Altruism and Spite in Games

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ILLC Workshop on Collective Decision Making
Amsterdam, April 11–12, 2013

Motivation

Situations of strategic decision making

Viewpoint: many real-world problems are complex and distributed in nature

- involve several independent decision makers (players)
- decision makers attempt to achieve their own goals (selfish)

Examples: network routing, Internet applications, auctions, ...

Phenomenon: strategic behavior leads to outcomes that are suboptimal for society as a whole

Need: gain fundamental understanding of the effect of strategic decision making in such applications

Algorithmic game theory:

- use game-theoretical foundations to study such situations
- focus on algorithmic and computational issues

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Criticism

1. Self-interest hypothesis: every player makes his choices based on purely selfish motives

   Assumption is at odds with other-regarding preferences observed in practice (altruism, spite, fairness).

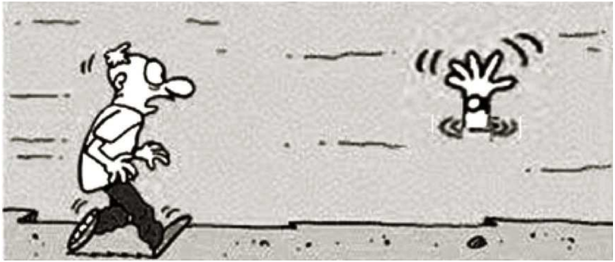
   ⇒ model such alternative behavior and study its impact on the outcomes of games

2. Most studies consider Nash equilibria as solution concept

   Assumption that computationally bounded players can reach such outcomes is questionable!

   ⇒ study inefficiency of more permissive solution concepts (correlated, coarse equilibria) and natural response dynamics

Criticism



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Overview

Motivation

Part I: Altruistic games

- modeling altruistic behavior in games
- inefficiency of equilibria

Part II: Smoothness technique

- smoothness and robust price of anarchy
- adaptations to altruistic games

Part III: Results in a nutshell

- linear congestion games
- fair cost-sharing games
- valid utility games

Concluding remarks

Altruistic Games

Cost minimization games

A **cost minimization game** $G = (N, (S_i)_{i \in N}, (C_i)_{i \in N})$ is a finite strategic game given by

- set of players $N = [n]$
- set of strategies S_i for every player $i \in N$
- cost function $C_i : S_1 \times \cdots \times S_n \to \mathbb{R}$

Every player $i \in N$ chooses his strategy $s_i \in S_i$ so as to minimize his individual cost $C_i(s_1, \ldots, s_n)$

Let $S = S_1 \times \cdots \times S_n$ be the set of strategy profiles.

Social cost of strategy profile $s = (s_1, \ldots, s_n) \in S$ is

$$C(s) = \sum_{i \in N} C_i(s)$$

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Equilibrium concepts

Nash equilibrium: $s = (s_1, \dots, s_n) \in S$ is a pure Nash equilibrium (PNE) if no player has an incentive to unilaterally deviate

$$\forall i \in N: \quad C_i(s_i, s_{-i}) \leq C_i(s'_i, s_{-i}) \qquad \forall s'_i \in S_i$$

(s_{-i} refers to $(s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$)

More general solution concepts:

- mixed Nash equilibrium (MNE)
- correlated equilibrium (CE)
- coarse correlated equilibrium (CCE)

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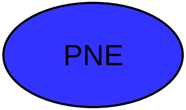

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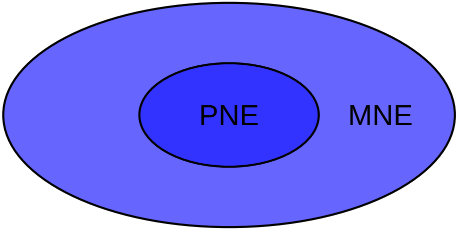

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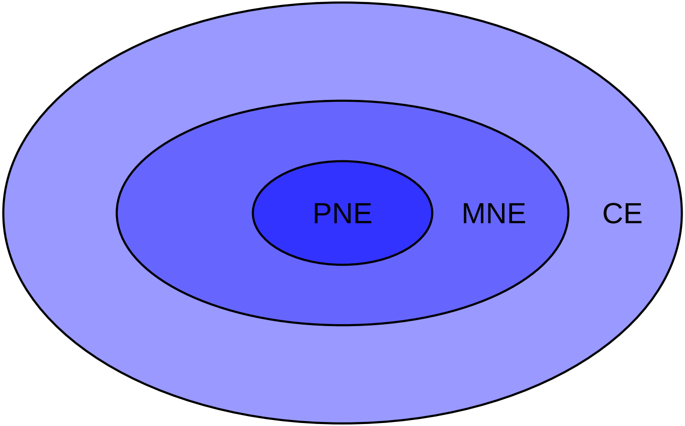

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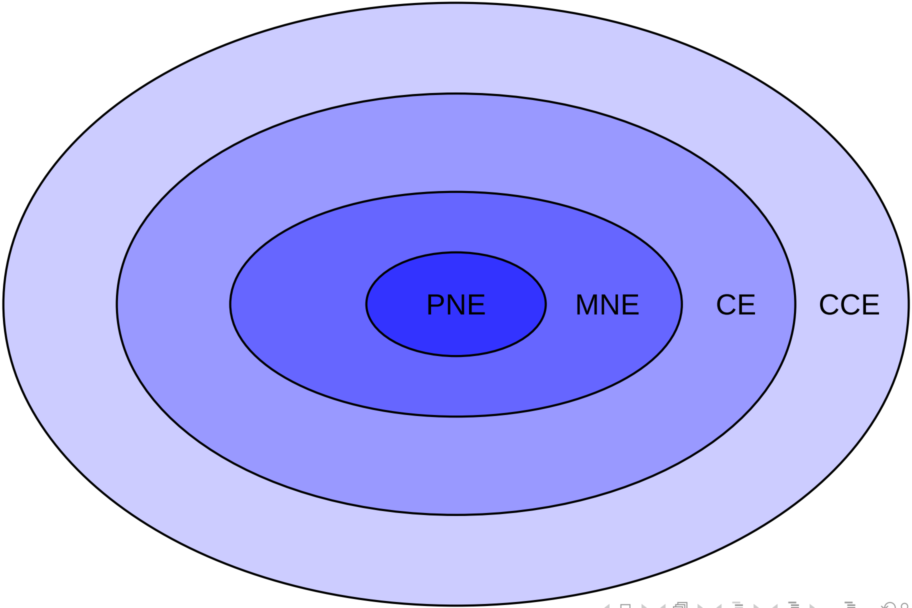
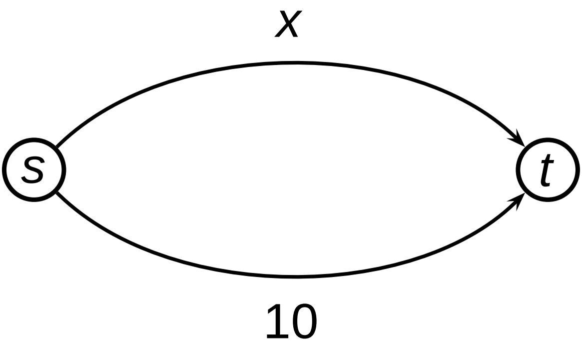


Example: Congestion game

$n = 10$



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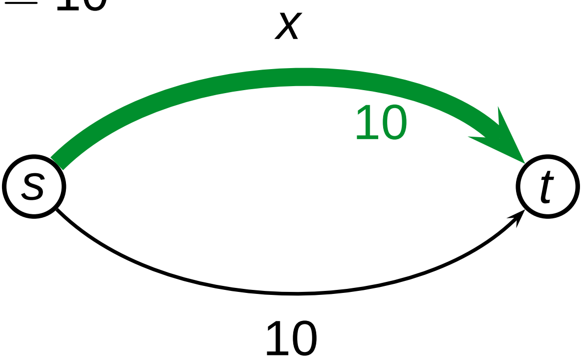

Nash equilibrium: $C(s) = 100$

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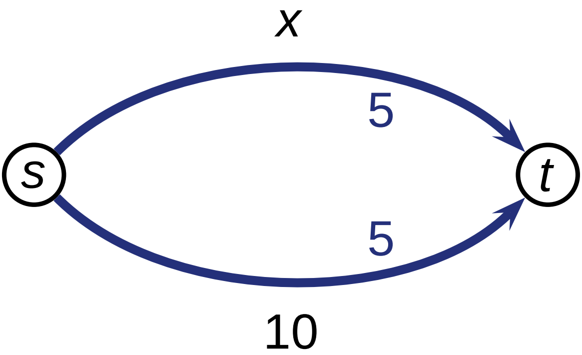

social optimum: $C(s^*) = 75$

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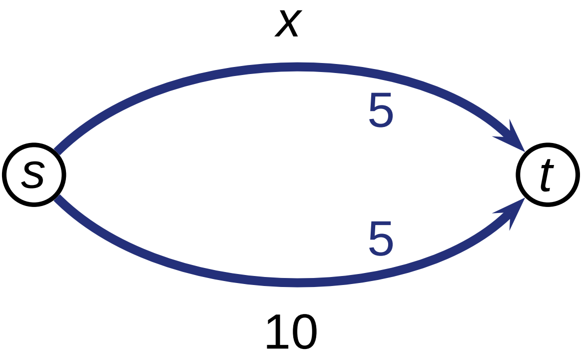

inefficiency: $\frac{C(s)}{C(s^*)} = \frac{100}{75} = \frac{4}{3}$

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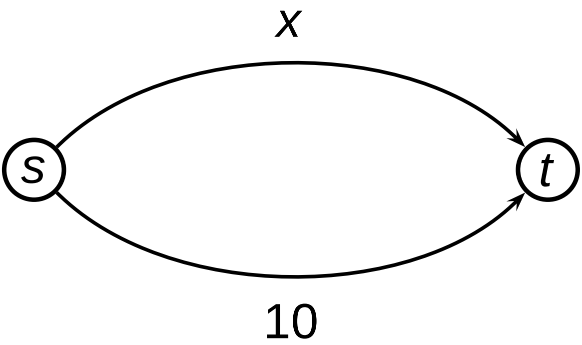

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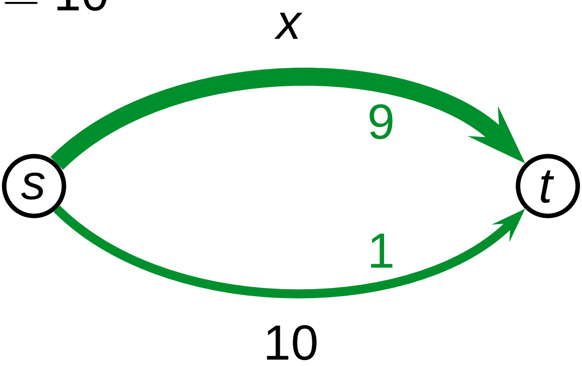

Nash equilibrium: $C(s) = 91$

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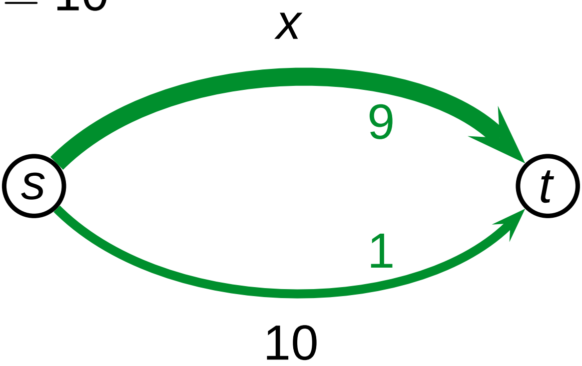

inefficiency: $\frac{C(s)}{C(s^*)} = \frac{91}{75} \approx 1.21$

Inefficiency of equilibria

Let s^* be a strategy profile that minimizes the social cost $C(s)$.

Price of anarchy: worst-case inefficiency of equilibria

$$POA(G) = \max_{s \in PNE(G)} \frac{C(s)}{C(s^*)}$$

[Koutsoupias, Papadimitriou, STACS '99]

Price of stability: best-case inefficiency of equilibria

$$POS(G) = \min_{s \in PNE(G)} \frac{C(s)}{C(s^*)}$$

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Remark: definitions extend to other solution concepts (such as MNE, CE, CCE) in the obvious way

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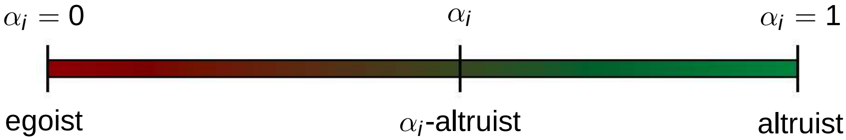

Some remarks

Viewpoint:

- C_i^α is the perceived cost of i (encodes i's altruistic behavior)
- outcome is determined by players minimizing their perceived costs
- C_i is the actual cost that player i contributes to the social cost
  $\Rightarrow$ consider unaltered social cost function

$$C(s) = \sum_{i \in N} C_i(s)$$

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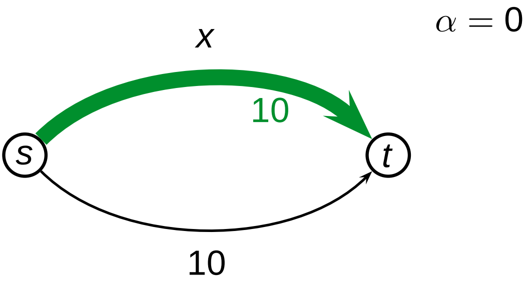
Other models

1. $C_i^{\alpha}(s) = (1-\alpha)C_i(s) + \alpha C(s)$ [Chen et al., WINE '11]
2. $C_i^{\beta}(s) = (1-\beta)C_i(s) + \frac{\beta}{n} C(s)$ [Chen, Kempe, EC '08]
3. $C_i^{\xi}(s) = (1-\xi)C_i(s) + \xi \sum_{j \neq i} C_j(s)$ [Caragiannis et al., TGC '10]
4. $C_i^{\alpha}(s) = C_i(s) + \alpha C(s)$ [Apt, Schäfer '12]
5. $\ldots$

Observation: above models are equivalent for suitable transformations of the altruism parameters

Example: Altruistic congestion game

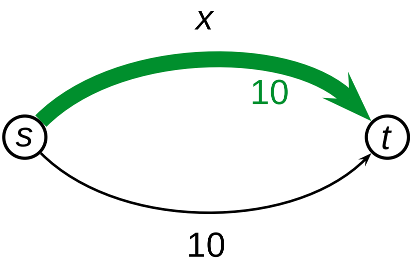


PNE conditions: s is Nash equilibrium of G^α if for every $i \in N$:

$$(1-\alpha)C_i(s_i, s_{-i}) + \alpha C(s_i, s_{-i}) \leq (1-\alpha)C_i(s_i', s_{-i}) + \alpha C(s_i', s_{-i})$$

Example: Altruistic congestion game

$\alpha = 0$



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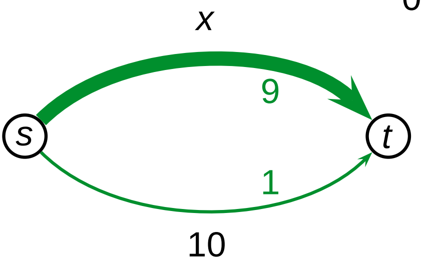
$$\Leftrightarrow (1-\alpha)10 + \alpha(10 \cdot 10) \leq (1-\alpha)10 + \alpha(9 \cdot 9 + 10)$$
$$\Leftrightarrow \alpha \leq 0$$

Example: Altruistic congestion game

$0 < \alpha \leq \cdot$



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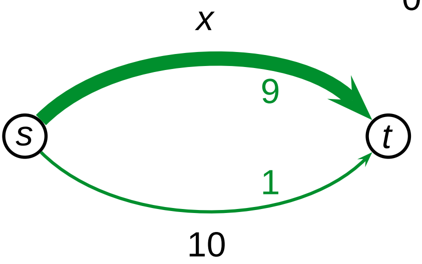

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$$\Leftrightarrow (1-\alpha)9 + \alpha(9\cdot 9 + 10) \leq (1-\alpha)10 + \alpha(8\cdot 8 + 2\cdot 10)$$

$$\Leftrightarrow \alpha \leq 1/8$$

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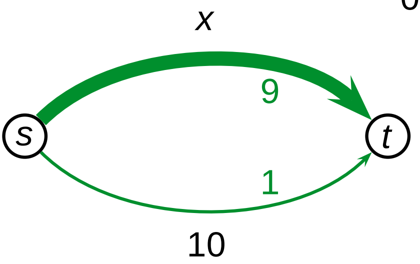

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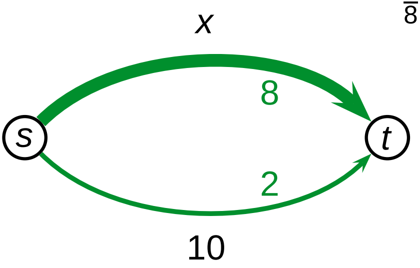

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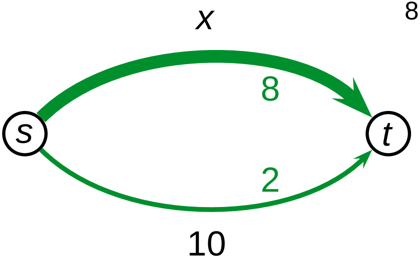

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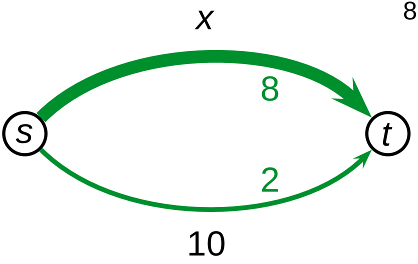

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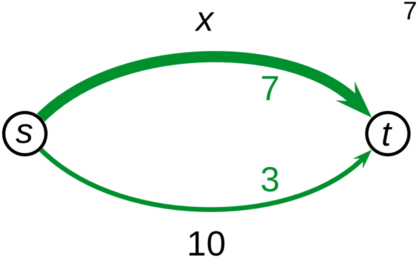
$\frac{2}{7} < \alpha \leq \frac{3}{6}$



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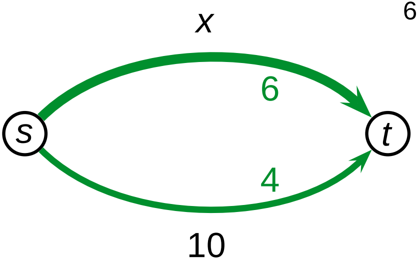
Example: Altruistic congestion game

$\frac{3}{6} < \alpha \leq \frac{4}{5}$



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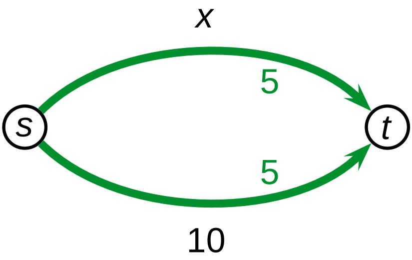

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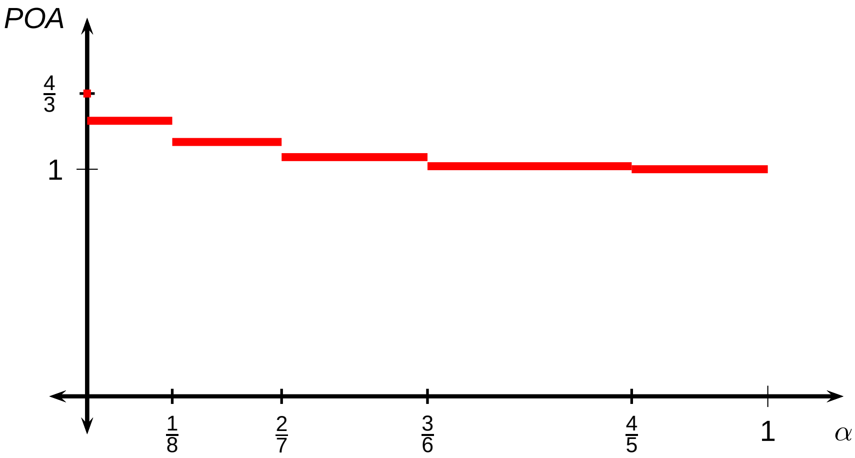
Example: Price of anarchy



Related Work

[Chen and Kempe, EC '08]: altruism and spite in non-atomic network routing games

- uniform altruism: POA $\leq 1/\beta$
- uniform spite/altruism, affine latencies: POA $\leq \frac{4}{3+2\beta+\beta^2}$
- non-uniform altruism, parallel links: POA $\leq 1/\bar{\beta}$

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Smoothness Technique

Smoothness

A strategic game G is (λ, μ)-smooth if for any two strategy profiles $s, s^* \in S$

$$\sum_{i=1}^{n} C_i(s_i^*, s_{-i}) \leq \lambda C(s^*) + \mu C(s).$$

[Roughgarden, STOC '09]

The robust price of anarchy of a game G is defined as

$$RPOA(G) = \inf \left\{ \frac{\lambda}{1-\mu} \; : \; G \text{ is } (\lambda, \mu)\text{-smooth with } \mu < 1 \right\}.$$

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Consequences in a nutshell

Theorem

Let G be a game with robust price of anarchy $RPOA(G)$.

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Glimpse: Pure price of anarchy

Suppose $s = (s_1, \dots, s_n) \in S$ is a pure Nash equilibrium. Fix an optimal strategy profile $s^* = (s_1^*, \dots, s_n^*) \in S$. Then

$$\begin{aligned} C(s) &= \sum_{i \in N} C_i(s_i, s_{-i}) \\ &\leq \sum_{i \in N} C_i(s_i^*, s_{-i}) && \text{(exploiting PNE conditions)} \\ &\leq \lambda C(s^*) + \mu C(s) && \text{(exploiting } (\lambda, \mu)\text{-smoothness)} \end{aligned}$$

By rearranging terms, we obtain

$$\frac{C(s)}{C(s^*)} \leq \frac{\lambda}{1-\mu} \qquad \text{and thus} \qquad POA \leq \frac{\lambda}{1-\mu}.$$

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Glimpse: No-regret sequences

Let $\sigma^1, \ldots, \sigma^T$ be a sequence of probability distributions over outcomes of G in which every player experiences vanishing average external regret, i.e., for every $i \in N$ and $s_i' \in S_i$:

$$\mathbf{E}\left[\sum_{t=1}^{T} C_i(s^t)\right] \le \mathbf{E}\left[\sum_{t=1}^{T} C_i(s_i', s_{-i}^t)\right] + o(T). \qquad (*)$$

$\rightarrow$ no-regret algorithms [Hart and Mas-Colell '00]

Exploiting the smoothness condition and $(*)$, it follows that the average cost of this sequence satisfies

$$\frac{1}{T}\sum_{t=1}^{T} \mathbf{E}\left[C(s^t)\right] \le RPOA(G) \cdot C(s^*) \quad \text{as} \quad T \to \infty.$$

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Adapted smoothness notion

For a given strategy profile $s \in S$, define

$$C_{-i}(s) = \sum_{j \neq i} C_j(s).$$

An altruistic game G^α is (λ, μ, α)-smooth if for any two strategy profiles $s, s^* \in S$

$$\sum_{i=1}^{n} C_i(s_i^*, s_{-i}) + \alpha_i(C_{-i}(s_i^*, s_{-i}) - C_{-i}(s)) \leq \lambda C(s^*) + \mu C(s).$$

Define the robust price of anarchy of an altruistic game G^α as

$$RPOA(G^\alpha) = \inf \left\{ \frac{\lambda}{1-\mu} \;:\; G^\alpha \text{ is } (\lambda, \mu, \alpha)\text{-smooth with } \mu < 1 \right\}.$$

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Implications

Can generalize most of the results of [Roughgarden, STOC '09] to altruistic extensions of games:

Theorem

Suppose the robust price of anarchy of G^α is $RPOA(G^\alpha)$.

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Results in a Nutshell

joint work:

Po-An Chen, Bart de Keijzer and David Kempe

Altruistic congestion games

Results in a nutshell:

1. The robust price of anarchy of α-altruistic linear congestion games is at most
$$\frac{5 + 2\hat{\alpha} + 2\check{\alpha}}{2 - \hat{\alpha} + 2\check{\alpha}},$$
where $\hat{\alpha}$ and $\check{\alpha}$ are the maximum and minimum altruism levels, respectively.

2. This bound specializes to $\frac{5+4\alpha}{2+\alpha}$ for uniformly α-altruistic congestion games and is tight even for pure NE.

[Caragiannis et al., TGC '10]

3. The pure price of stability of uniformly α-altruistic congestion games is at most $\frac{2}{1+\alpha}$.

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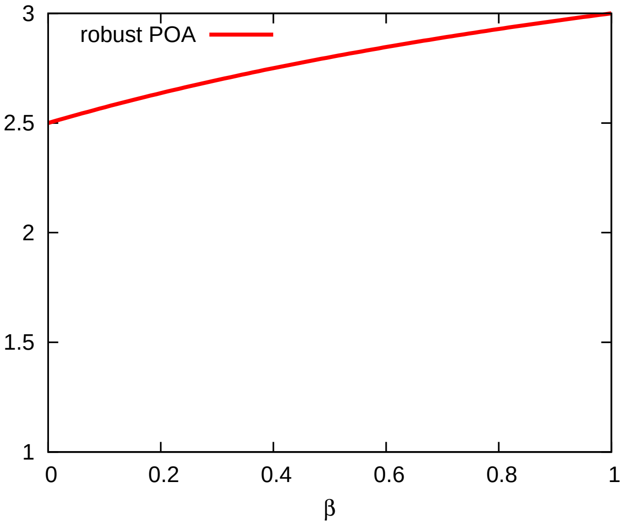
Bounds for uniform players



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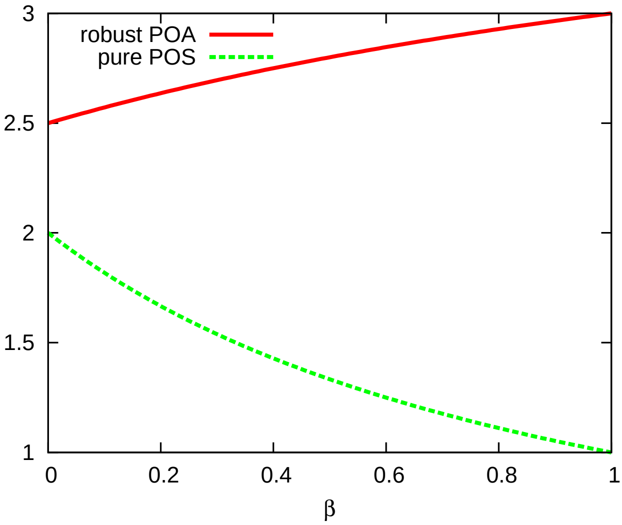

Altruistic singleton congestion games

4. The pure price of anarchy of uniformly α-altruistic extensions of symmetric singleton linear congestion games is $\frac{4}{3+\alpha}$.

   [Caragiannis et al., TGC '10]

5. The mixed price of anarchy of α-altruistic extensions of symmetric singleton linear congestion games is at least 2.

6. The pure price of anarchy of α-altruistic extensions of symmetric singleton linear congestion games with $\alpha \in \{0, 1\}^n$ is at most $\frac{4-2\bar{\alpha}}{3-\bar{\alpha}}$, where $\bar{\alpha}$ is the fraction of purely altruistic players.

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Altruistic cost-sharing games

Fair cost-sharing game: players choose facilities and the cost of each selected facility is evenly shared among the players using it

Results in a nutshell:

1. The robust price of anarchy of α-altruistic cost-sharing games is $\frac{n}{1-\hat{\alpha}}$ (with $n/0 = \infty$).
2. This bound is tight for the pure price of anarchy of uniformly α-altruistic extensions of network cost-sharing games.
3. The pure price of stability of uniformly α-altruistic cost-sharing games is at most $(1-\alpha)H_n + \alpha$.

Altruistic valid utility games

Valid utility games: model “two-sided market games” such as the facility location game

Results in a nutshell:

1. The robust price of anarchy of α-altruistic extensions of valid utility games is 2, independent of the altruism level distribution.
2. This bound is tight for the pure price of anarchy of α-altruistic extensions of valid utility games.

Ongoing Work

Ongoing work: together with Bart de Keijzer

- consider more general altruism models: every player $i \in N$ has a vector of altruism levels $\alpha_i \in \mathbb{R}^n_+$ and

$$C_i^{\alpha}(s) = \sum_{j \in N} \alpha_{ij} C_j(s)$$

(Our case: special case with $\alpha_{ii} = 1$ and $\alpha_{ij} = \alpha_i$ otherwise.)

- combine above idea with social networks, e.g., $\alpha_{ij} = 0$ for all players j that are not neighbors of i in a given social network
- preliminary results:
  - $RPOA \leq 7$ for linear congestion games
  - $RPOA = 4.236$ for singleton linear congestion games
  - $RPOA = \Theta(n)$ for generalized second price auctions

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Concluding remarks

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Summary:

- initiated the study of the impact of altruism in strategic games
- extended smoothness framework to altruistic games
- approach is powerful enough to derive tight bounds on the robust price of anarchy of altruistic extensions of congestion games, cost-sharing games and valid utility games

Conclusions:

- altruistic behavior may lead to an increase of inefficiency
- not a universal phenomenon: price of anarchy may decrease (singleton congestion games) or remain the same (valid utility games)

Thank you!