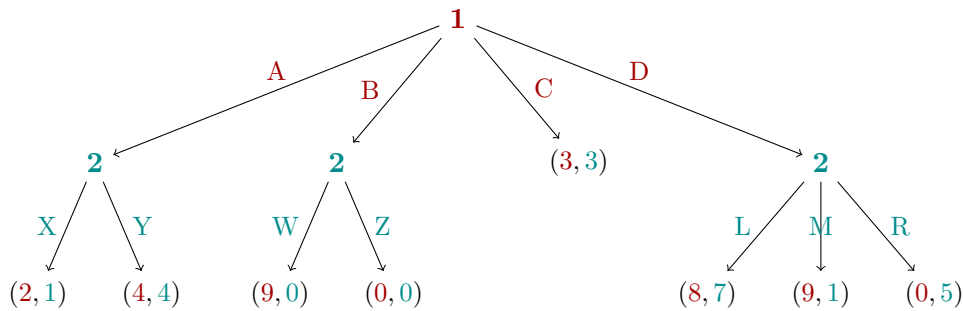# Tutorial #4

Exercise 1

Use backward induction to compute a subgame-perfect equilibrium for this game:

Exercise 2

Recall the simple two-player variant of Poker we played in class, where there are just three cards: King of Hearts, Queen of Hearts, and Jack of Hearts. Suppose we slightly change the rules and a player betting (or calling a bet) now must put down *three* extra dollars (rather than just *one* extra dollar). Thus, the rules of the game now are as follows:

- Each player puts one dollar into the pot.

- Each player draws one of the cards.

- The first player chooses whether she wants to *bet* three extra dollars or *fold*. If she folds, the second player gets the money in the pot and the game ends.

- If the game did not end yet, the second player chooses whether she wants to *call* the other player's bet (and also pay three extra dollars into the pot) or *fold*. If she folds, the first player gets the money in the pot and the game ends.

- If the game still did not end, we reach the showdown: both players reveal their cards and the one with the higher card (King $\succ$ Queen $\succ$ Jack) gets the money in the pot.

Analyse this game by answering the following questions:

(a) How much money can you win in the best case? How much money can you win in the best case once you know that you have drawn the Jack? Do the answers to these questions depend on whether you are the first or the second player?

(b) Intuitively, does one of the players have an advantage (before the cards are dealt)?

(c) We can model the dealing of the cards by means of a third player, *Nature*, who has six actions available to her at the start of the game (one for every possible ordering

of the three cards) and who plays each of them with the same probability. Draw the game tree for this game. Do not forget to indicate indistinguishability between nodes.

You may want to use the following template as a starting point:
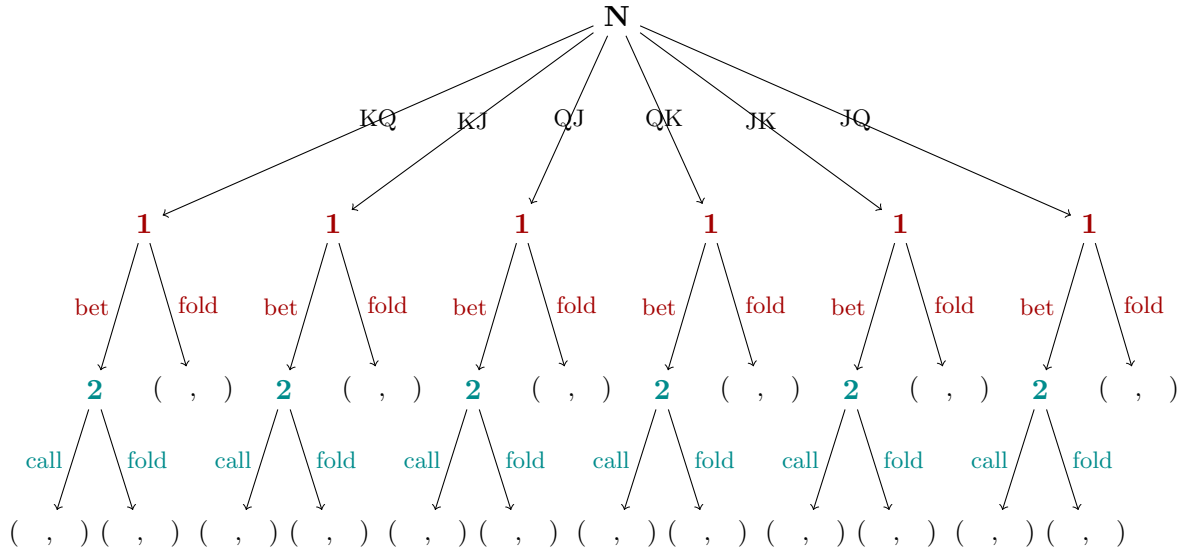
(d) There are three situations you might find yourself in as a player in which it is immediately obvious what you should do: If you draw the King, then you should bet (as the first player) or call (as the second player). If you draw the Jack in the role of the second player (and the game actually reaches the second round), you always should fold. Modify your game tree to reflect these insights.

(e) What should you do if you are the first player and you draw the Queen?

(f) Complete your analysis by considering all possible strategies as far as the remaining choices are concerned. That is, compute a Nash equilibrium for this game of Poker.

(g) What are the expected payoffs for the two players in the Nash equilibrium you obtained? Does this confirm your intuitions about whether one of the two players has an *a priori* advantage in this game?

(h) Discuss why the strategies you found are optimal in the sense that, if you play half of the time as the first player and half of the time as the second player, you will never lose money against any other player—at least not in expectation (of course, if you are very unlucky and always draw the Jack you will still lose money). Finally, does this mean that these strategies are the best choice against every kind of player?