

Homework #3

Deadline: Wednesday, 25 February 2015, 11:00

Question 1 (10 marks)

In class we have seen that the winner determination problem for the premise-based procedure for formula-based judgment aggregation is polynomial in case the set of premises is exactly the set of literals and the agenda is closed under propositional variables. That is, for a given profile \mathbf{J} and a given formula φ of the agenda, we can decide in polynomial time whether the premise-based procedure would accept φ under \mathbf{J} . This is so independently of whether φ is a premise or a conclusion. Recall that proving this required two separate arguments, one for premises and one for conclusions.

Analyse the computational complexity of the same winner determination problem when we drop all assumptions on the agenda and how it is divided into premises and conclusions (except that each of these two sets must still be closed under complementation).

Question 2 (10 marks)

Establish an upper bound for the complexity of the problem of the safety of the agenda for the majority rule under the restriction that all positive formulas in the agenda are 2-clauses. Your bound should be better than the one given in class for the case of agendas without any such syntactic restrictions.

Question 3 (10 marks)

For this question, restrict attention to judgment aggregation scenarios with an odd number of agents and an agenda without tautologies.

In class we have proved that an agenda Φ is safe for the family of aggregation rules that are anonymous, neutral, independent, complement-free, and complete if and only if Φ has the simplified median property. Clearly, the left-to-right direction must hold also for every larger family of rules, including those families we obtain when we drop some of these axioms. For the right-to-left direction, inspection of the proof reveals that we have in fact only invoked two of the axioms (neutrality and complement-freeness), so we could have dropped all the others for this direction as well. Thus, we know that an agenda Φ is safe for the (much larger) family of aggregation rules that are neutral and complement-free if and only if Φ has the simplified median property. (Of course, the version proved in class is the more interesting result.) The purpose of this question is to investigate what happens when we lower our requirements even further, from two axioms to just a single one:

- (a) Characterise the class of agendas that is safe for all complement-free aggregation rules.
- (b) Characterise the class of agendas that is safe for all neutral aggregation rules.