Homework #4

Deadline: Tuesday, 16 November 2010, 11:00

Question 1 (10 marks)

A *weak Condorcet winner* is a candidate that will win or draw against any other candidate in a pairwise majority contest. Show that a weak Condorcet winner always exists when voters express their preferences using the *language of single goals* introduced in the lecture on voting in combinatorial domains.

Question 2 (10 marks)

CP-nets are a natural and compact language for modelling preferences in combinatorial domains. However, they are not equally good at modelling all kinds of types of such preferences. Consider a *binary* combinatorial domain, i.e., a domain $\mathcal{D} = D_1 \times \cdots \times D_p$, in which each D_i is equal to the set $\{0, 1\}$. We call a partial order $\succeq$ defined on $\mathcal{D}$ *monotonic* if $(x_1, \ldots, x_p) \succ (y_1, \ldots, y_p)$ whenever $x_i \geqslant y_i$ for all $i \in \{1, \ldots, p\}$ and $x_i > y_i$ for some $i \in \{1, \ldots, p\}$. That is, a voter with monotonic preferences will always prefer exchanging a 0 for a 1. Characterise the class of monotonic partial orders on binary combinatorial domains that can be modelled using CP-nets. Note that we have not given a full definition of CP-nets in class; you will need to look this up in the literature (see Boutilier et al., 2004).

Hint: For example, the (monotonic) preference order $11 \succ {10 \atop 01} \succ 00$ (where 11 is better than both 10 and 01, which are incomparable to each other but are both better than 00) is expressible, while the (equally monotonic) preference order $11 \succ 10 \succ 01 \succ 00$ is not.

Reference: C. Boutilier, R.I. Brafman, C. Domshlak, H.H. Hoos, and D. Poole: CP-nets: A Tool for Representing and Reasoning with Conditional Ceteris Paribus Preference Statements. *Journal of Artificial Intelligence Research*, 21:135–191, 2004.