Homework #5

$\boxed{\textbf{Deadline: Tuesday, 30 November 2010, 11:00}}$

Question 1 (10 marks)

The purpose of this exercise is to analyse the computational complexity of the special case of the *possible winner problem* in which some voters have supplied ballots that rank all alternatives, while others (the "missing voters") have not yet provided any information at all. Recall that this is equivalent to the *constructive coalitional manipulation problem*. We are interested in the case where all voters have the same weight.

(a) Analyse the complexity of the possible winner problem for missing voters for the *Borda rule* for the special case of *three alternatives*. What about the general possible winner problem for three alternatives under the Borda rule?

   Remarks: Recall that for an arbitrary number of alternatives the case restricted to missing voters is an open problem that is widely considered to be difficult (while the general case is known to be NP-complete). Also recall that for three alternatives both the restricted and the general problem are NP-complete when votes are weighted.

(b) The voting procedure known as 2-*approval* is the positional scoring rule with the scoring vector $\langle 1, 1, 0, \ldots, 0 \rangle$. Analyse the complexity of the possible winner problem with missing voters for 2-approval.

Question 2 (10 marks)

Recall the concepts of *compilation complexity* and *communication complexity* of a voting procedure introduced in class. Briefly compare these two approaches to measuring the information requirements of a voting procedure and explain how they relate to each other. (Write at most two pages of text.)

Question 3 (10 marks)

Recall the definition of the (raw) Banzhaf index for weighted voting games as given in class.

(a) Give an example for a game where the Banzhaf indices of the voters add up to 1.

(b) Is it possible for the sum of the Banzhaf indices of the individual voters to be less than 1? Either give an example or prove that this is impossible.

(c) Is it possible for the sum of the Banzhaf indices of the individual voters to be greater than 1? Either give an example or prove that this is impossible.