes in equilibrium? Is there a Nash equilibrium in which the candidates tie and not everyone votes? Is there a Nash equilibrium in which one of the candidates wins by just one vote? Is there a Nash equilibrium in which one of the candidates wins by two or more votes?
- What is the set of Nash equilibria for $k < m$?

(Adapted from M.J. Osborne, *An Introduction to Game Theory*, OUP, 2004.)

(Please turn over)

Question 3 (10 marks)

The numbers in a game matrix mean slightly different things depending on whether we do or do not admit mixed strategies. If only pure strategies are considered, then the numbers represent ordinal preference relations for the players involved, and the cardinal intensities of payoffs do not matter. This question is about games with pure strategies only. Two such games are equivalent iff the game matrices involved represent the same ordinal preference relations. Answer the following questions (and justify your answers):

(a) How many different two-player games with two actions per player are there?

(b) How many of these do not have a (pure) Nash equilibrium?

Question 4 (10 marks)

Compute all mixed Nash equilibria for each of the following two games. Show your working.

(a)

	L	R
T	5/8	3/4
B	2/2	7/3

(b)

	L	R
T	3/7	4/8
B	1/4	8/3