Computational Social Choice: Spring 2017

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Plan for Today

So far we have only studied voting rules for elections with *one winner* (ties were considered a nuisance, not a desideratum).

Today we are going to discuss *multiwinner voting rules* designed to elect at set of k winners (so tie-breaking is still an issue):

- examples for multiwinner voting rules and design principles
- examples for properties satisfied and violated

In the *Handbook*, the chapter on voting in combinatorial domains covers the basics, but the topic is currently fast developing.

And: quick overview of all the *voting theory* covered in this course.

J. Lang and L. Xia. Voting in Combinatorial Domains. In F. Brandt et al. (eds.), *Handbook of Computational Social Choice*. Cambridge University Press, 2016.

Applications

All of these can be modelled as multiwinner elections:

- A hiring committee has to shortlist k out of m job candidates to invite to interviews (after which one of them will get an offer).
- An airline has to select k out of a pool of m movies for its inflight entertainment system, based on the preferences of passengers.
- In a national election, k out of m candidates running need to be chosen to form the new parliament, based on voter preferences.

Exercise: *What are good rules? What properties should they satisfy?*

Multiwinner Voting as Combinatorial Voting

Need to elect k committee members from a pool X of m candidates.

Could treat this as a problem of voting in combinatorial domains:

$$\textit{alternatives} = \textit{committee compositions}$$

Possible approaches:

- Ask voters to explicitly rank all $\binom{m}{k}$ committee compositions and apply a standard voting rule. *Not feasible in practice.*
- Use compact representation language to express preferences over committee compositions. *Nice idea. Not much done to date.*

Today we want to instead explore what we can do if voters submit *standard ballots* (linear orders on the m candidates):

$$F : \mathcal{L}(X)^n \to \{C \subseteq X \mid \#C = k\}$$

Approaches Based on Sequential Elimination

Need to elect k committee members from a pool X of m candidates, based on the ranked preferences reported by n voters.

Earlier we discussed *STV* as a single-winner voting rule, but in fact it is mostly used for multiwinner elections:

- If some candidate $x^\star$ is ranked first at least $q = \lfloor \frac{n}{k+1} \rfloor + 1$ times, then elect $x^\star$ and eliminate both $x^\star$ and q voters ranking $x^\star$ first.

  Otherwise, eliminate a plurality loser from the profile.

  Repeat until all k seats are filled.

Involves tie-breaking at all levels ($\hookrightarrow$ *parallel-universe tie-breaking*).

Could also use rules other than plurality to eliminate weak candidates.

From Simple Voting to Multiwinner Voting

Need to elect k committee members from a pool X of m candidates, using a standard voting rule F. Three approaches come to mind:

- *Rank-and-cut:* If F really is a social welfare function returning a ranking (e.g., Kemeny or Slater), elect its k top elements.
- *Score-and-cut:* If F comes with a notion of score for an alternative (e.g., Borda or Copeland), elect the k top-scoring alternatives.
- *Choose-and-repeat:* If F is resolute, elect the winner $x^\star$ under F, and repeat with $X := X \setminus \{x^\star\}$ until all seats are filled.

  Alternatively: If F is irresolute, in each round, choose *all* winners.

Of course, there are tie-breaking issues for all of these.

Exercise: *What do you think? Are these approaches any good?*

Example

Suppose we want to use the *plurality rule* to elect $k = 2$ winners:

3 voters:	$A \succ C \succ B$
2 voters:	$B \succ C \succ A$
1 voter:	$C \succ B \succ A$

We might proceed as follows:

- *Score-and-cut:* A gets 3, B gets 2, C gets 1. So $\{A, B\}$ wins.
- *Choose-and-repeat:* A wins first round, then C. So $\{A, C\}$ wins.

Thus: these really are very different rules!

Some Common Rules

Simple extensions to multiwinner voting rules are common in practice:

- Choose-and-repeat + plurality is known as *sequential plurality*. For $k = 3$ used to elect English bishops.
- Score-and-cut + plurality is known as *single nontransferable vote*. For $k = 3$ used to elect rectors of public universities in Brazil.
- Score-and-cut + k-approval PSR (where k is the committee size) is known as *bloc voting*. For $k = 3$ used to elect Irish bishops.

S. Barberà and D. Coelho. How to Choose a Non-controversial List with k Names. *Social Choice and Welfare*, 31(1):79–96, 2008.

E. Elkind, P. Faliszewski, P. Skowron, and A. Slinko. Properties of Multiwinner Voting Rules. *Social Choice and Welfare*, 48(3):599–632, 2017.

The Increasing-Committee-Size Paradox

Suppose we use *bloc voting* (score-and-cut + k-approval):

1 voter:	$A \succ B \succ C \succ D$
1 voter:	$B \succ A \succ D \succ C$
1 voter:	$A \succ C \succ D \succ B$
2 voters:	$B \succ D \succ C \succ A$

For committee size $k = 2$, candidates A and B are the winners.

But if we increase the committee size to $k = 3$, then A will lose!

The above is a simplified variant of a paradox due to Staring (1986), who presents a profile where the two committees even are disjoint.

Exercise: *Show that no choose-and-repeat rule has this problem.*

M. Staring. Two Paradoxes of Committee Elections. *Mathematics Magazine*, 59(3):158–159, 1986.

Condorcet Committees

Not clear or noncontroversial how to extend the Condorcet Principle to multiwinner elections. One proposal is due to Gehrlein (1985):

- Committee $C \subseteq X$ is a weak *Condorcet committee* under profile $\boldsymbol{R}$ if $|N^{\boldsymbol{R}}_{c \succ x}| \geqslant |N^{\boldsymbol{R}}_{x \succ c}|$ for all $c \in C$ and $x \in X \setminus C$.

So a committee that is *not* a weak Condorcet committee is *unstable:* a majority of voters would want to replace one of its members.

Call a multiwinner voting rule *stable* if it elects a weak Condorcet committee whenever such a committee exists.

Remark: Of course, weak Condorcet committees need not exist.

W.V. Gehrlein. The Condorcet Criterion and Committee Selection. *Mathematical Social Sciences*, 10(3):199–209, 1985.

Unstable and Stable Rules

Proposition 1 (Barberà and Coelho, 2008) *No PSR combined with either score-and-cut or choose-and-repeat is stable.*

Proof: Omitted. Similar to the proof (by inspection of a problematic profile) we have seen for all PSR's failing the Condorcet Principle.

Proposition 2 (Barberà and Coelho, 2008) *The Kemeny rule combined with rank-and-cut is stable.*

Proof: Omitted, but easy and unsurprising.

Proposition 3 (Barberà and Coelho, 2008) *Every stable multiwinner voting rule is subject to the increasing-committee-size paradox.*

Proof: Omitted (example where Condorcet com's for $k = 2, 3$ are disjoint).

S. Barberà and D. Coelho. How to Choose a Non-controversial List with k Names. *Social Choice and Welfare*, 31(1):79–96, 2008.

Axiomatic Analysis

Many more axioms besides *stability* (electing Condorcet committees) and *committee monotonicity* (avoiding paradox) have been considered.

What axioms to pick depends the features of your application.

For an in-depth discussion, refer to Elkind et al. (2017).

E. Elkind, P. Faliszewski, P. Skowron, and A. Slinko. Properties of Multiwinner Voting Rules. *Social Choice and Welfare*, 48(3):599–632, 2017.

Complex Rules: Minimising Misrepresentation

Suppose that, given profile $\boldsymbol{R}$, we have elected committee C of size k. Use a function $r : N \to C$ to assign each voter to "her" *representative*. The *misrepresentation* of i is how many candidates she prefers to $r(i)$.

The *Chamberlin-Courant rule* chooses a committee C that minimises total misrepresentation if voters pick their favourite representatives:

$$\text{elect } C \subseteq X \text{ minimising } \min_{r:N \to C} \sum_{i \in N} \#\{x \in X \mid x \succ_i r(i)\}$$

The *Monroe rule* does the same, but requires each committee member to represent the same number of voters (± 1): $\lfloor \frac{n}{k} \rfloor \leqslant |r^{-1}(x)| \leqslant \lceil \frac{n}{k} \rceil$

Procaccia et al. (2008) showed that, for both rules, deciding whether a given bound on misrepresentation can be respected is *NP-complete*.

A.D. Procaccia, J.S. Rosenschein, and A. Zohar. On the Complexity of Achieving Proportional Representation. *Social Choice and Welfare*, 30(3):353–362, 2008.

Voting for Parties

So far we have considered scenarios with m distinguishable candidates.

Suppose we instead vote for *political parties* (with plurality ballots).
Suppose there are 100 voters and 10 seats.

Party A:	47 votes
Party B:	27 votes
Party C:	26 votes

How many seats should each party get?

This question of *proportional representation* is similar to the problem of *apportionment:* in a federal system, how many seats in the house of representatives should go to each state, given its population?

M.L. Balinski and H.P. Young. *Fair Representation: Meeting the Ideal of One Man, One Vote.* 2nd edition, Bookings Institution Press, 2001.

Hamilton's Method

In the context of assigning seats in the US Congress to states, Alexander Hamilton proposed the following method in 1792:

- Compute the *quota* for each party i:

$$q_i \ := \ \frac{\#\text{votes for } i}{\#\text{votes in total}} \times \#\text{seats}$$

- To each party i, award (for now) $\lfloor q_i \rfloor$ seats.
- Award remaining seats to those parties with the largest $q_i - \lfloor q_i \rfloor$.

Remark: The last step may require tie-breaking.

The Alabama Paradox

Suppose there are 250 voters. Consider the outcome under Hamilton's Method when there are 25 seats *vs.* when there are 26 seats:

	#votes	$\frac{25 \cdot \#\text{votes}}{250}$	#seats	$\frac{26 \cdot \#\text{votes}}{250}$	#seats
Party A	24	2.400	3	2.496	2
Party B	113	11.300	11	11.752	12
Party C	113	11.300	11	11.752	12

That is, even though the total number of seats increases, the number of seats for Party A decreases.

In the context of apportionment, this paradox was observed in 1880 in the US when Congress had to fix the number of representatives based on the latest census data: Alabama would get 8 representatives out of 299 but only 7 out of 300.

Jefferson's Method

Let s be the number of seats to be allocated. Let p be the number of parties and let n_i be the number of votes received by party $i \leqslant p$.

Also in 1792, Thomas Jefferson proposed this method:

- Fix a divisor d such that

$$\lfloor n_1/d \rfloor + \lfloor n_2/d \rfloor + \cdots + \lfloor n_p/d \rfloor \quad = \quad s$$

- Award $\lfloor n_i/d \rfloor$ seats to party i.

Observation 1: The number of seats assigned to each party increases monotonically with the number of total seats, so Jefferson's Method does not suffer from the Alabama Paradox.

Observation 2: Jefferson's Method tends to favour larger parties.

Webster's Method

Let $\text{round}(x) := \lfloor x + 0.5 \rfloor$.

In 1832, Daniel Webster proposed this variant of Jefferson's Method:

- Fix a divisor d such that

$$\text{round}(n_1/d) + \text{round}(n_2/d) + \cdots + \text{round}(n_p/d) \quad = \quad s$$

- Award $\text{round}(n_i/d)$ seats to party i.

Summary

This has been an introduction to multiwinner voting rules.

Focus has been on designing rules for profiles of ranked preferences over individual candidates by *adapting single-winner voting rules*.

Remark: Much recent work instead deals with *approval ballots*.

We also briefly discussed fair representation when *voting for parties*.

What now? Brief reflection on voting theory in COMSOC as a whole.

What next? Two lectures on COMSOC topics other than voting.

Course Review: Voting Theory

Lots of voting rules (in particular: PSR's + Condorcet extensions).

Classical social choice theory (mostly axiomatic method):

- characterisation results (May, Young)
- impossibility theorems (Arrow, Sen, M-S, G-S, D-Sch)
- domain restrictions (in particular: single-peakedness)

Also: voting as truth-tracking + voting as distance minimisation

Nonstandard voting scenarios:

- multiwinner elections + voting in combinatorial domains
- iterative voting

Computational (or just: nonclassical) perspective on voting:

- complexity of winner determination
- computational and informational barriers against strategic manipulation
- possible winner problem + compilation complexity
- compact representation of voter preferences
- automated reasoning for verification and discovery