Coursework #4

Deadline: Wednesday, 29 April 2009, 15:00

Question 1 (10 marks)

Describe a discrete minimal-cut procedure for dividing a cake between 4 players that guarantees that each player believes they received at least $\frac{1}{6}$ of the cake. (Moving knifes are not allowed and "marks" count as cuts.)

(Adapted from J. Robertson and W. Webb, *Cake-Cutting Algorithms*, A.K. Peters, 1998.)

Question 2 (10 marks)

What is the computational complexity of (the decision variant of) the problem of finding an allocation of indivisible goods to agents that maximises elitist social welfare?

(a) First state your answer (and your proof) with respect to the explicit form of representing valuation functions (where the size of the representation of a function is taken to be proportional to the number of bundles to which it assigns a non-zero value).

(b) Then repeat the same exercise, this time assuming that valuation functions are expressed using the language of weighted propositional formulas (without restrictions).

Question 3 (10 marks)

Recall the distributed resource allocation framework discussed in class, where agents negotiate a sequence of individually rational deals. Without restrictions on the structure of deals, such sequences are known to always converge to an optimal allocation (here, an allocation that maximises utilitarian social welfare), but with structural restrictions this may not be so. The purpose of this exercise is to investigate what happens when all kinds of (individually rational) deals are allowed, except those that involve the complete set of agents within a single deal. From a result cited in class we know that convergence will not hold anymore in this case. The question is whether convergence can be maintained if we restrict the range of possible valuation functions. Check what happens if all agents have valuation functions that are (a) supermodular, (b) submodular, or (c) both super- and submodular.