Homework #3

Deadline: Tuesday, 2 November 2010, 11:00

Question 1 (10 marks)

The purpose of this exercise is to explore the boundaries of some of the impossibility theorems we have discussed.

(a) Show that Arrow's Theorem ceases to hold when we replace the weak Pareto condition by nonimposition.

(b) Show that the Muller-Satterthwaite Theorem ceases to hold when we replace strong monotonicity by weak monotonicity.

(c) Show that the Gibbard-Satterthwaite Theorem ceases to hold when we drop the condition of surjectivity.

(d) Show that the Duggan-Schwartz Theorem ceases to hold when we replace the condition of immunity against manipulation by both optimistic and pessimistic voters by immunity against manipulation by optimistic voters only.

(e) Show that the Duggan-Schwartz Theorem ceases to hold when we replace the condition of immunity against manipulation by both optimistic and pessimistic voters by immunity against manipulation by pessimistic voters only.

Question 2 (10 marks)

A voting procedure is called *single-winner manipulable* if it admits a situation where truthful voting would produce a single winner (no ties) and one of the voters could force a different and preferred single winner by not voting truthfully. Show that the Borda rule is not single-winner manipulable in the case of three candidates.

(Adapted from A.D. Taylor, *Social Choice and the Mathem. of Manipulation*, CUP, 2005.)

Question 3 (10 marks)

Recall the Copeland rule: each voter ranks all alternatives, and the alternative(s) that maximise the difference between won and lost majority contests, when compared to all other alternatives, win(s). Prove that the Copeland rule is easy to manipulate. This is in fact a corollary to a more general result by Bartholdi, Tovey and Trick (1989). Do not build on their general result, but rather give a direct proof for the Copeland rule only.

(See J.J. Bartholdi III, C.A. Tovey, and M.A. Trick. The Computational Difficulty of Manipulating an Election. *Social Choice and Welfare*, 6(3):227–241, 1989.)

(Please turn over)

Bonus Question (20 marks)

An important line of research in social choice theory is aimed at understanding the *frequency* with which certain undesirable situations, e.g., Condorcet cycles or opportunities for strategic manipulation, occur. While classical paradoxes and impossibility theorems show that these situations can never be ruled out entirely, it is conceivable that they might be very infrequent, in which case the situation would not actually be as bleak as the classical results suggest. To measure frequency we have to make assumptions regarding the likelihood of certain profiles of preferences (or ballots) to occur. The standard approach is to assume that every logically possible profile is equally likely to occur. This is known as the *impartial culture* (IC) assumption. Under the closely related *impartial anonymous culture* (IAC) assumption, each *anonymous* profile is taken to be equally likely to occur. For example, if there are two alternatives and two voters, then under the IC assumption each of the four possible profiles $(x \succ y, x \succ y)$, $(x \succ y, y \succ x)$, $(y \succ x, x \succ y)$ and $(y \succ x, y \succ x)$ has the same probability of $\frac{1}{4}$ to occur. Under the IAC assumption, on the other hand, we do not distinguish $(x \succ y, y \succ x)$ and $(y \succ x, x \succ y)$, and thus each of $(x \succ y, x \succ y)$, $(x \succ y, y \succ x)$ and $(y \succ x, y \succ x)$ has the same probability of $\frac{1}{3}$ to occur.

We can use assumptions such as the IC or the IAC assumption to generate a large number of profiles. For a given voting procedure, we can then check for each voter whether she would have an incentive to manipulate, if we assume that her true preferences are as indicated by the profile and all other voters' ballots are as indicated by the profile. This approach allows us to compare the *degree of manipulability* of different voting procedures. (Although much more difficult, in principle it is also possible to derive these degrees of manipulability using analytical methods, rather than to make use of simulations.) While interesting, this kind of approach has been criticised for being based on arguably unrealistic assumptions: the distribution of preferences in a real electorate will have little in common with either the IC or the IAC assumption.

Come up with a new probability distribution over profiles that can be used to automatically generate sample electorates. Argue why (and under what circumstances) your approach will produce realistic data. Then implement a framework for running simulations to approximate the degree of manipulability under your probability distribution. Run extensive tests for three voting procedures (and a suitable tie-breaking rule) of your choice and document your findings in a short report. Make sure that your experiments are reproducible by others. Any code you submit should run on the standard Linux environment provided by the FNWI.

Note: I will accept solutions for this question until 1 December 2010.