Homework #5

Deadline: Wednesday, 15 May 2024, 13:00

Submit your solutions for (up to) two of the following three exercises. If you solve all three, we will consult a random number generator to decide which two to look at and grade.

Exercise 1 (10 points)

Vickrey auctions are second-price sealed-bid auctions. We have seen the advantages of using second-price rather than first-price auctions in class. Maybe we can get further improvements by introducing a third-price auction?

(a) Describe how such a third-price sealed-bid auction would work.

(b) What would be a good bidding strategy for this type of auction?

(c) Recall that the dominant bidding strategy for private-value Vickrey auctions is to bid your true valuation. Is there a dominant strategy for third-price auctions?

(d) The four basic auction mechanisms discussed in class are Pareto efficient in the following sense: If all bidders know their own valuation and attempt to bid in a way that will maximise their expected utility, then (irrespective of how skilled they are at estimating the bids of their competitors), the winner will always enjoy a nonnegative utility and thus giving the item for the same price to some other bidder would leave the original winner worse off. Is this also the case for the third-price auction?

Exercise 2 (10 points)

Consider the following auction design problem. We want to sell k different goods, called $\alpha_1, \ldots, \alpha_k$, to n bidders (with $k < n$). Everyone agrees that the value of α_1 is exactly twice the value of α_2, that the value of α_2 is exactly twice the value of α_3, and so forth. But the bidders might disagree on the absolute values of the goods. Thus, we can fully describe the valuation of a bidder for all items by means of a single (nonnegative) number, her valuation for α_1. A bid consists of such a number. We are going to allocate α_1 to the highest bidder, α_2 to the second highest bidder, and so forth. Ties are broken in favour of bidders submitting their bids early (and we assume that no two bidders can bid at exactly the same time). Finally, the prices to be paid are determined as follows: For all $\ell \leqslant k$, the bidder receiving item α_ℓ must pay $\frac{1}{2^{\ell-1}}$ of the price corresponding to the next highest bid. For example, the highest bidder obtains item α_1 and pays the second highest bid; the second highest bidder gets α_2 and pays half of the third highest bid; and the third highest bidder gets α_3 and pays one quarter of the fourth highest bid. Is this mechanism incentive-compatible? Either prove that it is or provide a clear counterexample.

Hint: The issue of interest here has nothing to do with tie-breaking, so in your answer, if you find it helpful, you may assume that all bidders have mutually distinct valuations and also that all bidders always report mutually distinct valuations.

Exercise 3 (10 points)

Write a program to compute the outcome (allocation and prices) for a combinatorial auction with single-minded bidders under the VCG mechanism. Report on the performance of your algorithm for randomly generated auction instances of varying size.

Keep in mind that implementing such a combinatorial auction solver involves solving several NP-hard optimisation problems for each auction instance (one to compute the allocation and a further n of them to compute the prices), so a naïve algorithm is unlikely to work well in all cases for somewhat larger problem instances. Your report should include a discussion of the limitations of your algorithm.