

Coursework #1

Deadline: Monday, 20 February 2006, 11:15am**Question 1** (10 marks)

- (a) Is it possible that a Pareto optimal allocation does not have maximal utilitarian social welfare? If yes, give an example; if no, explain why not.
- (b) Is it possible that an allocation with maximal utilitarian social welfare is not Pareto optimal? If yes, give an example; if no, explain why not.
- (c) Is it possible that an allocation with maximal egalitarian social welfare is not Pareto optimal? If yes, give an example; if no, explain why not.
- (d) Is it possible that an allocation that is optimal with respect to the leximin-ordering is not Pareto optimal? If yes, give an example; if no, explain why not.
- (e) Is it possible that an allocation that is optimal with respect to the leximin-ordering does not have maximal utilitarian social welfare? If yes, give an example; if no, explain why not.

Question 2 (10 marks)

Choose four pairs of axioms for SWOs amongst those introduced in class (ANO, UNA, ZI, SI, ICP, SEP, ICZ, ICS, PD). For each of those pairs of axioms show whether (1) one of them implies the other, (2) they are inconsistent, or (3) they are logically independent.

Question 3 (10 marks)

Suppose three towns, A , B and C , are located in the plane \mathbb{R}^2 . We have to decide where to build a new hospital H . Any point in the plane is feasible. The *disutility* of a town is the distance of that town to H .

- (a) Show that the Pareto optimal locations for H are precisely those that are lying within the triangle ΔABC .
- (b) Show that we have an equality-efficiency dilemma iff that triangle is obtuse angled (that is, iff it has an angle of more than 90 degrees).
- (c) Give a geometric characterisation of the optimum of the egalitarian CUF in case the triangle is obtuse angled.

(Adapted from H. Moulin, *Axioms of Cooperative Decision Making*, CUP, 1988.)

(Please turn over)

Question 4 (10 marks)

Suppose there are n agents located anywhere on the interval $[0, 1]$. We have to decide where to build an amusement park A , also anywhere on the same interval. The *disutility* of an agent is its distance to A .

- (a) What is the solution selected by the egalitarian CUF?
- (b) What is the solution selected by the elitist (n -rank dictator) CUF?
- (c) For arbitrary $k \leq n$, give a general procedure to compute the solution that would be optimal with respect to the k -rank dictator CUF.