Homework #1

Deadline: Monday, 6 November 2023, 19:00

Exercise 1 (10 points)

Provide a one-paragraph definition in plain English of the voting rule you presented in class, as well as one sentence each on one positive and one negative property of that rule. If you did not present, instead pick a paper published in 2023 on any topic in computational social choice you like and submit a one-page summary written in your own words. This exercise will be graded with a 10 for any reasonable attempt and a 1 in all other cases.

Exercise 2 (10 points)

For the sake of simplicity, for your solution restrict yourself to the case of $m = 3$ alternatives.

In analogy to the definition of a Condorcet winner, a *Condorcet loser* is an alternative that would lose against any other alternative in a pairwise majority contest. It seems uncontroversial to say that a Condorcet loser, when it exists, is a particularly unattractive alternative. So we should hope that the voting rule we use will not elect it.

(a) Give an example that shows that the plurality rule *can* elect a Condorcet loser.

(b) Prove that the Borda rule *never* elects a Condorcet loser.

(c) Is the Borda rule the only positional scoring rule that never elects a Condorcet loser? Provide either a proof of the uniqueness of Borda or an example for another such rule.

Hint: For part (c), do not confuse different representations of a rule with different rules. For instance, the positional scoring rule induced by the scoring vector $(7, 5, 3)$ is the same rule as the positional scoring rule induced by $(2, 1, 0)$. Both are representations of the Borda rule.

Exercise 3 (10 points)

Recall that the divide-and-conquer cake-cutting protocol guarantees each of the n agents a proportional share of the cake (i.e., a share of at least $\frac{1}{n}$ according to her own valuation), while requiring a total of $O(n \log n)$ cut-queries (and no evaluation-queries). Given that we demand that a cut may only be made in a location returned as the answer to a cut-query, the smallest number of cut-queries any protocol requires is $n - 1$ (otherwise we would end up with fewer than n pieces of cake). The purpose of this exercise is to see that we can achieve a reasonable approximation of the ideal of proportional fairness by means of a protocol that requires only $n - 1$ cut-queries. Answer the following questions:

(a) Design a protocol for four agents that, using three cut-queries (and an arbitrary but bounded number of evaluation queries), guarantees each agent a share of at least $\frac{1}{6}$.

(b) Design a protocol for $n \geqslant 2$ agents that, using $n-1$ cut-queries (and an arbitrary but bounded number of evaluation queries), guarantees each agent a share of at least $\frac{1}{2(n-1)}$.

Make sure it is clear from your presentation that your protocols really use no more than the allowed number of cut-queries and that they really provide the required fairness guarantees. Present your solutions to the two parts of the exercise in the order indicated.