Homework #1

Deadline: Wednesday, 21 September 2011, 11:00

Question 1 (10 marks)

A social welfare function F is *nonimposed* (or *surjective*) if for every preference relation R there exists a profile $(R_1, \ldots, R_n)$ such that $F(R_1, \ldots, R_n) = R$. That is, if F is nonimposed, then for every possible social preference order R there exists a profile of individual preferences under which R will be chosen: R is not excluded as the social preference order *a priori*. The purpose of this exercise is to investigate what happens to Arrow's Theorem when we replace the Pareto condition by the axiom of nonimposition.

(a) Show that the Pareto condition is strictly stronger than nonimposition. That is, show that every Pareto efficient social welfare function is nonimposed and that there exists a nonimposed social welfare function that is not Pareto efficient.

(b) Show that Arrow's Theorem ceases to hold when we replace the Pareto condition by nonimposition. That is, show that there exists a social welfare function that satisfies IIA and that is both nonimposed and nondictatorial.

Question 2 (10 marks)

In the proof of Arrow's Theorem given in *Logic and Social Choice Theory* some details are "left as an exercise to the reader". State clearly what claims remain to be proven to complete the proof of the theorem and provide your own proofs for these claims.