

Johan van Benthem

Fanning the flames of reason

VALEDICTORY LECTURE

Credits

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Valedictory lecture

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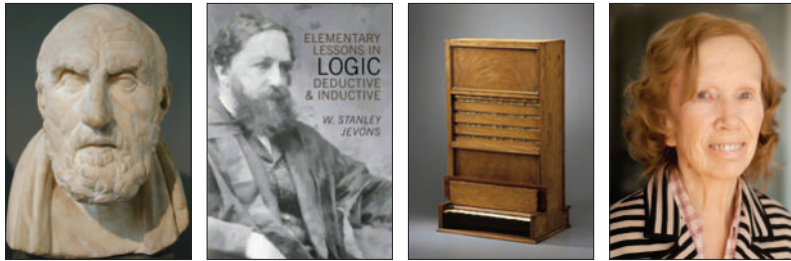
by

Johan van Benthem

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Arguments

When people hear that you are a logician, their eyes start looking for the nearest exit. But now and then, there are creatures like me, whose encounter with logic changed their life. Already in high school, I liked rules and formulas, which I jotted down in a special notebook. And when I arrived at the UvA in its revolutionary 1960s, I loved to engage in argumentation about the future shape of Dutch society – to be determined of course by us, the student elite. My argumentative skills were not particularly successful, however, and I well remember the sinking feeling of slowly, but inexorably talking myself into a corner with an indefensible claim. A kind fellow student advised me to buy a book about logic, and that is what I did.



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Reasoning patterns

What I bought was a pocket edition of a 19th century classic by William Stanley Jevons, available on the internet today. And what I learnt was that reasoning has *patterns*, and that some patterns are correct, while others are not.

Consider the following statement, where capital letters stand for arbitrary sentences:

If B, then not-H,

¹ I will explain the point of these and all further pictures as far as possible during my talk.

and ask yourself if it follows logically that

If not-B, then H.

You might be tempted to agree. But what if the doctor on duty in the emergency room where you have just been rushed in after a collapse, is trying to figure out what is wrong with you? Let us say that, if the cause is in your brain, it is not in your heart – but nothing is wrong with your brain scan. Is it correct to conclude that something is the matter with your heart? I do not think so, and neither should you. I would prefer a doctor whose reasoning goes differently, using a correct reasoning pattern like this:

from *if B, then not-H* to *if H, then not-B.*

Of course there is more to proper diagnosis than just these few rules – but at least, on the lighter side, you now see that logical correctness may be a matter of life or death.

Explaining and teaching

In general, logical reasoning patterns lie behind how we find the truth, and explain it to others, and ourselves. I always felt a need to explain things I think I understood as a private test. As a student, I would walk at night in the De Lairesestraat in Amsterdam talking to myself to explain what I had learnt from the day's classes. Again, people around me quickened their step and increased their distance – and this time, even without my having told them that I was a logician.²

Reasoning and information

But logic is more than seeing patterns and improving reasoning practice. With the study of correctness and incorrectness, we enter a much broader world of exact scientific ideas. One of these ideas is the notion of *information*.

In principle, there are 4 ways the world might be given the above statements *B* and *H*:

$+B+H, +B-H, -B+H, -B-H$

² A word of advice to the students. Do not give up weird behaviors too quickly by consulting the cool UvA student psychologists. Compulsive explainers often become successful teachers.

If we then learn the above rule *if B, then not-H*, then we lose the possibility that $+B+H$. That is, we eliminate 1 out of the 4 options to obtain the information.

$$+B-H, -B+H, -B-H$$

If we now learn that *not-B*, we lose the further possibility $+B-H$, but two still remain:

$$-B+H, -B-H$$

This final state does not have enough information to determine that H is the case.

You can check for yourself how this scenario of successive information updates differs from that for the logically valid inference from *if B, then not-H* to *if H, then not-B*.

Here the information does force the conclusion *not-B* through the following stages:

- | | | |
|-------|--------------------------|--|
| (i) | $+B+H, +B-H, -B+H, -B-H$ | |
| (ii) | $+B-H, -B+H, -B-H$ | (after learning that <i>if B, then not-H</i>) |
| (iii) | $-B+H$ | (after learning that H) |

So, reasoning is tied up with information, the lubricant of our whole society, and logic suddenly starts looking like a much bigger field than you might have thought.

Logical systems

And there is still more than reasoning patterns and their underlying ideas. Quickly, I learnt that patterns form larger *systems*, that can be studied on their own intrinsic merits – all the way back to Aristotle’s system of syllogistic inference. Logical systems have mathematical structure, which leads to new avenues of its own.

Reasoning and computation

The criterion for correctness or incorrectness of simple rules like those I gave for implication and negation is manipulating the ‘truth tables’ of basic logic, and thus reasoning gets tied to mathematical structures in *computation*. This is a general thread in the history of ideas: reasoning is connected to binary arithmetic, or as Hobbes said: “reasoning is reckoning”. And once we see this computational angle in its general light, the idea emerges naturally, already in the Middle Ages, that reasoning might be done by machines, provided we find

suitable logical languages for them to work with. My second picture for Stanley Jevons was his ‘logical piano’, one of the pioneering attempts at creating mechanical computing devices.³

This mixture of reasoning, information, and computation is the thriving conglomerate of logic, philosophy, mathematics, linguistics, and computer science that has put logic firmly at the crossroads of the modern university, and our modern information society. This, indeed, is the world of our Institute for Logic, Language and Computation.

The mathematical stance: logical systems and their theory

As we have seen by now, logical patterns and systems capture major ideas that can be analyzed and developed as such. And the great engine for pursuing that analysis and development has been the *mathematical* turn of modern logic since the 19th century, turning it into an exact discipline that unleashes the power of mathematical precision to study systems of reasoning and everything that comes in their wake. This is how modern logic works, and nothing that I will say in the remainder of this lecture about broadening the scope of the discipline is intended to depart from this methodology.

Paradoxes, self-reflection, and foundational theorems

In particular, the power of mathematical thinking allowed the great logicians of the 1930s to reflect on the basic nature of reasoning itself, both its strengths and its limitations. A celebrated instance of this style of ‘confronting reason with itself’, to use a sonorous philosophical phrase, are Gödel’s Incompleteness Theorems that capture surprising essential features of the mathematical method par excellence, namely, *deductive proof*.

Consider one of the oldest instances of logical thinking that has come down to us in various cultures, from ancient Greece to ancient China, ‘self-referential’ statements that talk about themselves. A famous case is the Liar Paradox from Antiquity where someone asserts that the very statement she is making is false. But for here, think of a related but subtly different statement L that says about itself that it cannot be *proved*:

$$L \text{ if and only if } L \text{ is not provable,}$$

³ Here I am not endorsing the old AI mantra that all reasoning can be done by machines. The ultimate goal of logic to me is not a world where reasoning and judgment are left to technology, but one where we understand ‘agency’, in the broad sense to come in this lecture.

or in Amsterdam-style formula notation:⁴

$$L \leftrightarrow \neg \Box L.$$

Languages have several ways of achieving such self-reference. If you want a dramatic historical instance, the Hebrew God told the people seeking his essence:

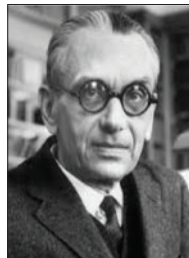
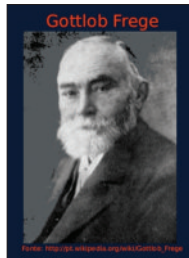
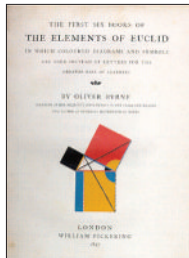
“I am the unknowable one”.⁵

However, our logical intellect does not stop at such a statement, and the liar sentence L admits mathematical reasoning with startling consequences.

Here is a piece of this reasoning at the edge of reason. It is well within your capacities to appreciate – assuming the famous motto of John Perry’s and Ken Taylor’s radio show Philosophy Talk: “the only thing that we do *not* question is the hearers’ intelligence”.

Let us draw some quick consequences in a first, rather simple pass:

Gödel lite. Is L true or false? If we assume that L is false, then L is provable.⁶ But if L is provable, then L is true: and we have contradicted our assumption. We conclude from this episode that L cannot be false. In other words: L is true. But given what L says, it follows that L is a *true statement that is unprovable*.



⁴ The strange box \Box is not a pdf conversion error, but the logical symbol for ‘provability’.

⁵ Yes, there is much more to the ‘unknowability’ in this example than mere formal logic.

⁶ Here we use earlier rules: we have $\neg \Box L \rightarrow L$ by the meaning of L , and hence also $\neg L \rightarrow \Box L$.

Living at the edge of reason

This simple piece of reasoning lies at the heart of Gödel’s famous First Incompleteness Theorem. But it is clearly just a very first pass, which invites questions and refinement at once. Indeed, the art of ‘living dangerously’ displayed here is another major attraction of logic, especially to minds of a somewhat absolutist bent, and it involves a continuous practice of refining our styles of thinking.

To turn the screws of exactness a little bit more, how reasonable, really, was the above step where we assumed that provable statements L are true? Does not that represent an unwarranted confidence in the reliability of our proof methods? The great logicians working in the foundations of mathematics in the 1930s such as Gödel, Hilbert and Gentzen thought so, and dropped this assumption in their analysis of the power of proof. Let me sketch a more sophisticated version for you then:

Assume we have $\Box(L \leftrightarrow \neg \Box L)$ (*). Next, assume that $\Box L$. Then using (*), we can prove that $\Box \neg \Box L$. But at the same time, inspecting the proof for L that exists by $\Box L$, we can prove $\Box \Box L$. Combining the two, a contradiction $\neg \Box L \ \& \ \Box L$ is provable. So, our proof system is inconsistent, and unreliable.

This time our conclusion gets more sophisticated. *If our mathematical proof methods are reliable*, then there have got to be true unprovable statements beyond their reach.

The process of refining our understanding of truth, proof, and reliability is still going on. Masters of the art in Amsterdam included my thesis supervisor Martin Löb, in this Aula we have Sergei Artemov, Dick de Jongh, and Albert Visser – and still last year, I read delightful new perspectives on the mechanics of Gödel’s reasoning discovered by Lev Beklemishev at the Steklov Institute in Moscow. We can keep reflecting on our understanding of mathematical proof, and go deeper and deeper, until the mind reels.

Science, religion, or common sense?

With all this, we are at the heart of classical logic and foundations of mathematics, and its deep results that keep inspiring religious awe in every new generation of students worldwide that gets exposed to logic.

Yet, this depth comes with a somewhat narrow focus on mathematical proof, a very specialized human activity. But I started this lecture in our daily reasoning practice, and want to return there eventually. As a student, I had to memorize a list of differences between

scientific and common sense reasoning – with the former much deeper and better than the latter. But the striking thing to me is the very opposite. The logical steps for Gödel’s Theorem are the same ones we employ in daily life, and this fits the facts of human cognitive evolution, where bands of hunters with survival skills eventually produced the greatest art and science, all using the same brain.

So, here is where this lecture is going to go, charting further aspects of what our discipline is, or can become. Eventually, I want to understand the laws of what actual *reasoning agents* can and should do: in terms of knowledge, action, learning, making mistakes, and all of that in mutual social dependencies. And while setting our bounds that widely, I still hope to convince you that we are doing logic.

But before we enter this enticing Realm of Agency, let me first talk about the intellectual development that took me there. I will take things slowly, introducing new ideas one by one.

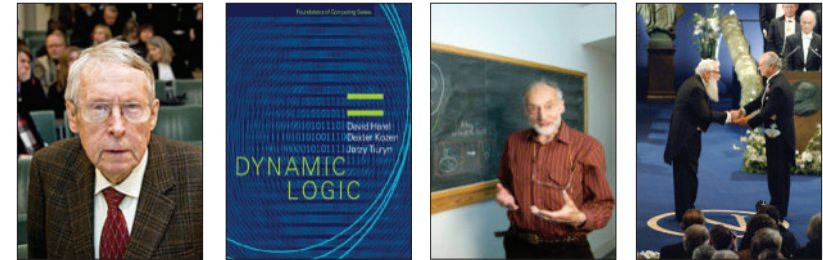
Proof as seeing and as doing

I start with two essential aspects of mathematical proofs that have occurred entangled ever since Euclid’s “Elements” in Antiquity. A proof is *evidence* that a theorem is true: that is, it provides information – but at the same time, it is a method for computing solutions and creating situations where the theorem holds.

The same duality pervades general learning and knowledge, where ‘knowing that’ and ‘knowing how’ are deeply intertwined. For instance, we have truly learnt a language when we recognize when given sentences are correct, but are *also* able to produce discourse that makes sense, and the same is true for logical systems. Thus, studying information also means studying action, and vice versa. The two notions are two sides of the same coin. How can logic do justice to this deep duality?

Reasoning about information and action

Logicians have long studied reasoning about both information and action – though they may come to these pursuits from different academic fields. Logical systems analyzing information often come from the philosophical study of knowledge and belief, but in recent years they have entered computer science and economics. ‘Dynamic logics’ of action come from computer science, in the analysis of programs and processes, but have spread to philosophy and linguistics. These contacts have blossomed over the years, and taken their practitioners to Nobel Prizes, Turing Awards, or their equivalents in philosophy.



Modal logic

The abstract common structure of information and action that underlies large parts of philosophy, computer science, or game theory is studied in a technical field called *modal logic*. A modal notion states how things might be, insofar as not ruled out by our information – or how things might become, given the actions at our disposal. Very similar laws govern such notions. One example is this:

$$\text{Distribution Law} \quad \Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$$

In terms of information, this principle expresses that

if I have evidence for $A \rightarrow B$ as well as evidence for A ,
then I also have evidence for B .⁷

In terms of actions, the same principle says that

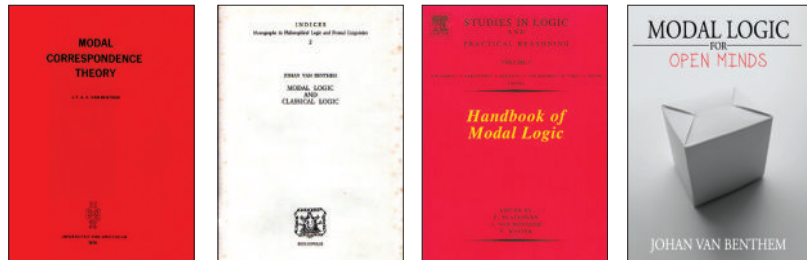
if the outcomes of an action satisfy $A \rightarrow B$, and also satisfy A ,
then all outcomes of that action satisfy B .

There are many other, and technically much more sophisticated, laws of modal logic, but the simple example of the Distribution Law will have to do here.⁸

Modal reasoning patterns have been a focus in my work over the years, which offers a systematic study of modal laws, but also of connections between modal patterns and other

⁷ This use of the box notation is naturally related to our earlier formal analysis of provability.

⁸ In particular, if the actions considered can be different, then Distribution must be adapted.



basic logical systems, measured by semantic invariances such as ‘bisimulation’. These matters can be found in any good textbook, but let me stress one point. While modal logic is ‘applied’ as a theory of information and action, it is at the same time ‘pure’ as a study of basic aspects of reasoning that had not come to light before, and thus it falls squarely within the mathematical methodology that I have emphasized.⁹

Information-driven agency

But let us now move beyond the agenda stated so far. For a start, what about the actors getting the information and engaging in actions? What do they do, and why? My next step toward the study of agency is an arena where knowledge and action meet naturally, the basic *informational acts* themselves.

Basic informational acts

While reasoning may seem the only informational action of interest to logicians, in reality, inference steps usually occur together with other equally fundamental acts that make information flow. For instance, the following scenario plays out day by day in Amsterdam cafés.

The Restaurant Three people order three drinks: water, beer, and wine. A new waiter comes to the table carrying three glasses. There are 6 ways these drinks could be distributed, and here is what you will see happen.

⁹ Modal logic is not the only field that does justice to the information–action duality. Other such paradigms that I admire include process algebra or category-theoretic approaches.

The waiter *asks* who has the beer, say, and after hearing the answer, puts down that glass in the right place. This reduces his uncertainty from the initial 6 options to 2, much as in our example of diagnostic rules. The waiter then asks a second question, and puts, say, the glass of water. Now his options are down to 1, and he does not ask any more, but can *infer* who has the wine, putting it down without a question.

Thus, basic informational acts include, at least, both questions and inferences. But there is more. As it happens, a broad view of informational action was already present with the ancient Chinese logicians in the Moist school, active from around 500 BC. It stressed the interplay of three sources that come together in producing information:

知闻说亲 (*zhi wen shuo qin*)

an elegant compact statement which says roughly, in a more modern phrasing, that “knowledge comes from three sources: hearing from others, proof, or experience”.

Here the third category of ‘experience’ is one more major source of information, in daily life as well as in the natural sciences: viz. just *observing* things.¹⁰ Observation is what a logical detective like Sherlock Holmes needs in addition to pure deduction, and it is the wellspring of the empirical sciences, on a par with mathematical proof.



Logic of information change

My own view of logic has come to resemble this ancient paradigm. We must study all basic informational acts on a par: inference, observation, and communication, and their interplay

¹⁰ The Moists’ example was a Dark Room, with an object inside whose color you do not know. However, you see a white object outside of the room, and someone now tells you that the object in the room has the same color. You then conclude that the object in the room is white.

in how we deliberate, plan, and decide. A major test for such a venture is a search for formal laws governing these additional informational actions that retain the precision logicians are used to. Here is one:

$$[! \varphi] K_i \psi \leftrightarrow (\varphi \rightarrow K_i(\varphi \rightarrow [! \varphi] \psi))$$

This law describes the new knowledge an agent i will possess after having received the information that φ is the case.¹¹ It does so in terms of the ‘conditional knowledge’ that the agent had before the new information $! \varphi$ came in. Typically, it is ‘recursion equations’ like this law that drive the logical study of information-driven agency.

As an aside, please note how the above principles show the cooperation of ideas from different fields: the modality $K_i \psi$ for knowledge in this law comes from philosophical logic, while the dynamic modality $[! \varphi]$ for action comes from logics of computation.

Still, we are only at the start of a logical study of agency. The actors in the above could still be instinctive devices recording information from the environment, like the bees sucking up nectar in the flowers in my garden. So let me turn to further intelligent activities that rational agents engage in, of which I will mention a few in this lecture.

Consistency?

The first new aspect of agency concerns the quest for certainty we saw earlier. The founding fathers of modern logic were concerned with the power of proof, and its reliability. Is *consistency* our ultimate goal? Gottlob Frege thought so, saying that mathematics would ‘crumble like a house of cards’ if a contradiction were to arise.¹² But there can be logical grandeur in fragility, being wrong, and a volte face.

¹¹ These formulas are mainly for display: I ruthlessly omit technical details in this lecture.

¹² In daily life we also value ‘consistency’, a more subtle virtue of applying the same standards to similar matters, without self-serving shifts, a delicate form of harmony in judgment.

Learning is correction

As I said before, proofs give reasons, and a rational agent can be described as someone driven by reasons.¹³ But at this point, I part company with the original foundational enterprise. Our drivers should be *good* reasons – but there is no guarantee that they are correct. We make mistakes and often form incorrect *beliefs*, and the task of logic cannot be to prevent that from happening once and for all. Indeed, our beliefs and expectations, even when wrong, represent a creative human talent in navigating a world where knowledge is a currency that is often unavailable.

But this non-eliminable, and perhaps even desirable, potential for error comes with another creative talent, our ability to *correct ourselves* when beliefs are shown wrong by an observation, or when theories explode after a contradiction comes to light. There is no house of cards that crumbles: indeed, human intelligence shows at its finest in a clever recovery after we have been shown incorrect. So, a true logic of rational agency tolerates errors and highlights methods for revising beliefs. What this points at more generally is an intimate connection between reasoning and *learning*.

While all this may be true, are beliefs and their revision a fit theme for logic? As it happens, thanks to the work of pioneers like Peter Gärdenfors present here today, we know that this more delicate error-tolerant process of adjusting to new information still satisfies logical laws, that now involve operators for belief and acts of belief revision. Of such laws, I just display two at this point, to show you that, once again, we need not sacrifice the mathematical precision of our foundationalist founding fathers:

$$[! \varphi] B_i \psi \leftrightarrow (\varphi \rightarrow B_i^q [! \varphi] \psi)$$

$$[\uparrow \varphi] B^w \chi \leftrightarrow (E(\varphi \wedge [\uparrow \varphi] \psi) \wedge B^{q \wedge [\uparrow \varphi] \psi} [\uparrow \varphi] \chi) \vee (\neg E(\varphi \wedge [\uparrow \varphi] \psi) \wedge B^{[\uparrow \varphi] \psi} [\uparrow \varphi] \chi)$$

Such laws describe which new beliefs arise as new information comes in, either in a ‘hard’ totally reliable form ($! \varphi$) or as a more ‘soft’ plausibility ($\uparrow \varphi$). I am not going to try to explain to you what these formulas say. Their typography may well scare you. These laws of learning are clearly more complex than the simple rules of reasoning that I started with. But then, I never said that our broader logical skills are simple!

¹³ The common idea that rationality means ‘serving one’s own interests’ seems an aberration. Good reasons involve much more, such as an ability to put oneself in *someone else’s* place.

Let me just state the upshot. Correctness forms a tandem with correction – and as some philosophers have said long ago, logic is also about learning from errors.¹⁴

My favorite analogy for this comes again from health. Creating a World free from disease seems a sterile and unappealing goal: much more impressive is the dynamics of the human *immune system* that deals with challenges as they occur. To me, logic is not the guardian of consistency proofs, but rather, the ‘immune system of the mind’.¹⁵

Even so, I should not play up the difference with foundational research too much. There might well be very interesting extensions of the classical theorems by Gödel or Turing when, in addition to what is known with total certainty, we also explicitly represent agent’s reason-based, but fallible beliefs about what is true and provable.¹⁶

Many minds and social interaction

Beliefs may need correction for all three reasons in the earlier picture of information dynamics: a surprising observation, a paradoxical inference, or also, being contradicted by somebody else. This leads me to my next fundamental theme in the logical study of agency, which can be summarized as:

“intelligence seldom comes alone”.

My example of argumentation was a social scenario where different agents interact. This is significant. Rational behavior is a *many-mind talent* rather than an isolated skill – just as in modern science, the key is many-body interactions, not single bodies in their natural place, as in Aristotle’s physics. This social theme of deliberation and argumentation runs throughout the history of logic, in both Greek and Non-Western traditions. Logic is also about how we convince, refute, or otherwise influence others.

Just as earlier, this social dimension has two dual aspects: *information*, and *action*.

¹⁴ This tandem view again illustrates the earlier point of the duality of information and action.

¹⁵ Students ask why I still grade exams for correct answers, instead of their responses to their errors. For lack of a good answer, let me just step aside, and say that exams are an ancient Chinese invention that has conquered the world, on a par with paper, gunpowder, and silk.

¹⁶ E.g., while it might be provably inconsistent to know that a given statement is true and unknowable, it could well be consistent for an agent to believe that it has these two properties.

Theory of mind

The social aspect of information involves a crucial human ability often called ‘Theory of Mind’ where we reason about the facts, but also about other people’s information, what they know about the facts and about us, and so on. Theory of Mind underlies many informational acts. My asking you a *question* normally conveys I do not know the answer, and also that I believe you might know the answer.¹⁷

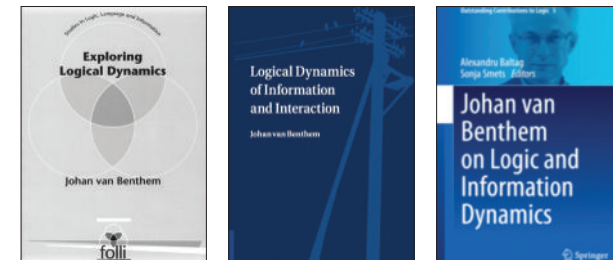
Social behavior is kept in place by a web of such complex iterated information about others that can be studied as before. Again, it has been found that crucial properties of rational social equilibrium then show up in logical laws. For instance, a proposition φ is ‘common knowledge’ in a group of agents G , i.e., it is known up to any depth of iteration (written as $C_G\varphi$), if and only if the following ‘fixed-point law’ holds

$$C_G\varphi \leftrightarrow (\varphi \wedge E_G C_G\varphi)$$

where an operator $E_G\psi$ says that everybody in the group G knows that ψ is the case. This looks like a vicious circle that foolishly tries to define common knowledge $C_G\varphi$ in terms of that notion itself, but in fact, the above equivalence is a ‘luscious circle’ where the notion at issue is well-defined, and supports precise reasoning.¹⁸

The program of logical dynamics

The preceding samples suggest a more systematic logical study of information-driven social agency. Over the past decades it has become clear that the methods of logic are well up to



¹⁷ Of course, both presuppositions are off when a teacher asks a question to a student.

¹⁸ The status of common knowledge is widely debated: it may represent a ‘useful fiction’.

this agenda extension, creating systems with beautiful laws, not just for inferences, but also observations, questions, correction, learning, and social interaction. And in doing so, the grand style of our founding fathers does not lose its luster. Its thinking about strengths and limitations also applies to what diverse sources of information do for rational agents.

In what follows, I discuss one current highlight of this ‘Logical Dynamics’ program.

Games

Social action is at the heart of the fast-developing area of logic and strategic behavior in *games*. Our actions toward each other congregate in longer-term strategies, where what I do next depends on what you do now, and so on. For examples close to logic, just think of the timing of playing your cards in argumentation, or your points in giving a valedictory lecture. The logical laws of strategies or plans form a natural continuation of the earlier-mentioned logics of actions, and they often concern situations where strategies are in a rational *equilibrium*, meaning that no agent has an incentive for deviating from her current plan.

Only one such law will have to serve as an example here, again mostly just in display. It is very similar qua form to the earlier equivalence for common knowledge, and it describes when someone has a strategy for playing a game G that always achieves the effect φ (this is written here as $\{G^*\}\varphi$) in terms of φ 's being true and what the player can achieve by her currently available moves (the latter ability is written as $\{G\}$):

$$\{G^*\}\varphi \leftrightarrow (\varphi \wedge \{G\}\{G^*\}\varphi)$$

The analogy with the earlier law for knowledge again shows the fundamental duality of information and action that governs logic of agency. They are two sides of the same coin, and often, the shape of their mathematical laws reflects their similarities.

Harmony of information and desire

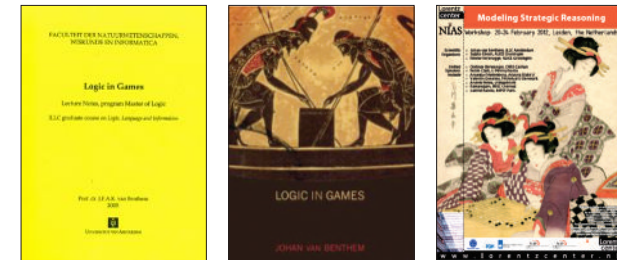
However, in the fundamental notion of game-theoretic equilibrium in behavior between different agents, a new notion comes in sight that you might have thought far outside of the province of logic, namely, our *preferences*, intentions, and *desires*. We are not purely informational agents attuned to how the world is by knowledge or belief: everything we do is colored by what we want, hope, fear, or regret. While some people view the latter as a realm of blind impulses and irrationality, nothing could be further from the truth. What we want to know explains how we seek information, and what we want to achieve determines

our choices of actions. Instead of a separating barrier between these two realms, a logical perspective on rationality involves understanding their delicate balance.^{19,20}

A reasonable person keeps her information in harmony with her goals and desires, plus those of others. That quality shows in the activity of taking *decisions*, a growing interest among logicians. Incidentally, it is that broader balance of information, desires, and decisions that we should also seek to instill in a true academic education.

Logic in games

Noble causes indeed, but can logic offer any illumination in such a broad arena? What I can say for now is found in my new book “Logic in Games”. It covers two major lines of research, that have been developing over the past decades.



The first line is *logic of games*, being the study of the structure of games and everything that happens before, during and after their play by rational agents. This line may be considered a joint offspring of logic, game theory, and modern computer science in its interactive-systems mode. Logic of games is now engendering a joint research program that might called ‘Theory of Play’, where we analyze the reasoning of players in games and related social scenarios – and sometimes, more ambitiously, try to design new games that implement desirable forms of social interaction.

¹⁹ Public audiences often say that emotion rules the world, not logic. My response is finding lipstick on the collar of your spouse: observation, inference, and emotion entangle in a flash.

²⁰ One of my readers wondered if one should also query whether one’s goals are *worthwhile*. I agree that taking nothing for granted is a logical virtue, and another aspect of learning.

But at the same time, there is a second intimate connection here, that may be called *logic as games*. As we have hinted at in the beginning of this lecture, paradigmatic logical phenomena such as argumentation themselves have a game-like character, and this metaphor can be made very precise. For instance, mathematical proofs turn out to be very much like ‘winning strategies’ in suitably defined dialogue games.

Thus, once more, logical laws acquire a dual character we have seen several times. Viewed in an informational perspective, they tell us what the world is like. But in a dual dynamic procedural perspective, they express fundamental facts about strategies and equilibria in basic scenarios of interactive computation or social behavior.

This duality is not an annoying ambiguity, but rather the true face of logic as I see it.²¹

What is next?

Ladies and gentlemen, I have shown you glimpses and glimmers of my dynamic program for logic as a broad study of information-driven social agency. As one of my readers put it nicely: “classical logic handled the straight lines, now you want logic to also do the curves”. Many of those curves are found in my own work and that of my colleagues and students. What is left for *me* to do?

The empirical facts

In my declining years, I sometimes stray from the glorious path of theory, and feel the pull of the brute *facts*. What I told you about information and agency can be seen as theory-bound and normative: this is the information that is truly available in the world, these are the conclusions that agents *should* draw, these are the game-theoretic equilibria they should attain. In this classical perspective, the laws of logical dynamics have no descriptive content, and how people really act, presumably stupid and ill-mannered, can be left to cognitive or behavioral scientists.

But I fear that this common cozy analytic separation of the normative and descriptive reflects intellectual poverty rather than safety. Logical theory that ignores actual cognitive behavior seems dangerously empty, lacking focus. On the other hand, I do not want to lose

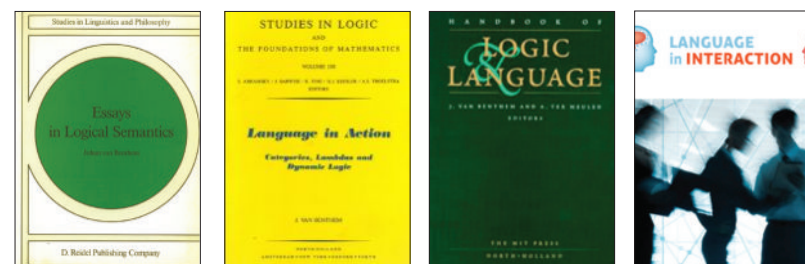
²¹ The third picture in the sequence on the preceding page is of a book on logics for strategic reasoning, bringing together many communities, that I am editing with Sujata Ghosh and Rineke Verbrugge.

the potential of normative thinking either, that can help us *improve* performance, or design better ways of dealing with the world and with one another. In other words, things are complicated, and I have no definitive line to offer.²²

Let me give an example where these two pressures in my current life come to a head.

Natural languages and real brains

In the 1980s, I was deeply engaged in the logical analysis of natural language, and people sometimes ask where this love has gone. My work then was driven by static perspectives on language as reflecting the world, with hardly a trace of dynamic agency. But natural language is a major interface where logical intuitions meet empirical facts. One project on my desk is the *language of agency*, studied in ways that will be familiar to you after my discussion of logic and games. One prong is a study of the patterns of agency that constitute language use, the other prong is the ‘language of agency’, and all the subtle ways that we use for describing what we do, plus the laws of the natural logic governing this natural language.



At the same time, I am involved with empirical research on language in the brain in our national Gravity Project “Language in Interaction” where we will use logic games to see what really happens, and how new language arises when people communicate.

²² This has not prevented me from publishing papers on the interface of logic and psychology, a fascinating border region where much is happening, also at the University of Amsterdam.

Large numbers and probability

The second project waiting on my desk concerns another aspect of the facts, namely, that there are *so many* of them. Logical reasoning and rational agency play in a thin zone of conscious deliberation where what we think and decide matters. This zone is like the thin realm of our physical abilities, hemmed in between objects that are too large, or too small, for our hands to manipulate. Above us is the statistical world of social behavior and public opinion, where each of our well-considered views is just a drop in the ocean. Below us is the statistical neural machinery of our brains, busily clicking away as you are listening to me, but not under your conscious control. Here logic meets *probability theory*, as two full-fledged mathematical paradigms with very different views of what are relevant patterns.²³

The interface of logic and probability has a long history: even Stanley Jevons included it in his book. It is also very much in evidence at our institute the ILLC, and I believe that it poses many fundamental problems that go far beyond what we know so far.

Foundational results after all

Factual interests are by no means opposed to the classical modus operandi in logic. Some neuroscience even intriguingly repeats ideals from the foundations of mathematics, such as a ‘Universal Scanner’ that tells us exactly what an experimental subject is thinking, a goal with a whiff of Inquisition. Fortunately, one can modify famous technical arguments in logic to show that universal scanners are as impossible as universal decision methods for reasoning or computation.

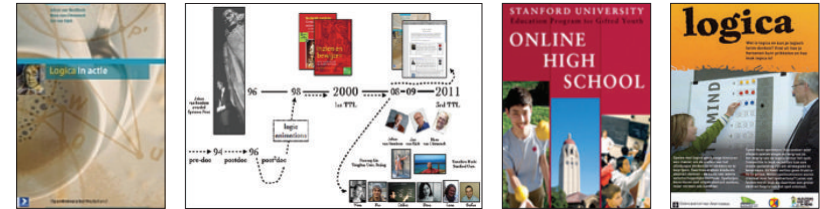
Spreading the word in education

The best test of a proposed scientific perspective may not be whether it pleases one’s senior colleagues, but whether it is *teachable* to audiences with fresh open minds, such as students, or the general public.

I wish I could jump into things here and start teaching you the topics of this lecture, but there are other resources for that, such as our on-line course *Logic in Action*:

www.logicinaction.org

²³ There are many more reasons for combining logic and probability, but these may suffice.



I recommend a tour of this educational world, which we are still creating and testing out at several sites worldwide, including Amsterdam, Stanford, Beijing, and Seville.

This reflects the earlier ties between logic and learning. People can get better at argumentation, and many other themes in this lecture point at skills that can be taught.

Logic and culture

As for further outreach, the view of logic in this lecture does justice to many historical sources for our field that are coming to light in different cultures, as archives yield their secrets. What I told you today is not a Western preoccupation: it stands in a worldwide movement including ancient Greece, China, India, as well as the Islamic tradition, and I am involved in exploring Chinese strands in all of this.



What is logic?

Let me return to the beginning of this lecture. What subject did I choose as a student, and was it the right choice? My general view of taking decisions is quite unlike the paradigm in decision theory of choosing optimal actions given one’s present expectations. The latter perspective is too passive. In my own life, decisions have turned out to be good, not because

of superior a priori reasoning, but because I *made* them good afterwards by committing to the actions that I had taken. This view is also true for choosing logic.

Perhaps the field as it was when I chose it in the 1960s was not an optimal choice for me.²⁴ But I have told you what logic can *become* if we take it to its full potential as a science of information in action, performed by agents engaged in meaningful activities, doing justice to both the classical ideals and a modern future of the field. Indeed, this view is at the heart of my Logical Dynamics program: *becoming* is as important as *being*.

Deep thoughts and noble sentiments: but at the end of this long lecture, you may well prefer a lighter take home message. Let me try. Throughout my presentation, I have extolled the virtues of *reason*, as the light that, sometimes, relieves the boredom and darkness all around us. But that light has to come at the right time, in the right style.

One autumn night, fifteen years ago, I had a heart to heart conversation with my 85-year-old mother – and I put the question I had never dared ask before, why she had ever decided to marry my father. Her response was this:

“because you were not there to advise against it”.

There is a logical paradox lurking here, not unlike the ones I discussed in this lecture – but my mother’s response also gives me my final definition of logic for you today:

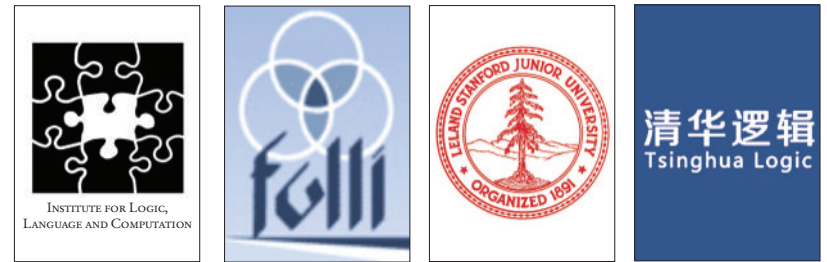
logic is the science of *wit*.

Thanks

In line with what I said about interfaces of logic and probability, my personal views and life’s decisions have ridden on a statistical sea of behavior by many others. Those seas were on the whole sustaining and pleasant. I cannot imagine my own work and views without interactions with many people, many of them present today.

²⁴ My sons occasionally wonder if I would not have made a much more productive member of Dutch society as a physicist, the field of study that I started with at the UvA.

Colleagues have been crucial in shaping my work, in Amsterdam, Groningen, Stanford, and many other places – and so have the many students that I see as my extended family, even when your father may sometimes seem a bit distant and preoccupied. All of you have made me, and I find it hard to say where my own views and contributions end and where those of so many others start. I thank you for that.



My formative interactions included helping shape new organizations and institutions in our field whose lifespan may well exceed our own. I mention the Institute for Logic, Language and Computation in Amsterdam, the European Association for Logic, Language and Information, the Center for the Study of Language and Information at Stanford, and as a most recent initiative, our new Joint Research Center in Logic between the UvA and Tsinghua University in Beijing. Perhaps not accidentally, counting their time zones, they form a realm where the sun never sets. May you all prosper!

Of course, in this Aula, we are at the University of Amsterdam, the matrix where all these scientific ventures were made possible, starting with my thesis defense in 1977. I thank the successive rectores, presidents, deans, and administrative staff of our university that have always supported and appreciated what I tried to do. Many of my fond memories have to do with our common search for the elusive public good.

Finally, there is my family circle. You know what you mean to me. That says it all.

Coda

Having emphasized what went well, like many academic colleagues, I feel compelled to add that ‘all remaining errors are my own’. I have made mistakes in my life, and if you were affected by them, please accept my apologies as well as the thanks.