Homework #2

Deadline: Wednesday, 5 October 2011, 11:00

Question 1 (10 marks)

This question concerns two alternative definitions of the property of *strong monotonicity* of a *resolute* voting rule $F : \mathcal{L}(\mathcal{X})^{\mathcal{N}} \to \mathcal{X}$. Recall the definition given in class:

(a) F is called strongly monotonic if $x^\star = F(\boldsymbol{R})$ implies $x^\star = F(\boldsymbol{R}')$ for any alternative $x^\star$ and any two profiles $\boldsymbol{R}$ and $\boldsymbol{R}'$ with $N^{\boldsymbol{R}}_{x^\star \succ y} \subseteq N^{\boldsymbol{R}'}_{x^\star \succ y}$ for all alternatives $y \in \mathcal{X} \setminus \{x^\star\}$.

An alternative definition to be found in the literature is the following:

(b) F is called strongly monotonic if $F(\boldsymbol{R'}) = F(\boldsymbol{R})$ or $F(\boldsymbol{R'}) = x^\star$ for any alternative $x^\star$ and any two profiles $\boldsymbol{R}$ and $\boldsymbol{R'}$ satisfying $N^{\boldsymbol{R}}_{x^\star \succ y} \subseteq N^{\boldsymbol{R'}}_{x^\star \succ y}$ and $N^{\boldsymbol{R}}_{y \succ z} = N^{\boldsymbol{R'}}_{y \succ z}$ for all alternatives $y, z \in \mathcal{X} \setminus \{x^\star\}$.

Explain each definition in plain English and briefly argue why it is a reasonable definition. Then check whether the two definitions are equivalent (proof or counterexample).

Notation: Recall that $N^{\boldsymbol{R}}_{x \succ y}$ is the set of individuals who rank alternative x above alternative y under profile $\boldsymbol{R}$.

Question 2 (10 marks)

Prove May's Theorem for the case of an even number of voters.

Question 3 (10 marks)

Under the *antiplurality rule*, also known as the *veto rule*, the voters rank the alternatives, and the alternative(s) ranked last the least often win(s). The purpose of this question is to find a number of different characterisations of this rule.

(a) Find a consensus criterion such that the antiplurality rule is characterised by that criterion and the *discrete distance*.

(b) Find a way of measuring distances such that the antiplurality rule is characterised by the *unanimous winner* consensus criterion and that distance.

(c) Find a noise model such that the corresponding maximum likelihood estimator is equivalent to the antiplurality rule.