Trying to please everyone

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Classical ILLC themes: Logic, Language, Computation

Also interesting: Social Choice Theory

In this talk I want to

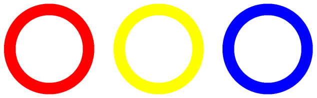
- illustrate what kind of questions are studied in SCT; and
- argue that the "ILLC approach" can make a contribution.

Social Choice?

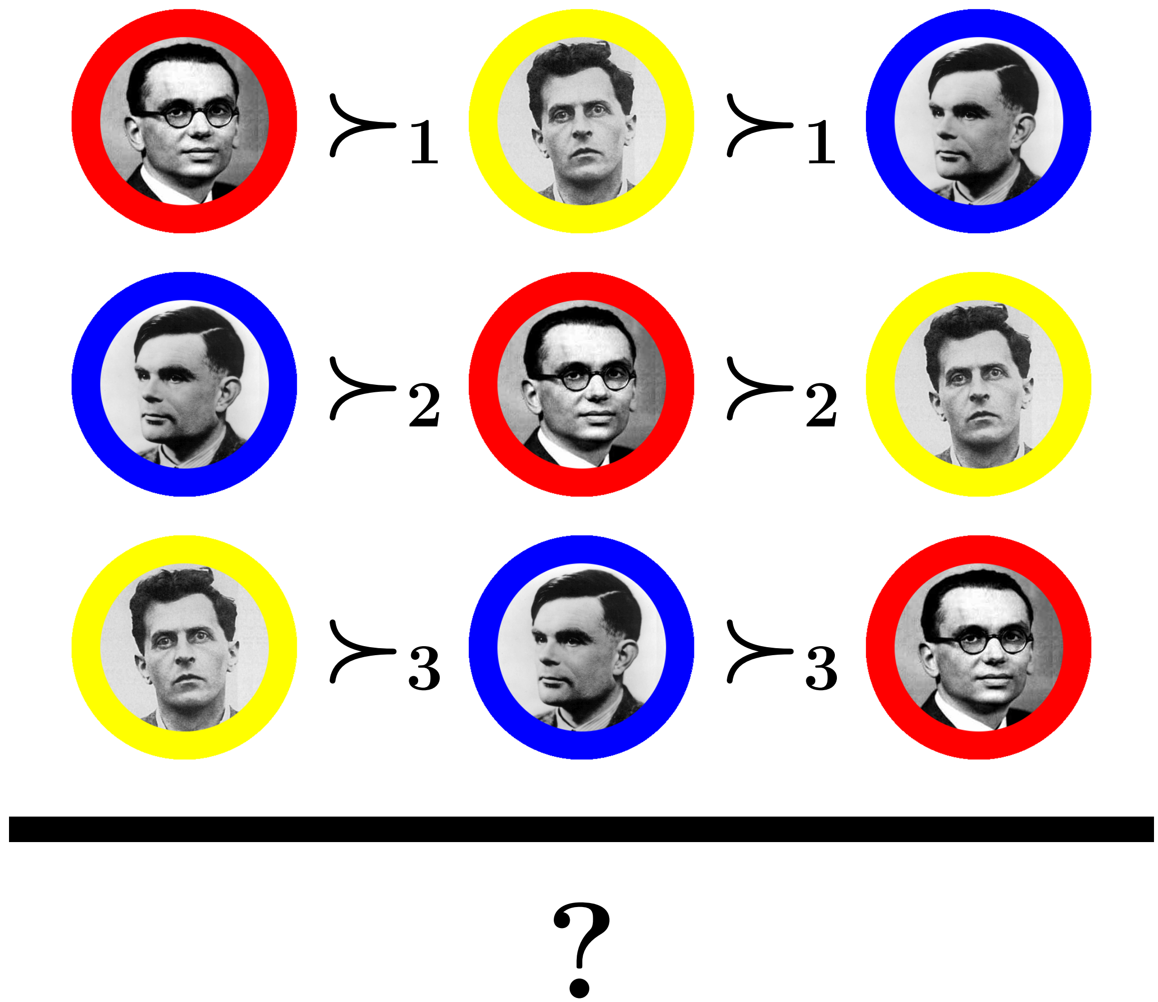
Several candidates or alternatives.

Each of us has their own preferences.

What would be an appropriate "social preference"?



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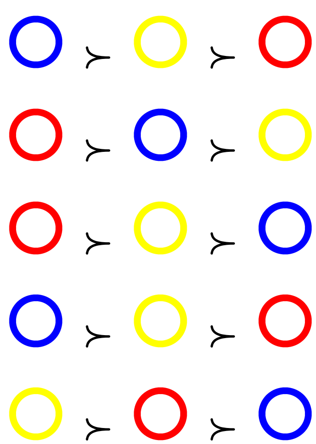
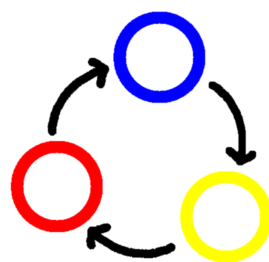
Condorcet Paradox

The fact that using the majority rule to aggregate preferences can lead to a cyclic preference is known as the Condorcet Paradox.





Marie Jean Antoine Nicolas de Caritat (1743–1794), better known as **Marquis de Condorcet**: Mathematician, Philosopher, Political Scientist; Member, *Académie Royale des Sciences*; Inspector General of the *Monnaie de Paris*; opponent of the death penalty (Louis XVI); died under mysterious circumstances while imprisoned.

Arrow's Theorem

In 1951, Kenneth J. Arrow published his famous Impossibility Theorem:

Any preference aggregation mechanism for three or more alternatives that satisfies unanimity and IIA must be dictatorial.

- Unanimity: If everyone says $A \succ B$, then so should society.
- Independence of Irrelevant Alternatives (IIA): If society says $A \succ B$ and someone changes their ranking of C, then society should still say $A \succ B$.

Kenneth J. Arrow (born 1921): American Economist; currently Professor Emeritus of Economics at Stanford University; Recipient of the 1972 Nobel Prize in Economics (youngest ever recipient of this award). His 1951 PhD thesis started modern Social Choice Theory. Google Scholar lists 6340 citations of the thesis.

"*We study here [Arrow's] celebrated theorem [...]*

This theorem is in fact false in general, as a counterexample shows."

— J.H. Blau, 1957

"*Arrow's proof is somewhat opaque, [...]*

his Condition 3 [...] is never even mentioned in the proof."

— A.K. Sen, 1970

"*[...] numerous proofs [...] have been given in the literature.*

Yet logicians, as usual, have found fault with (some of) these proofs."

— T. Nipkow, 2008

- **Can we do something about this?**

Formalising Social Choice

Try to formalise social choice theorems and mechanisms in logic:

- Application of automated reasoning tools: automatically check or even produce proofs of theorems; vary axioms and try to search for niches for possibility results.
- Logic has long been used to formally specify computer systems, facilitating formal or even automatic verification. Maybe the same is possible for social choice mechanisms ("social software").

Various approaches are possible and promising:

- Classical, nonmonotonic, modal/dynamic, higher-order logics, ...
- One such approach is ongoing work with Umberto Grandi.

$$\forall x.\forall y.\forall s.[Alt(x) \wedge Alt(y) \wedge Sit(s) \wedge [\forall i.(Ind(i) \rightarrow Pref(i,x,y,s))] \rightarrow Soc(x,y,s)]$$

Voting

49% think  $\succ$  $\succ$ 

20% think  $\succ$  $\succ$ 

20% think  $\succ$  $\succ$ 

11% think  $\succ$  $\succ$ 

- How should you vote?

Manipulation

There are better voting rules than Plurality/Majority:

- Single Transferable Vote
- Borda's Rule
- Approval Voting

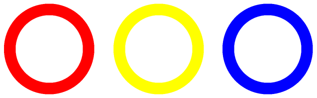
But all of them can be manipulated!

By a classical theorem (Gibbard-Satterthwaite), *every* conceivable voting rule can be manipulated (unless it is dictatorial or random).

"*My scheme is only intended for honest men.*"

— J.-C. de Borda, $\sim$1785

- **Can we do better?**



Reframing the Problem of Social Choice

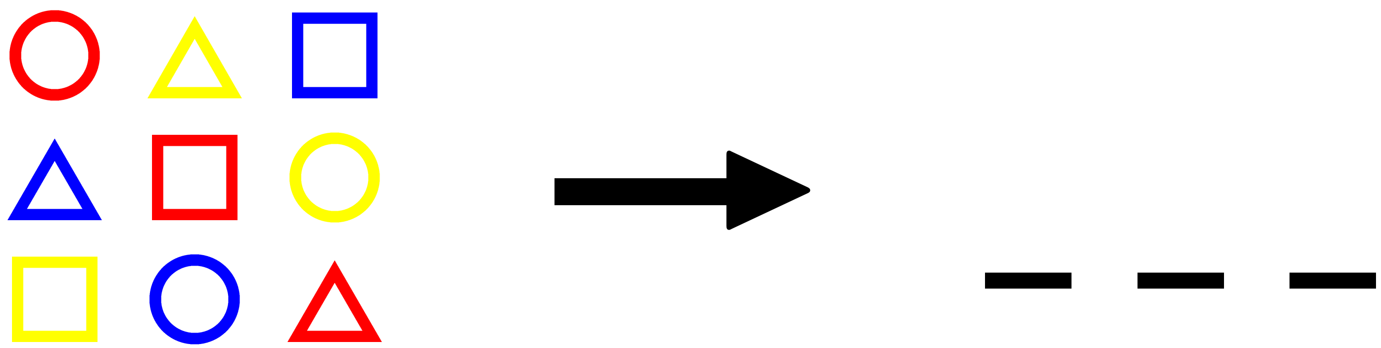
Sometimes, a change of perspective can help:

- So it's always *possible* to manipulate, but maybe it's *difficult!?* We can use complexity theory to make this idea precise.
  - Does *not* work for Plurality: easy to compute my best ballot.
  - Does work for STV: manipulation is known to be NP-complete.
  - Seminal work by Bartholdi *et al.*; now a major research area.
- Switch to a different model, where (true and reported) preferences could be something else than just strict total rankings:
  - Several possibility results: voters can be sincere *and* effective.
  - Ongoing work with colleagues from the University of Padova.

J.J. Bartholdi III, C.A. Tovey, and M.A. Trick. The Computational Difficulty of Manipulating an Election. *Social Choice and Welfare*, 6(3):227–241, 1989.

U. Endriss, M.S. Pini, F. Rossi, and K.B. Venable. *Preference Aggregation over Restricted Ballot Languages: Sincerity and Strategy-Proofness* (in preparation).

Electing a Committee



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Social Choice in Combinatorial Domains

Many social choice problems have a combinatorial structure;

- Elect a committee of k members from amongst n candidates.
- Find the best matching of offices to people in the *Nieuwbouw*.
- Decide on a fair allocation of n indivisible goods to agents.

Seemingly small problems generate huge numbers of alternatives:

- 15 offices and 15 people: $15! = 1307674368000$ matchings
- Allocating 10 goods to 5 agents: $5^{10} = 9765625$ allocations and $2^{10} = 1024$ bundles for each agent to think about

▶ **Can we do something about this?**

Representing Preferences

Simple logic-based languages can be used to model preferences:

$$\big({\color{red}\bigcirc} \text{ OR } {\color{yellow}\bigcirc} \text{ OR } {\color{blue}\bigcirc}, 5\big), \big({\color{red}\bigcirc} \text{ AND } {\color{yellow}\bigcirc}, 12\big), \big({\color{red}\square} \text{ AND } {\color{blue}\square}, 8\big)$$

$\blacktriangleright$ then my utility for $\big\{{\color{red}\bigcirc}, {\color{yellow}\bigcirc}, {\color{blue}\square}\big\}$ will be 17 $\blacktriangleleft$

Relevant research questions:

- Expressivity? / Succinctness? / Complexity?
- How to exploit such representations in voting, auctions, ...

For the full story: *watch out for Joel Uckelman's PhD thesis!*

J. Uckelman, Y. Chevaleyre, U. Endriss, and J. Lang. Representing Utility Functions via Weighted Goals. *Mathematical Logic Quarterly*, 2009 (astmr's).

Trying to please everyone . . .

. . . won't always succeed. Trying to please as many people as much as possible means designing social choice mechanisms. Difficult!

With our ILLC approach, we can make a contribution:

- Full Formalisation of Social Choice Mechanisms
- Reframing Social Choice Problems
- Social Choice in Combinatorial Domains

These are all examples for work in Computational Social Choice.

To find out more: read our expository article in the latest copy of the *AI Magazine* or visit `http://www.illc.uva.nl/~ulle/COMSOC/`.

Y. Chevaleyre, U. Endriss, J. Lang, and N. Maudet. *Preference Handling in Combinatorial Domains: From AI to Social Choice*. AI Magazine, Winter 2008.