Homework #3

Deadline: Monday, 24 September 2012, 13:00

Question 1 (10 marks)

Under the *antiplurality rule*, also known as the *veto rule*, the voters rank the alternatives, and the alternative(s) ranked last the least often win(s). The purpose of this question is to find a number of different characterisations of this rule.

(a) Find a consensus criterion such that the antiplurality rule is characterised by that criterion and the *discrete distance*.

(b) Find a way of measuring distances such that the antiplurality rule is characterised by the *unanimous winner* consensus criterion and that distance.

(c) Find a noise model such that the corresponding maximum likelihood estimator is equivalent to the antiplurality rule.

Question 2 (10 marks)

Recall the Copeland rule: each voter ranks all alternatives, and the alternative(s) that maximise the difference between won and lost majority contests, when compared to all other alternatives, win(s). Prove that the Copeland rule is easy to manipulate. This is in fact a corollary to a more general result by Bartholdi, Tovey and Trick (1989). Do not build on their general result, but rather give a direct proof for the Copeland rule only.

(See J.J. Bartholdi III, C.A. Tovey, and M.A. Trick. The Computational Difficulty of Manipulating an Election. *Social Choice and Welfare*, 6(3):227–241, 1989.)

(Please turn over)

Question 3 (10 marks)

An important line of research in social choice theory is aimed at understanding the *frequency* with which certain undesirable situations, e.g., Condorcet cycles or opportunities for strategic manipulation, occur. While classical paradoxes and impossibility theorems show that these situations can never be ruled out entirely, it is conceivable that they might be very infrequent, in which case the situation would not actually be as bleak as the classical results suggest. To measure frequency we have to make assumptions regarding the likelihood of certain profiles of preferences to occur. The standard approach is to assume that every logically possible profile is equally likely to occur. This is known as the *impartial culture* (IC) assumption. Under the closely related *impartial anonymous culture* (IAC) assumption, each *anonymous* profile is taken to be equally likely to occur. For example, if there are two alternatives and two voters, then under the IC assumption each of the four possible profiles $(x \succ y, x \succ y)$, $(x \succ y, y \succ x)$, $(y \succ x, x \succ y)$ and $(y \succ x, y \succ x)$ has the same probability of $\frac{1}{4}$ to occur. Under the IAC assumption, on the other hand, we do not distinguish $(x \succ y, y \succ x)$ and $(y \succ x, x \succ y)$, and thus each of $(x \succ y, x \succ y)$, $(x \succ y, y \succ x)$ and $(y \succ x, y \succ x)$ has the same probability of $\frac{1}{3}$ to occur.

We can use assumptions such as the IC or the IAC assumption to generate a large number of profiles. For a given voting rule, we can then check for each voter whether she would have an incentive to manipulate, if we assume that her true preferences are as indicated by the profile and all other voters' ballots are as indicated by the profile. This approach allows us to compare the *degree of manipulability* of different voting rules. (Although much more difficult, in principle it is also possible to derive these degrees of manipulability using analytical methods, rather than to make use of simulations.) While interesting, this kind of approach has been criticised for being based on arguably unrealistic assumptions: the distribution of preferences in a real electorate will have little in common with either the IC or the IAC assumption.

The purpose of this question is to explore the frequency-based approach further:

(a) Find out about one further approach to generating test data for election simulations proposed in the literature. Explain the approach and briefly discuss its advantages and disadvantages. Write at most one page of text.

(b) Suggest a new approach to automatically generating profiles. Argue why (and under what circumstances) your approach will produce realistic or otherwise interesting data. Also discuss the limitations of your approach. Write at most one page of text.