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Part I: Hartley and Zisserman Appendix 6:

Iterative estimation methods

Part II: Zhengyou Zhang:

A Flexible New Technique for Camera Calibration

Presented by Daniel Fontijne

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HZ Appendix 6: Iterative estimation methods

Topics:

- Basic methods: Newton, Gauss-Newton, gradient descent.
- Levenberg-Marquardt.
- Sparse Levenberg-Marquardt.
- Applications to homography, fundamental matrix, bundle adjustment.
- Sparse methods for equations solving.
- Robust cost functions.
- Parameterization.

Lecture notes which I found useful

(methods for non-linear least squares problems):

http://www2.imm.dtu.dk/pubdb/views/edoc_download.php/3215/pdf/imm3215.pdf

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Iterative estimation methods

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Examples of HZ problems:
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Central approach of Appendix 6: Levenberg-Marquardt.

Questions: Pronunciation? Why LM?

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Newton iteration

Goal: minimize $\mathbf{X} = \mathbf{f}(\mathbf{P})$ for $\mathbf{P}$.

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We assume $\mathbf{f}$ is locally linear at each $\mathbf{P}_i$, then
$\mathbf{f}(\mathbf{P}_i + \Delta_i) = \mathbf{f}(\mathbf{P}_i) + \mathsf{J}_i\Delta_i$,
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Find Δ_i either using normal equations: $\mathsf{J}_i^\mathsf{T}\mathsf{J}_i\Delta = -\mathsf{J}_i^\mathsf{T}\boldsymbol{\epsilon}_i$
or using pseudo-inverse: $\Delta_i = -\mathsf{J}_i^+\boldsymbol{\epsilon}_i$.

Iterate until convergence . . .

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Gauss-Newton method

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We may expand in a Taylor series up to second degree
$g(\mathbf{P}+\Delta) = g + g_{\mathbf{P}}\Delta + \Delta^{\mathtt{T}} g_{\mathbf{PP}}\Delta/2$,
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Putting it all together we get $\mathtt{J}^\mathtt{T}\mathtt{J}\Delta = -\mathtt{J}^\mathtt{T}\boldsymbol{\epsilon}$.
So we arrive at the normal equations again.
(So what was the point?)

Gradient descent

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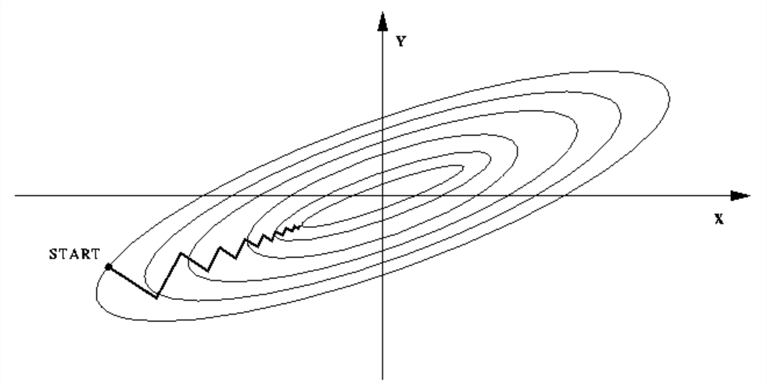
A problem is zig-zagging which can cause slow convergence:



Levenberg-Marquardt

Levenberg-Marquardt is a blend of Gauss-Newton and gradient descent. Update equation:

$$(\mathtt{J}^{\mathtt{T}}\mathtt{J} + \lambda \mathtt{I})\Delta = -\mathtt{J}^{\mathtt{T}}\boldsymbol{\epsilon}.$$

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- Initially set $\lambda = 10^{-3}$.
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The idea is (?):
-take big gradient descent steps far away from minimum.
-take Gauss-Newton steps near (hopefully quadratic) minimum.

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Sparse Levenberg-Marquardt 1/2

In many estimation problems, the Jacobian is sparse.
One should this to lower the time complexity (sometimes even from $O(n^3)$ to $O(n)$).

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In the example, the parameters are partitioned into two blocks:
$\mathbf{P} = (\mathbf{a}^T, \mathbf{b}^T)^T$

The Jacobian then has the form $\mathtt{J} = [A|B]$, with
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$A = [\partial\hat{\mathbf{X}}/\partial\mathbf{a}], \qquad B = [\partial\hat{\mathbf{X}}/\partial\mathbf{b}]$.

Using A and B, the normal equations $(\mathtt{J}^\mathtt{T}\mathtt{J})\Delta = -\mathtt{J}^\mathtt{T}\boldsymbol{\epsilon}$ take on the the form

$$\left[\begin{array}{c|c} A^T A & A^T B \\ \hline B^T A & B^T B \end{array}\right] \left(\begin{array}{c} \boldsymbol{\delta}_\mathbf{a} \\ \hline \boldsymbol{\delta}_\mathbf{b} \end{array}\right) = \left(\begin{array}{c} A^T \boldsymbol{\epsilon} \\ \hline B^T \boldsymbol{\epsilon} \end{array}\right).$$

Sparse Levenberg-Marquardt 2/2

If the normal equations are written as (what's with the *?)

$$\begin{bmatrix} \mathtt{U}^* & \mathtt{W} \\ \mathtt{W}^T & \mathtt{V}^* \end{bmatrix} \begin{pmatrix} \boldsymbol{\delta}_{\mathbf{a}} \\ \boldsymbol{\delta}_{\mathbf{b}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\epsilon}_A \\ \boldsymbol{\epsilon}_B \end{pmatrix},$$

we can rewrite this to

$$\begin{bmatrix} \mathtt{U}^* - \mathtt{W}\mathtt{V}^{*-1}\mathtt{W}^T & 0 \\ \mathtt{W}^T & \mathtt{V}^* \end{bmatrix} \begin{pmatrix} \boldsymbol{\delta}_{\mathbf{a}} \\ \boldsymbol{\delta}_{\mathbf{b}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\epsilon}_A - \mathtt{W}\mathtt{V}^{*-1}\boldsymbol{\epsilon}_B \\ \boldsymbol{\epsilon}_B \end{pmatrix}$$

by multiplying on the left by $\begin{bmatrix} \mathtt{I} & \mathtt{W}\mathtt{V}^{*-1} \\ 0 & \mathtt{I} \end{bmatrix}$.

Now first solve the top half, then the lower half using back-substitution.

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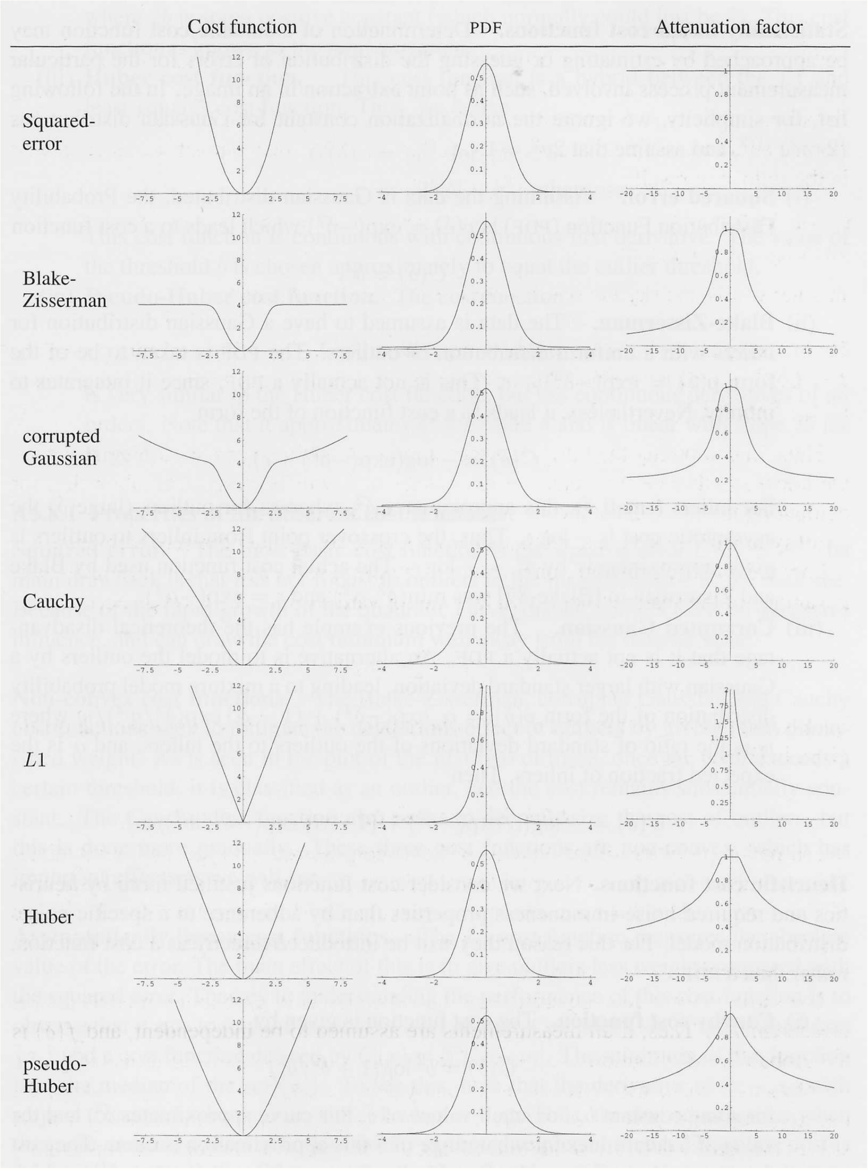
Robust cost functions 1/5



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Robust cost functions 2/5

Squared-error is not usable unless outliers are filtered out.

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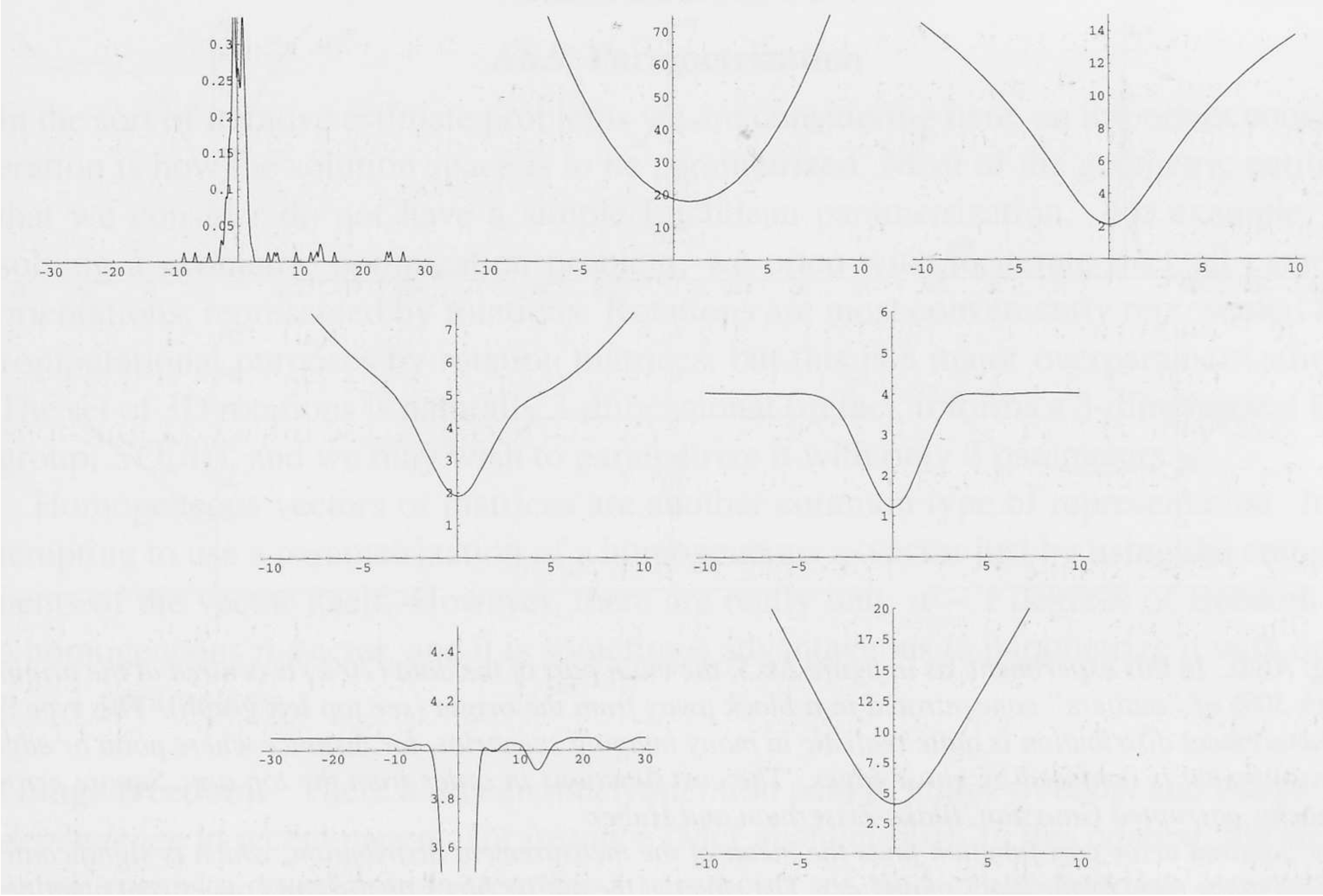
- Pseudo Huber: like Huber, but with continuous derivatives.

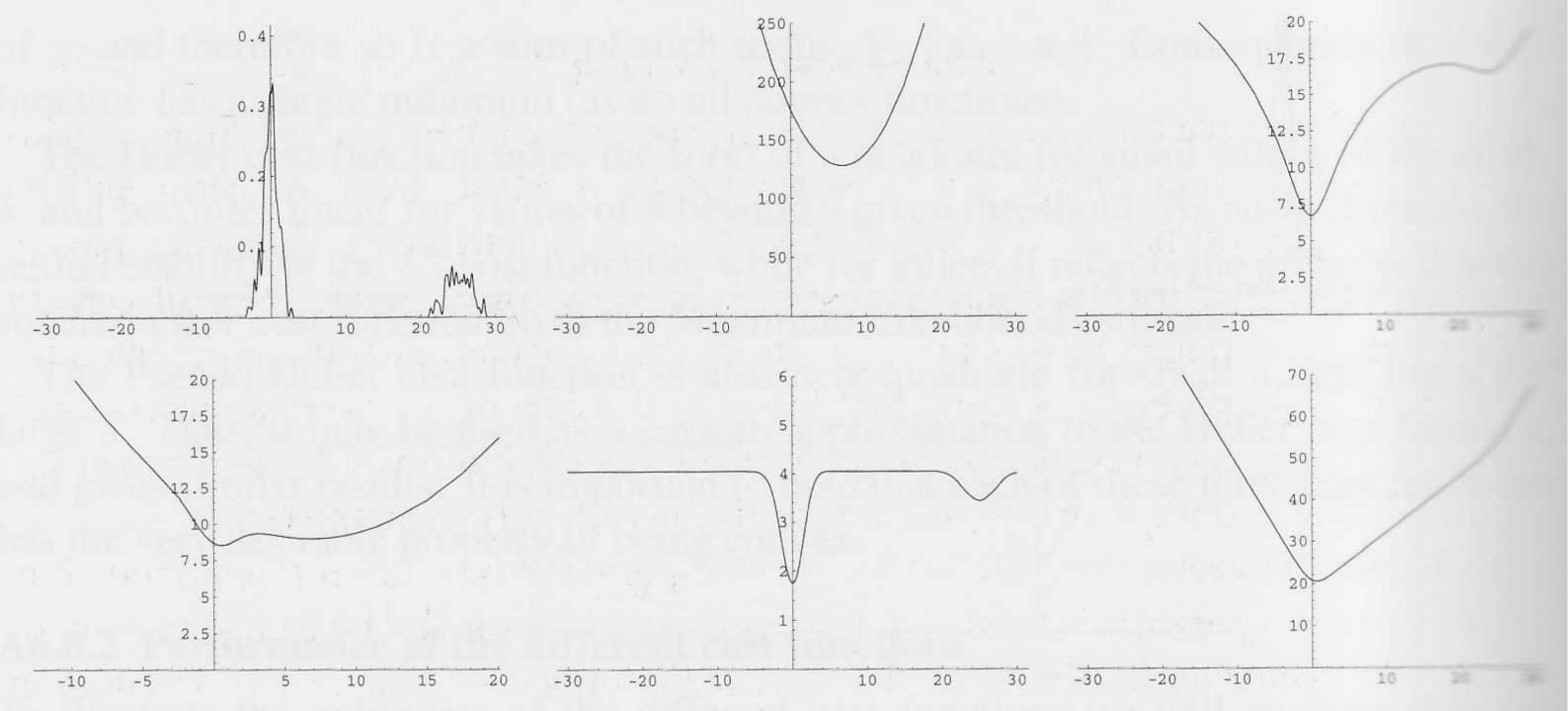
Robust cost functions 3/5

Figure A.6.5



Robust cost functions 4/5

Figure A.6.6



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Robust cost functions 5/5

Summary:

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Robust cost functions 5/5

Summary:

- Squared-error cost function is very susceptible to outliers.
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- Best: Huber and Pseudo-Huber.

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Fooling LM implementations

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Thus

$$w_i = \frac{\sqrt{C(\|\delta_i\|)}}{\|\delta_i\|}.$$

(confusion about δ being a vector? why not scalar?)

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Variance?

Parameterization of 3-D rotations

3-D rotation matrix: 9 elements, only 3 degrees of freedom.

Angle-axis (3-vector) representation: 3 elements, 3 d.o.f.

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- Normalization: stay away from $\|\mathbf{t}\| = 2\pi$.

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Parameterization of homogeneous vectors

Let $\mathbf{v}$ be a n-D-vector (already stripped of ‘extra’ homogeneous coordinate?).

Then parameterize it as $n+1$ vector:
$\bar{v} = (\text{sinc}(\|\mathbf{v}\|/2)\mathbf{v}^T, \cos(\|\mathbf{v}\|/2))^T$.

Parameterization of the n-sphere

How to parameterize unit vectors $\mathbf{x}$?

Compute Householder matrix (reflection) such that $\mathtt{H}_{\mathbf{v}(\mathbf{x})}\mathbf{x} = (0, \ldots, 0, 1)^{\mathtt{T}}$.

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(ii) $f(\mathbf{y}) = (\mathrm{sinc}(\|\mathbf{y}\|/2)\mathbf{y}^T, \cos(\|\mathbf{y}\|/2))^T$ (?).
both have a Jacobian $\partial f/\partial \mathbf{y} = [\mathtt{I}|\mathbf{0}]^T$.

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So ‘constrained’ Jacobian can be computed

$$\mathtt{J} = \frac{\partial C}{\partial \mathbf{y}} = \frac{\partial C}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{y}} = \frac{\partial C}{\partial \mathbf{x}}\mathtt{H}_{\mathbf{v}(\mathbf{x})}\mathbf{x}[\mathtt{I}|\mathbf{0}]^T.$$

Zhang Paper

Zhengyou Zhang

A Flexible New Technique for Camera Calibration

(1998)

Zhang Paper

Zhengyou Zhang

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(1998)

As implemented for:

Matlab The Camera Calibration Toolbox for Matlab

C++ Intel OpenCV

Internal Camera Calibration 1/4

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In short, the camera intrinsic matrix:

$$\mathbf{A} = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

Internal Camera Calibration 2/4

The Zhang algorithm also computes radial lens distortion parameters $[k_1, k_2, k_3, k_4]$.

The original paper uses
$x_d = x + x\,(k_1\,(x^2 + y^2) + k_2\,(x^2 + y^2)^2)$,
$y_d = y + y\,(k_1\,(x^2 + y^2) + k_2\,(x^2 + y^2)^2)$,
where x and y are normalized image coordinates and x_d and y_d are the distorted coordinates.

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But the implementations use a more complex model
$x_d = x + x\,(k_1(x^2 + y^2) + k_2\,(x^2 + y^2)^2) + x_{td}$,
$y_d = y + y\,(k_1(x^2 + y^2) + k_2\,(x^2 + y^2)^2) + y_{td}$,
where
$x_{td} = 2k_3\,x\,y + k_4\,(3\,x^2 + y^2)$,
$y_{td} = 2k_4\,x\,y + k_3\,(x^2 + 3\,y^2)$.

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Internal Camera Calibration 3/4

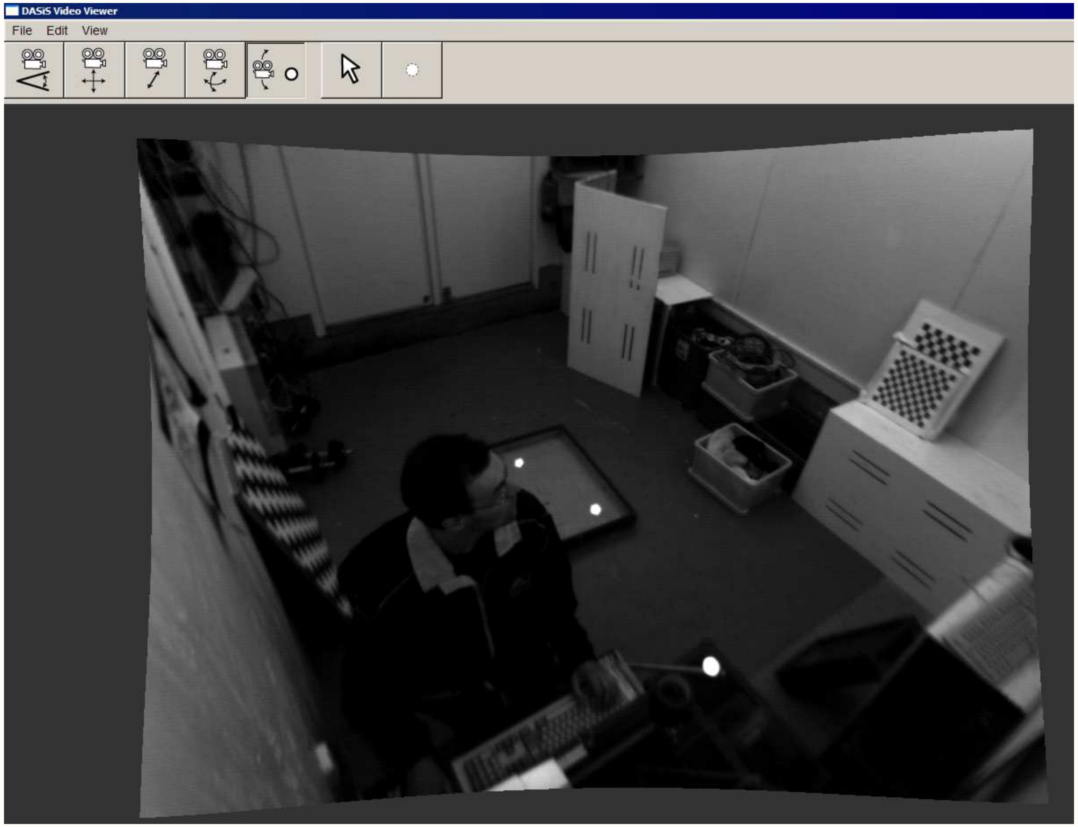
Example of internal camera calibration parameters.

Camera: PixeLINK A741, 2/3 inch CMOS sensor, 1280x1024.
Lens: Cosmicar 8.5mm fixed focal length.

$$f_x = 1272.872 \,\text{pixels} = 8.528\text{mm}$$
$$f_y = 1272.988 \,\text{pixels} = 8.529\text{mm}$$
$$c_x = 632.740$$
$$c_y = 507.648$$
$$k_1 = -0.204$$
$$k_2 = 0.171$$
$$k_3 = -0.00074896$$
$$k_4 = 0.00008878$$

Internal Camera Calibration 4/4

Show lens distortion in DASiS video viewer. . .



External Camera Calibration

The Zhang algorithm may also be used for external camera calibration.

Camera rotation and translation are computed as side-product of internal calibration.

If two cameras see the same calibration pattern at the same time, their relative position and orientation may be computed.

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- Setup equations in order to estimate camera intrinsics.
- Given camera intrinsics, estimate extrinsics.
- Estimate radial distortion.
- Use Levenberg-Marquardt to optimize initial estimates.

Basic Equations

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Projection:

$$s\widetilde{\mathbf{m}} = \mathbf{A}[\mathbf{R} \;\; \mathbf{t}]\widetilde{\mathtt{M}} =$$

$$\begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} [\mathbf{r}_1 \;\; \mathbf{r}_2 \;\; \mathbf{r}_3 \;\; \mathbf{t}] \, [X \;\; Y \;\; 0 \;\; 1]^T =$$

$$\mathbf{A} \, [\mathbf{r}_1 \;\; \mathbf{r}_2 \;\; \mathbf{t}] \, [X \;\; Y \;\; 1]^T$$

Homography, constraints

An homography $\mathbf{H}$ can be estimated between known points on the calibration object and the measured world points.

$$\mathbf{H} = [\mathbf{h}_1 \ \ \mathbf{h}_2 \ \ \mathbf{h}_3] = \lambda \mathbf{A} [\mathbf{r}_1 \ \ \mathbf{r}_2 \ \ \mathbf{t}]$$

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We demand:

C1: $\mathbf{r}_1^T \mathbf{r}_2 = 0$ ($\mathbf{r}_1$, $\mathbf{r}_2$ orthogonal),

C2: $\mathbf{r}_1^T \mathbf{r}_1 = \mathbf{r}_2^T \mathbf{r}_2$ ($\mathbf{r}_1$, $\mathbf{r}_2$ have same length).

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We know:
$$\mathbf{h}_1 = \lambda \mathbf{A}\, \mathbf{r}_1 \quad \rightarrow \quad \mathbf{r}_1 = \lambda^{-1} \mathbf{A}^{-1} \mathbf{h}_1$$
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So the constraints are:

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Closed-form solution using constraints 1/4

Using the constraints, we can first find $\mathbf{A}$, followed by $\mathbf{R}$ and $\mathbf{t}$. Let

$$\mathbf{B} = \mathbf{A}^{-T}\mathbf{A}^{-1} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\alpha^2} & -\frac{\gamma}{\alpha^2\beta} & \frac{v_0\gamma - u_0\beta}{\alpha^2\beta} \\ -\frac{\gamma}{\alpha^2\beta} & \frac{\gamma^2}{\alpha^2\beta^2} + \frac{1}{\beta^2} & -\frac{\gamma(v_0\gamma - u_0\beta)}{\alpha^2\beta^2} - \frac{v_0}{\beta^2} \\ \frac{v_0\gamma - u_0\beta}{\alpha^2\beta} & -\frac{\gamma(v_0\gamma - u_0\beta)}{\alpha^2\beta^2} - \frac{v_0}{\beta^2} & \frac{(v_0\gamma - u_0\beta)^2}{\alpha^2\beta^2} + \frac{v_0^2}{\beta^2} + 1 \end{bmatrix}.$$

This allows to solve for α, β, etc.

Closed-form solution using constraints 2/4

If we reshuffle the six unique elements of $\mathbf{B}$ into a vector $\mathbf{b} = [B_{11}, B_{12}, B_{22}, B_{13}, B_{23}, B_{33}]$,

we can rewrite both constraints as $\mathbf{h}_i^T \mathbf{B} \mathbf{h}_j = \mathbf{v}_{ij}^T \mathbf{b}$,

where

$$\mathbf{v}_{ij} = [h_{i1}h_{j1}, h_{i1}h_{j2} + h_{i2}h_{j1}, h_{i2}h_{j2}, \\ h_{i3}h_{j1} + h_{i1}h_{j3}, h_{i3}h_{j2} + h_{i2}h_{j3}, h_{i3}h_{j3}]^T,$$

ultimately resulting in

$$\begin{bmatrix} \mathbf{v}_{12}^T \\ (\mathbf{v}_{11} - \mathbf{v}_{22})^T \end{bmatrix} \mathbf{b} = 0.$$

Closed-form solution using constraints 3/4

Next, stack all the equations from n measurements (estimated homographies) of the plane ('checkerboard'):

$$\mathbf{V}\mathbf{b} = 0,$$

where $\mathbf{V}$ is a $2n \times 6$ matrix. Solve as usual using the SVD.

Closed-form solution using constraints 4/4

Once $\mathbf{A}$ is known, we can obtain $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{t}$:

$$\mathbf{r}_1 = \lambda^{-1}\mathbf{A}^{-1}\mathbf{h}_1,$$
$$\mathbf{r}_2 = \lambda^{-1}\mathbf{A}^{-1}\mathbf{h}_2,$$
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Now Zhang says
$\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$,
and use SVD to make matrix $\mathbf{R}$ orthogonal, i.e.,
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I say:

Make $\mathbf{r}_1$, $\mathbf{r}_2$ orthogonal in least-squares sense.

The compute $\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$.

Is simpler and boils down to the same thing.

Radial distortion

Using the camera intrinsics and extrinsics undistorted coordinates of points (corners on the checkerboard) can be approximated. These is used to solve for k_1, k_2:

$$\begin{bmatrix} (u-u_0)(x^2+y^2) & (u-u_0)(x^2+y^2)^2 \\ (v-v_0)(x^2+y^2) & (v-v_0)(x^2+y^2)^2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \breve{u}-u \\ \breve{v}-v \end{bmatrix}.$$

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These equations are stacked ($\mathbf{D}[k_1 \;\; k_2]^T = \mathbf{d}$) and we solve least squares $[k_1 \;\; k_2]^T = (\mathbf{D}^T\mathbf{D})^{-1}\mathbf{D}^T\mathbf{d}$.

Then iterate both algorithm (internal+external, radial) until convergence.

Maximum likelihood estimation

Optimize: use Levenberg-Marquardt to find minimum of

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \|\mathbf{m}_{ij} - \breve{\mathbf{m}}(\mathbf{A}, k_1, k_2, \mathbf{R}_i, \mathbf{t}_i, \mathtt{M}_i)\|^2$$

(n images, m points per image)

All done . . .

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- Even with systematic non-planarity, results still usable.
- Error in compute sensor center seems not to have too much effect in 3-D reconstruction.